

# **Mark Scheme 4724 January 2007**

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1	Factorise numerator and denominator Num = $(x + 6)(x - 4)$ or denom = $x(x - 4)$  Final answer = $\frac{x + 6}{x}$ or $1 + \frac{6}{x}$	M1 A1 A1 3	or Attempt long division Result = $1 + \frac{6x - 24}{x^2 - 4x}$ $= 1 + \frac{6}{x}$
2	Use parts with $u = \ln x, dv = x$  Obtain $\frac{1}{2}x^2 \ln x - \int \frac{1}{x} \cdot \frac{1}{2}x^2 (dx)$ $= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 (+c)$ Use limits correctly Exact answer $2 \ln 2 - \frac{3}{4}$	M1 A1 A1 M1 A1 5	& give 1 <sup>st</sup> stage in form $f(x) + / - \int g(x)(dx)$ or $\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x(dx)$  AEF ISW
3	(i) Find $a - b$ or $b - a$ irrespective of label Method for magnitude of any vector $\sqrt{161}$ or 12.7(12.688578) (ii) Using $(\overline{AO}$ or $\overline{OA})$ and $(\overline{AB}$ or $\overline{BA})$ $\cos \theta = \frac{\text{scalar product of any two vectors}}{\text{product of their moduli}}$ 43 or better (42.967...), 0.75 or better (0.7499218...)	M1 M1 A1 3 B1 M1 A1 3	(expect $11i - 2j - 6k$ or $-11i + 2j + 6k$ )  Do not class angle $AOB$ as MR  If 137 obtained, followed by 43, award A0 Common answer 114 probably $\rightarrow$ B0 M1 A0
4	Attempt to connect $dx$ and $du$ For $du = 2 dx$ AEF correctly used $\int u^8 + u^7 (du)$ Attempt new limits for $u$ at any stage (expect 0,1) $\frac{17}{72}$ S.R. If M1 A0 A0 M1 A0, award S.R. B1 for answer $\frac{68}{72} \cdot \frac{34}{36}$ or $\frac{17}{18}$	M1 A1 A1 M1 A1 5	but not just $dx = du$ sight of $\frac{1}{2} (du)$ necessary or $\int u^7 (u + 1)(du)$ or re-substitute & use $(\frac{5}{2}, 3)$ AG WWW ISW
5	(i) Show clear knowledge of binomial expansion  $= 1 + x$ $+ 2x^2$ $+ \frac{14}{3}x^3$ (ii) Attempt to substitute $x + x^3$ for $x$ in (i) Clear indication that $(x + x^3)^2$ has no term in $x^3$ $\frac{17}{3}$	M1 B1 A1 A1 4 M1 A1 $\sqrt{A1}$ 3	$-3x$ should appear but brackets can be missing; $-\frac{1}{3} \cdot -\frac{4}{3}$ should appear, not $-\frac{1}{3} \cdot \frac{2}{3}$ Correct first 2 terms; not dep on M1  Not just in the $\frac{14}{3}x^3$ term  f.t. $cf(x) + cf(x^3)$ in part (i)
6	(i) $2x + 1 = / \equiv A(x - 3) + B$ $A = 2$ $B = 7$ (ii) $\int \frac{1}{x-3} (dx) = \ln(x - 3)$ or $\ln x - 3 $ $\int \frac{1}{(x-3)^2} (dx) = -\frac{1}{x-3}$  $6 + 2 \ln 7$ Follow-through $\frac{6}{7}B + A \ln 7$	M1 A1 A/B 1 3 B1 B1 $\sqrt{B2}$ 4	Cover-up rule acceptable for B1 Accept A or $\frac{1}{A}$ as a multiplier  Accept B or $\frac{1}{B}$ as a multiplier

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7	$\frac{d}{dx}(xy) = x \frac{dy}{dx} + y$ $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$ $4x + x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0$ <p>Put <math>\frac{dy}{dx} = 0</math>                      Obtain <math>4x + y = 0</math> AEF                      Attempt to solve simultaneously with eqn of curve</p> <p>Obtain <math>x^2 = 1</math> or <math>y^2 = 16</math> from <math>4x + y = 0</math>  <math>(1, -4)</math> and <math>(-1, 4)</math> and no other solutions</p>	B1 B1 B1 *M1 A1 dep*M1 A1 A1	and no other (different) result 8 Accept $(\pm 1, \mp 4)$ but not $(\pm 1, \pm 4)$
8	<p>(i) Use <math>\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx}</math> and <math>-\frac{1}{m}</math> for grad of normal  <math>= -p</math> AG WWW</p> <p>(ii) Use correct formula to find gradient of line                      Obtain <math>\frac{2}{p+q}</math> AG WWW</p> <p>(iii) State <math>-p = \frac{2}{p+q}</math>                      Simplify to <math>p^2 + pq + 2 = 0</math> AG WWW</p> <p>(iv) <math>(8, 8) \rightarrow t</math> or <math>p</math> or <math>q = 2</math> only                      Subst <math>p = 2</math> in eqn (iii) to find <math>q_1</math>                      Subst <math>p = q_1</math> in eqn (iii) to find <math>q_2</math>  <math>q_2 = \frac{11}{3} \rightarrow (\frac{242}{9}, \frac{44}{3})</math></p>	M1 A1 M1 A1 M1 A1 B1 M1 M1 A1	or change to cartesian., diff & use $-\frac{1}{m}$ 2 Not $-t$ . 2 Minimum of denom = $2(p-q)(p+q)$ Or find eqn normal at P & subst $(2q^2, 4q)$ 2 With sufficient evidence No possibility of $-2$ Or eqn normal, solve simult with cartes/param Ditto 4 No follow-through; accept $(26.9, 14.7)$
9	<p>(i) Separate variables as <math>\int \sec^2 y \, dy = 2 \int \cos^2 2x \, dx</math>                      LHS = <math>\tan y</math>                      RHS; attempt to change to double angle                      Correctly shown as <math>1 + \cos 4x</math>  <math>\int \cos 4x \, dx = \frac{1}{4} \sin 4x</math>                      Completely correct equation (other than +c)                      +c on either side</p> <p>(ii) Use boundary condition  <math>c</math> (on RHS) = 1                      Substitute <math>x = \frac{1}{6}\pi</math> into their eqn, produce <math>y = 1.05</math></p>	M1 A1 M1 A1 A1 A1 A1 M1 A1 A1	seen or implied   $\tan y = x + \frac{1}{4} \sin 4x$ 7 <u>not</u> on both sides unless $c_1$ and $c_2$ provided a sensible outcome would ensue or $c_2 - c_1 = 1$ ; not fortuitously obtained or 4.19 or 7.33 etc. Radians only 3
10	<p>(i) For (either point) + <math>t</math> (diff between posn vectors)  <math>\mathbf{r} = (\text{either point}) + t(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k} \text{ or } -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})</math></p> <p>(ii) <math>\mathbf{r} = s(\mathbf{i} + 2\mathbf{j} - \mathbf{k})</math> or <math>(\mathbf{i} + 2\mathbf{j} - \mathbf{k}) + s(\mathbf{i} + 2\mathbf{j} - \mathbf{k})</math>                      Eval scalar product of <math>\mathbf{i} + 2\mathbf{j} - \mathbf{k}</math> &amp; their dir vect in (i)                      Show as <math>(1 \times 1 \text{ or } 1) + (2 \times -2 \text{ or } -4) + (-1 \times -3 \text{ or } 3)</math>  <math>= 0</math> and state perpendicular AG</p> <p>(iii) For at least two equations with diff parameters                      Obtain <math>t = -2</math> or <math>s = 3</math> (possibly -3 or 2 or -2)                      Subst. into eqn AB or OT and produce <math>3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}</math></p> <p>(iv) Indicate that <math> \overline{OC} </math> is to be found  <math>\sqrt{54}</math>; f.t. <math>\sqrt{a^2 + b^2 + c^2}</math> from <math>a\mathbf{i} + b\mathbf{j} + c\mathbf{k}</math> in (iii)</p>	M1 A1 B1 M1 A1 A1 M1 A1 A1 M1 A1 A1 M1	"r =" not necessary for the M mark 2 ... but it is essential for the A mark Accept any parameter, including $t$  This is just one example of numbers involved 4 e.g. $5 + t = s, 2 - 2t = 2s, -9 - 3t = -s$ Check if $t = 2, 1$ or $-1$ 3 where C is their point of intersection 2

In the above question, accept any vectorial notation

$t$  and  $s$  may be interchanged, and values stated above need to be treated with caution.

In (iii), if the point of intersection is correct, it is more than likely that the whole part is correct – but check.

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