

General Certificate of Education
January 2008
Advanced Subsidiary Examination



MATHEMATICS
Unit Pure Core 1

MPC1

Wednesday 9 January 2008 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.



Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The triangle ABC has vertices $A(-2, 3)$, $B(4, 1)$ and $C(2, -5)$.

(a) Find the coordinates of the mid-point of BC . (2 marks)

(b) (i) Find the gradient of AB , in its simplest form. (2 marks)

(ii) Hence find an equation of the line AB , giving your answer in the form $x + qy = r$, where q and r are integers. (2 marks)

(iii) Find an equation of the line passing through C which is parallel to AB . (2 marks)

(c) Prove that angle ABC is a right angle. (3 marks)

2 The curve with equation $y = x^4 - 32x + 5$ has a single stationary point, M .

(a) Find $\frac{dy}{dx}$. (3 marks)

(b) Hence find the x -coordinate of M . (3 marks)

(c) (i) Find $\frac{d^2y}{dx^2}$. (1 mark)

(ii) Hence, or otherwise, determine whether M is a maximum or a minimum point. (2 marks)

(d) Determine whether the curve is increasing or decreasing at the point on the curve where $x = 0$. (2 marks)

3 (a) Express $5\sqrt{8} + \frac{6}{\sqrt{2}}$ in the form $n\sqrt{2}$, where n is an integer. (3 marks)

(b) Express $\frac{\sqrt{2} + 2}{3\sqrt{2} - 4}$ in the form $c\sqrt{2} + d$, where c and d are integers. (4 marks)

4 A circle with centre C has equation $x^2 + y^2 - 10y + 20 = 0$.

(a) By completing the square, express this equation in the form

$$x^2 + (y - b)^2 = k \quad (2 \text{ marks})$$

(b) Write down:

(i) the coordinates of C ; (1 mark)

(ii) the radius of the circle, leaving your answer in surd form. (1 mark)

(c) A line has equation $y = 2x$.

(i) Show that the x -coordinate of any point of intersection of the line and the circle satisfies the equation $x^2 - 4x + 4 = 0$. (2 marks)

(ii) Hence show that the line is a tangent to the circle and find the coordinates of the point of contact, P . (3 marks)

(d) Prove that the point $Q(-1, 4)$ lies inside the circle. (2 marks)

5 (a) Factorise $9 - 8x - x^2$. (2 marks)

(b) Show that $25 - (x + 4)^2$ can be written as $9 - 8x - x^2$. (1 mark)

(c) A curve has equation $y = 9 - 8x - x^2$.

(i) Write down the equation of its line of symmetry. (1 mark)

(ii) Find the coordinates of its vertex. (2 marks)

(iii) Sketch the curve, indicating the values of the intercepts on the x -axis and the y -axis. (3 marks)

Turn over for the next question

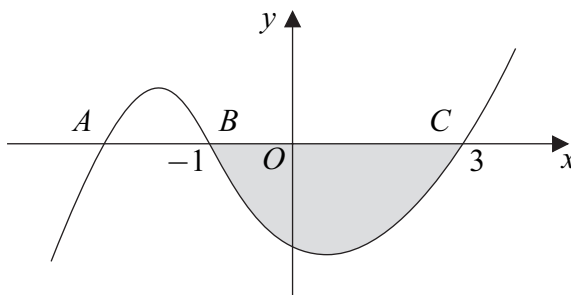
Turn over ►

6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 - 7x - 6$.

(i) Use the Factor Theorem to show that $x + 1$ is a factor of $p(x)$. (2 marks)

(ii) Express $p(x) = x^3 - 7x - 6$ as the product of three linear factors. (3 marks)

(b) The curve with equation $y = x^3 - 7x - 6$ is sketched below.



The curve cuts the x -axis at the point A and the points $B(-1, 0)$ and $C(3, 0)$.

(i) State the coordinates of the point A . (1 mark)

(ii) Find $\int_{-1}^3 (x^3 - 7x - 6) dx$. (5 marks)

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - 7x - 6$ and the x -axis between B and C . (1 mark)

(iv) Find the gradient of the curve $y = x^3 - 7x - 6$ at the point B . (3 marks)

(v) Hence find an equation of the normal to the curve at the point B . (3 marks)

7 The curve C has equation $y = x^2 + 7$. The line L has equation $y = k(3x + 1)$, where k is a constant.

(a) Show that the x -coordinates of any points of intersection of the line L with the curve C satisfy the equation

$$x^2 - 3kx + 7 - k = 0 \quad (1 \text{ mark})$$

(b) The curve C and the line L intersect in two distinct points. Show that

$$9k^2 + 4k - 28 > 0 \quad (3 \text{ marks})$$

(c) Solve the inequality $9k^2 + 4k - 28 > 0$. (4 marks)

END OF QUESTIONS