



Monday 10 June 2013 – Morning

A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4727/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

2

- 1 The plane Π passes through the points with coordinates $(1, 6, 2)$, $(5, 2, 1)$ and $(1, 0, -2)$.
- (i) Find a vector equation of Π in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$. [2]
- (ii) Find a cartesian equation of Π . [4]
- 2 G consists of the set $\{1, 3, 5, 7\}$ with the operation of multiplication modulo 8.
- (i) Write down the operation table and, assuming associativity, show that G is a group. [5]
- (ii) State the order of each element. [1]
- (iii) Find all the proper subgroups of G . [1]
- The group H consists of the set $\{1, 3, 7, 9\}$ with the operation of multiplication modulo 10.
- (iv) Explaining your reasoning, determine whether H is isomorphic to G . [2]
- 3 The differential equation
- $$3xy^2 \frac{dy}{dx} + 2y^3 = \frac{\cos x}{x}$$
- is to be solved for $x > 0$. Use the substitution $u = y^3$ to find the general solution for y in terms of x . [8]
- 4 The complex numbers 0 , 3 and $3e^{\frac{1}{3}\pi i}$ are represented in an Argand diagram by the points O , A and B respectively.
- (i) Sketch the triangle OAB and show that it is equilateral. [3]
- (ii) Hence express $3 - 3e^{\frac{1}{3}\pi i}$ in polar form. [2]
- (iii) Hence find $(3 - 3e^{\frac{1}{3}\pi i})^5$, giving your answer in the form $a + b\sqrt{3}i$ where a and b are rational numbers. [3]
- 5 Find the solution of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}$ for which $y = \frac{dy}{dx} = 0$ when $x = 0$. [11]
- 6 The plane Π has equation $x + 2y - 2z = 5$. The line l has equation $\frac{x-1}{2} = \frac{y+1}{5} = \frac{z-2}{1}$.
- (i) Find the coordinates of the point of intersection of l with the plane Π . [3]
- (ii) Calculate the acute angle between l and Π . [3]
- (iii) Find the coordinates of the two points on the line l such that the distance of each point from the plane Π is 2. [5]

3

- 7 A commutative group G has order 18. The elements a , b and c have orders 2, 3 and 9 respectively.
- (i) Prove that ab has order 6. [4]
 - (ii) Show that G is cyclic. [3]
- 8 (i) Use de Moivre's theorem to show that $\cos 5\theta \equiv 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta$. [5]
- (ii) Hence find the roots of $16x^4 - 20x^2 + 5 = 0$ in the form $\cos \alpha$ where $0 \leq \alpha \leq \pi$. [4]
 - (iii) Hence find the exact value of $\cos \frac{1}{10}\pi$. [3]

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.



Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.