General Certificate of Education June 2008 Advanced Subsidiary Examination

# ASSESSMENT and QUALIFICATIONS ALLIANCE

## MATHEMATICS Unit Pure Core 2

MPC2

Thursday 15 May 2008 9.00 am to 10.30 am

#### For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

#### **Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer all questions.
- Show all necessary working; otherwise marks for method may be lost.

#### **Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

#### **Advice**

• Unless stated otherwise, you may quote formulae, without proof, from the booklet.

P5324/Jun08/MPC2 6/6/ MPC2

群尧咨询

#### Answer all questions.

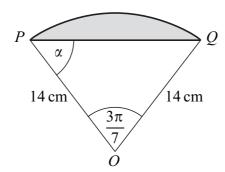
1 (a) Write  $\sqrt{x^3}$  in the form  $x^k$ , where k is a fraction. (1 mark)

(b) A curve, defined for  $x \ge 0$ , has equation

$$y = x^2 - \sqrt{x^3}$$

(i) Find  $\frac{dy}{dx}$ . (3 marks)

- (ii) Find the equation of the tangent to the curve at the point where x = 4, giving your answer in the form y = mx + c. (5 marks)
- 2 The diagram shows a shaded segment of a circle with centre O and radius 14 cm, where PQ is a chord of the circle.



In triangle OPQ, angle  $POQ = \frac{3\pi}{7}$  radians and angle  $OPQ = \alpha$  radians.

- (a) Find the length of the arc PQ, giving your answer as a multiple of  $\pi$ . (2 marks)
- (b) Find  $\alpha$  in terms of  $\pi$ . (2 marks)
- (c) Find the **perimeter** of the shaded segment, giving your answer to three significant figures. (2 marks)

#### 3 A geometric series begins

$$20 + 16 + 12.8 + 10.24 + \dots$$

(a) Find the common ratio of the series.

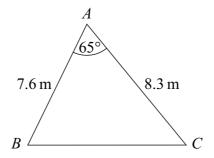
(1 mark)

(b) Find the sum to infinity of the series.

- (2 marks)
- (c) Find the sum of the first 20 terms of the series, giving your answer to three decimal places. (2 marks)
- (d) Prove that the *n*th term of the series is  $25 \times 0.8^n$ .

(2 marks)

#### 4 The diagram shows a triangle ABC.



The size of angle BAC is 65°, and the lengths of AB and AC are 7.6 m and 8.3 m respectively.

- (a) Show that the length of BC is 8.56 m, correct to three significant figures. (3 marks)
- (b) Calculate the area of triangle ABC, giving your answer in  $m^2$  to three significant figures. (2 marks)
- (c) The perpendicular from A to BC meets BC at the point D.

Calculate the length of AD, giving your answer to the nearest 0.1 m.

(3 marks)

#### 5 (a) Write down the value of:

(i)  $\log_a 1$ ;

(1 mark)

(ii)  $\log_a a$ .

(1 mark)

(b) Given that

$$\log_a x = \log_a 5 + \log_a 6 - \log_a 1.5$$

find the value of x.

(3 marks)

6 The *n*th term of a sequence is  $u_n$ .

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = -8$$
  $u_2 = 8$   $u_3 = 4$ 

- (a) Show that q = 6 and find the value of p. (5 marks)
- (b) Find the value of  $u_4$ . (1 mark)
- (c) The limit of  $u_n$  as n tends to infinity is L.
  - (i) Write down an equation for L. (1 mark)
  - (ii) Hence find the value of L. (2 marks)
- 7 (a) The expression  $\left(1 + \frac{4}{x^2}\right)^3$  can be written in the form

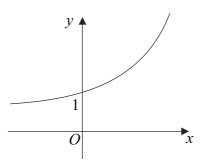
$$1 + \frac{p}{x^2} + \frac{q}{x^4} + \frac{64}{x^6}$$

By using the binomial expansion, or otherwise, find the values of the integers p and q.

(3 marks)

- (b) (i) Hence find  $\int \left(1 + \frac{4}{x^2}\right)^3 dx$ . (4 marks)
  - (ii) Hence find the value of  $\int_{1}^{2} \left(1 + \frac{4}{x^2}\right)^3 dx$ . (2 marks)

8 The diagram shows a sketch of the curve with equation  $y = 6^x$ .



- (a) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for  $\int_0^2 6^x dx$ , giving your answer to three significant figures. (4 marks)
  - (ii) Explain, with the aid of a diagram, whether your approximate value will be an overestimate or an underestimate of the true value of  $\int_0^2 6^x dx$ . (2 marks)
- (b) (i) Describe a single geometrical transformation that maps the graph of  $y = 6^x$  onto the graph of  $y = 6^{3x}$ . (2 marks)
  - (ii) The line y = 84 intersects the curve  $y = 6^{3x}$  at the point A. By using logarithms, find the x-coordinate of A, giving your answer to three decimal places.

    (4 marks)
- (c) The graph of  $y = 6^x$  is translated by  $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$  to give the graph of the curve with equation y = f(x). Write down an expression for f(x).
- 9 (a) Solve the equation  $\sin 2x = \sin 48^\circ$ , giving the values of x in the interval  $0^\circ \le x < 360^\circ$ . (4 marks)
  - (b) Solve the equation  $2 \sin \theta 3 \cos \theta = 0$  in the interval  $0^{\circ} \le \theta < 360^{\circ}$ , giving your answers to the nearest  $0.1^{\circ}$ .

### END OF QUESTIONS

There are no questions printed on this page

There are no questions printed on this page

There are no questions printed on this page