General Certificate of Education
June 2007
Advanced Subsidiary Examination

MATHEMATICS **MPC2**
Unit Pure Core 2

Monday 21 May 2007 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Simplify:

(i) $x^{\frac{3}{2}} \times x^{\frac{1}{2}}$; *(1 mark)*

(ii) $x^{\frac{3}{2}} \div x$; *(1 mark)*

(iii) $\left(x^{\frac{3}{2}}\right)^2$. *(1 mark)*

(b) (i) Find $\int 3x^{\frac{1}{2}}\,dx$. *(3 marks)*

(ii) Hence find the value of $\int_1^9 3x^{\frac{1}{2}}\,dx$. *(2 marks)*

2 The nth term of a geometric sequence is u_n, where

$$u_n = 3 \times 4^n$$

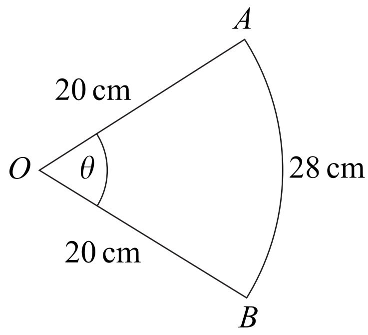
(a) Find the value of u_1 and show that $u_2 = 48$. *(2 marks)*

(b) Write down the common ratio of the geometric sequence. *(1 mark)*

(c) (i) Show that the sum of the first 12 terms of the geometric sequence is $4^k - 4$, where k is an integer. *(3 marks)*

(ii) Hence find the value of $\sum_{n=2}^{12} u_n$. *(1 mark)*

3 The diagram shows a sector OAB of a circle with centre O and radius 20 cm. The angle between the radii OA and OB is θ radians.

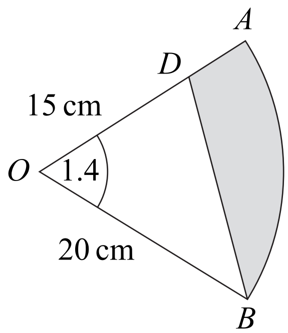


The length of the arc AB is 28 cm.

(a) Show that $\theta = 1.4$. *(2 marks)*

(b) Find the area of the sector OAB. *(2 marks)*

(c) The point D lies on OA. The region bounded by the line BD, the line DA and the arc AB is shaded.



The length of OD is 15 cm.

(i) Find the area of the shaded region, giving your answer to three significant figures. *(3 marks)*

(ii) Use the cosine rule to calculate the length of BD, giving your answer to three significant figures. *(3 marks)*

4 An arithmetic series has first term a and common difference d.

The sum of the first 29 terms is 1102.

(a) Show that $a + 14d = 38$. *(3 marks)*

(b) The sum of the second term and the seventh term is 13.

Find the value of a and the value of d. *(4 marks)*

5 A curve is defined for $x > 0$ by the equation

$$y = \left(1 + \frac{2}{x}\right)^2$$

The point P lies on the curve where $x = 2$.

(a) Find the y-coordinate of P. *(1 mark)*

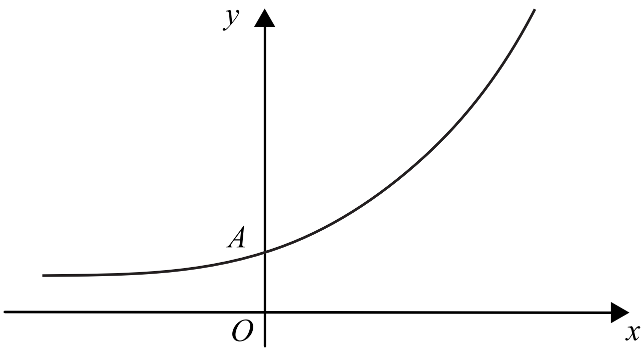
(b) Expand $\left(1 + \frac{2}{x}\right)^2$. *(2 marks)*

(c) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(d) Hence show that the gradient of the curve at P is -2. *(2 marks)*

(e) Find the equation of the normal to the curve at P, giving your answer in the form $x + by + c = 0$, where b and c are integers. *(4 marks)*

6 The diagram shows a sketch of the curve with equation $y = 3(2^x + 1)$.



The curve $y = 3(2^x + 1)$ intersects the y-axis at the point A.

(a) Find the y-coordinate of the point A. *(2 marks)*

(b) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^6 3(2^x + 1)\,dx$. *(4 marks)*

(c) The line $y = 21$ intersects the curve $y = 3(2^x + 1)$ at the point P.

(i) Show that the x-coordinate of P satisfies the equation

$$2^x = 6$$ *(1 mark)*

(ii) Use logarithms to find the x-coordinate of P, giving your answer to three significant figures. *(3 marks)*

Turn over for the next question

7 (a) Sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

(b) Write down the **two** solutions of the equation $\tan x = \tan 61°$ in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(c) (i) Given that $\sin\theta + \cos\theta = 0$, show that $\tan\theta = -1$. *(1 mark)*

(ii) Hence solve the equation $\sin(x - 20°) + \cos(x - 20°) = 0$ in the interval $0° \leqslant x \leqslant 360°$. *(4 marks)*

(d) Describe the single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x - 20°)$. *(2 marks)*

(e) The curve $y = \tan x$ is stretched in the x-direction with scale factor $\frac{1}{4}$ to give the curve with equation $y = \mathrm{f}(x)$. Write down an expression for $\mathrm{f}(x)$. *(1 mark)*

8 (a) It is given that n satisfies the equation

$$\log_a n = \log_a 3 + \log_a(2n - 1)$$

Find the value of n. *(3 marks)*

(b) Given that $\log_a x = 3$ and $\log_a y - 3\log_a 2 = 4$:

(i) express x in terms of a; *(1 mark)*

(ii) express xy in terms of a. *(4 marks)*

END OF QUESTIONS

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