

# 4724 Core Mathematics 4

<p>1 Method for finding magnitude of any vector Method for finding scalar prod of any 2 vectors Using <math>\cos \theta = \frac{\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} \cdot 2\mathbf{i} + \mathbf{j} + \mathbf{k}}{ \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}   2\mathbf{i} + \mathbf{j} + \mathbf{k} }</math> 70.9 (70.89, 70.893) WWW; 1.24 (1.237)</p>	<p>M1 M1 M1 A1</p>	<p>Expect <math>\sqrt{14}</math> and <math>\sqrt{6}</math> Expect <math>1.2 + (-2)1 + 3.1 = 3</math> Correct vectors only. Expect <math>\cos \theta = \frac{3}{\sqrt{14}\sqrt{6}}</math> 4 Condone answer to nearest degree (71)</p>
<p>2 (i) Correct format <math>\frac{A}{x+1} + \frac{B}{x+2}</math> <math>-\frac{1}{x+1}</math> or <math>A = -1</math> <math>+\frac{2}{x+2}</math> or <math>B = 2</math></p> <hr style="border-top: 1px dashed black;"/> <p>(ii) <math>\int \frac{1}{x+1} dx = \ln(x+1)</math> or <math>\ln x+1 </math> or <math>\int \frac{1}{x+2} dx = \ln(x+2)</math> or <math>\ln x+2 </math> <math>A \ln x+1  + B \ln x+2  + c</math> ISW</p>	<p>M1 A1 A1  B1 <math>\sqrt{A1}</math></p>	<p>stated or implied by answer    3   2 Expect <math>-\ln x+1  + 2 \ln x+2  + c</math></p>
<p>3 <u>Method 1 (Long division)</u> Clear correct division method at beginning  Correct method up to &amp; including <math>x</math> term in quot <u>Method 2 (Identity)</u> Writing <math>(x^2 + 2x - 1)(x^2 + bx + 2) + cx + 7</math> Attempt to compare cfs of <math>x^3</math> or <math>x^2</math> or <math>x</math> or const  Then: <math>b = -4</math> <math>c = -1</math> <math>a = 5</math></p>	<p>M1 M1 M1 M1  A1 A1 A1</p>	<p><math>x^2</math> in quot, mult back &amp; attempt subtraction [At subtraction stage, cf <math>(x^4) = 0</math>] [At subtraction stage, cf <math>(x^3) = 0</math>]  Probably equated to <math>x^4 - 2x^3 - 7x^2 + 7x + a</math>   5</p>
<p>4 <math>\frac{d}{dx}(x^2 y) = x^2 \frac{dy}{dx} + 2xy</math> <math>\frac{d}{dx}(y^3) = 3y^2 \frac{dy}{dx}</math> Substitute <math>(x,y) = (1,1)</math> and solve for <math>\frac{dy}{dx}</math>  <math>\frac{dy}{dx} = -\frac{11}{7}</math> WWW Gradient normal = <math>-\frac{1}{\frac{dy}{dx}}</math> <math>7x - 11y + 4 = 0</math> AEF</p>	<p>B1 B1 M1 M1 A1 M1 A1</p>	<p>s.o.i.;  or v.v. Solve now or at normal stage. [This dep on either/both B1 earned] Implied if grad normal = <math>\frac{7}{11}</math>  Numerical or general, awarded at any stage  6 No fractions in final answer.</p>

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<p>5 (i) Use <math>3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}</math> and <math>2\mathbf{i} - \mathbf{j} - 5\mathbf{k}</math> only</p> <p>Use correct method for scalar prod of <u>any</u> 2 vectors</p> <p>Obtain <math>6 + 4 - 10</math>, state = 0 &amp; deduce perp <b>AG</b></p>	<p>M1</p> <p>M1</p> <p>A1 <b>3</b></p>	<p>(indep) May be as part of <math>\cos \theta = \frac{a \cdot b}{ a  b }</math></p>
<p>(ii) Produce 3 equations in <math>s</math> and <math>t</math></p> <p>Solve 2 of the equations for <math>s</math> and <math>t</math></p> <p>Obtain <math>(s,t) = \left(\frac{3}{5}, \frac{12}{5}\right)</math> or <math>\left(\frac{9}{22}, \frac{18}{11}\right)</math> or <math>\left(\frac{3}{19}, \frac{33}{19}\right)</math></p> <p>Substitute their values in 3<sup>rd</sup> equation</p> <p>State/show inconsistency &amp; <u>state non-parallel</u> ∴ skew</p>	<p>*M1</p> <p>dep*M1</p> <p>A1</p> <p>dep*M1</p> <p>A1 <b>5</b></p>	<p>of the type <math>5 + 3s = 2 + 2t</math>, <math>-2 - 4s = -2 - t</math> and <math>-2 + 2s = 7 - 5t</math></p> <p><u>Or</u> Eliminate <math>s</math> (or <math>t</math>) from 2 pairs dep*M1</p> <p><math>(5t=12, 11t=18, 19t=33)</math> or <math>(5s=3, 22s=9, 19s=3)</math> A1,A1</p> <p>State/show inconsistency &amp; <u>state non-parallel</u> ∴ skew WWW A1</p>
<p>6 (i) <math>1 - 4ax + \dots</math></p> <p><math>\frac{-4. - 5}{1.2}(ax)^2</math> or <math>\frac{-4. - 5}{1.2}a^2x^2</math> or <math>\frac{-4. - 5}{1.2}ax^2</math></p> <p><math>\dots + 10a^2x^2</math></p> <p>(ii) f.t. (their cf <math>x</math>) + <math>b</math>(their const cf) = 1</p> <p>f.t. (their cf <math>x^2</math>) + <math>b</math>(their cf <math>x</math>) = -2</p> <p>Attempt to eliminate 'b' and produce equation in 'a'</p> <p>Produce <math>6a^2 + 4a = 2</math> AEF</p> <p><math>a = \frac{1}{3}</math> and <math>b = \frac{7}{3}</math> only</p>	<p>B1</p> <p>M1</p> <p>A1 <b>3</b></p> <p>√B1</p> <p>√B1</p> <p>M1</p> <p>A1</p> <p>A1 <b>5</b></p>	<p>Do not accept <math>\begin{pmatrix} -4 \\ 2 \end{pmatrix}</math> unless 10 also appears</p> <p>Expect <math>b - 4a = 1</math></p> <p>Expect <math>10a^2 - 4ab = -2</math></p> <p>Or eliminate 'a' and produce equation in 'b'</p> <p>Or <math>6b^2 + 4b = 42</math> AEF</p> <p>Made clear to be only (final) answer</p>
<p>7 (i) Perform an operation to produce an equation connecting <math>A</math> and <math>B</math> (or possibly in <math>A</math> or in <math>B</math>)</p> <p><math>A = 2</math></p> <p><math>B = -2</math></p> <p>(ii) Write <math>4 \sin \theta</math> as <math>A(\sin \theta + \cos \theta) + B(\cos \theta - \sin \theta)</math></p> <p>and re-write integrand as <math>A + \frac{B(\cos \theta - \sin \theta)}{\sin \theta + \cos \theta}</math></p> <p><math>\int A d\theta = A\theta</math></p> <p><math>\int \frac{B(\cos \theta - \sin \theta)}{\sin \theta + \cos \theta} d\theta = B \ln(\sin \theta + \cos \theta)</math></p> <p>Produce <math>\frac{1}{4}A\pi + B \ln \sqrt{2}</math> f.t. with their <math>A, B</math></p>	<p>M1</p> <p>A1</p> <p>A1 <b>3</b></p> <p>M1</p> <p>√B1</p> <p>√A2</p> <p>√A1 <b>5</b></p>	<p>Probably substituting value of <math>\theta</math>, or comparing coefficients of <math>\sin x</math>, and/or <math>\cos x</math></p> <p>WW scores 3</p> <p><math>A</math> and <math>B</math> need not be numerical – but, if they are, they should be the values found in (i).</p> <p>general or numerical</p> <p>general or numerical</p> <p>Expect <math>\frac{1}{2}\pi - \ln 2</math> (Numerical answer only)</p>
<p>8 (i) <math>\frac{dx}{dt}</math> or <math>-kx^{\frac{1}{2}}</math> or <math>kx^{\frac{1}{2}}</math> seen</p> <p><math>\frac{dx}{dt} = -kx^{\frac{1}{2}}</math> or <math>\frac{dx}{dt} = kx^{\frac{1}{2}}</math></p> <p>(ii) Separate variables or invert, + attempt to integrate</p> <p>Correct result for their equation after integration</p> <p>Subst <math>(t, x) = (0, 2)</math> into eqn containing <math>k</math> &amp;/or <math>c</math> dep*M1</p> <p>Subst <math>(t, x) = (5, 1)</math> into eqn containing <math>k</math> &amp; <math>c</math> dep*M1</p> <p>Subst <math>x = 0.5</math> into eqn with their <math>k</math> &amp; <math>c</math> subst dep*M1</p> <p><math>t = 8.5</math> (8.5355339)</p>	<p>M1</p> <p>A1 <b>2</b></p> <p>* M1</p> <p>A1</p> <p>dep*M1</p> <p>dep*M1</p> <p>dep*M1</p> <p>A1 <b>6</b></p>	<p><math>k</math> non-numerical; i.e. 1 side correct</p> <p>i.e. both sides correct</p> <p>Based <u>only</u> on above eqns or <math>\frac{dx}{dt} = x^{\frac{1}{2}}</math>, <math>-x^{\frac{1}{2}}</math></p> <p>Other than omission of 'c' or substitute (5,1) or substitute (0,2)</p> <p>[1 d.p. requested in question]</p>

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9	<p>(i) Use <math>\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}</math> or <math>\frac{\frac{dy}{dp}}{\frac{dx}{dp}}</math>  <math>= \frac{2t}{3t^2}</math> or <math>\frac{2p}{3p^2}</math>                      Find eqn tgt thro <math>(p^3, p^2)</math> or <math>(t^3, t^2)</math>, their gradient  <math>3py - 2x = p^3</math> <b>AG</b></p>	<p>M1 A1 M1 A1</p>	<p>Or conv to cartes form &amp; att to find <math>\frac{dy}{dx}</math> at P                      Using <math>y - y_1 = m(x - x_1)</math> or <math>y = mx + c</math>  <b>4</b> Do not accept <math>t</math> here</p>
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	<p>(ii) Substitute <math>(-10,7)</math> into given equation                      Satis attempt to find at least 1 root/factor                      Any one root                      All 3 roots  <math>(-1,1), (-64,16)</math> and <math>(125,25)</math></p>	<p>*M1 dep*M1 A1 A1 A1</p>	<p>to produce a cubic equation in <math>p</math>                      Inspection/factor theorem/rem theorem/t&amp;i  <math>-1</math> or <math>-4</math> or <math>5</math>  <math>-1, -4</math> and <math>5</math>  <b>5</b> All 3 sets; no f.t.</p>
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10	<p>(i) <math>(1-x^2)^{\frac{3}{2}} \rightarrow \cos^3 \theta</math>  <math>dx \rightarrow \cos \theta d\theta</math>  <math>\frac{1}{(1-x^2)^{\frac{3}{2}}} dx \rightarrow \sec^2 \theta (d\theta)</math> or <math>\frac{1}{\cos^2 \theta} (d\theta)</math>  <math>\int \sec^2 \theta (d\theta) = \tan \theta</math>                      Attempt change of limits (expect <math>0</math> &amp; <math>\frac{1}{6}\pi / 30</math>)  <math>\frac{1}{\sqrt{3}}</math> AEF</p>	<p>B1 B1 B1 B1 M1 A1</p>	<p>May be implied by <math>\int \sec^2 \theta d\theta</math>                      Use with <math>f(\theta)</math>; or re-subst &amp; use <math>0</math> &amp; <math>\frac{1}{2}</math>  <b>6</b> Obtained with no mention of <math>30</math> anywhere</p>
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	<p>(ii) Use parts with <math>u = \ln x, \frac{dv}{dx} = \frac{1}{x^2}</math>  <math>-\frac{1}{x} \ln x + \int \frac{1}{x^2} (dx)</math> AEF  <math>-\frac{1}{x} \ln x - \frac{1}{x}</math>                      Limits used correctly  <math>\frac{2}{3} - \frac{1}{3} \ln 3</math>  <u>If substitution attempted in part (ii)</u>  <math>\ln x = t</math>                      Reduces to <math>\int t e^{-t} dt</math>                      Parts with <math>u = t, dv = e^{-t}</math>  <math>-te^{-t} - e^{-t}</math>  <math>\frac{2}{3} - \frac{1}{3} \ln 3</math></p>	<p>*M1 A1 A1 dep*M1 A1 B1 B1 M1 A1 A1</p>	<p>obtaining a result <math>f(x) + / - \int g(x)(dx)</math>                      Correct first stage result                      Correct overall result  <b>5</b></p>