Version 1.0 0110

General Certificate of Education

Mathematics 6360

MPC1 Pure Core 1

Mark Scheme

2010 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

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Q ... [1]

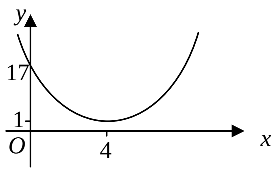
MPC1

Q	Solution	Marks	Total	Comments
1(a)	$p(-3)=(-3)^3-13(-3)-12$	M1		must attempt $p(-3)$ NOT long division
	$=-27+39-12$ $=0 \quad \Rightarrow x+3$ is factor	A1	2	shown $=0$ plus statement
(b)	$(x+3)(x^2+bx+c)$	M1		Full long division, comparing coefficients or by inspection either $b=-3$ or $c=-4$
	(x^2-3x-4) obtained	A1		or M1A1 for either $(x-4)$ or $(x+1)$ clearly found using factor theorem
	$(x+3)(x-4)(x+1)$	A1	3	CSO; must be seen as a product of 3 factors NMS full marks for correct product SC B1 for $(x+3)(x-4)(\ )$ or $(x+3)(x+1)(\ )$ or $(x+3)(x+4)(x-1)$ NMS
	Total		**5**	
2(a)(i)	grad $AB=\dfrac{7-3}{3-1}$	M1		$\dfrac{\Delta y}{\Delta x}$ correct expression, possibly implied
	$=2$ (must simplify 4/2)	A1	2	
(ii)	grad $BC=\dfrac{7-9}{3+1}=-\dfrac{2}{4}$	M1		Condone one slip **NOT** Pythagoras or cosine rule etc
	grad AB × grad BC $=-1$ $\Rightarrow \angle ABC=90^\circ$ or AB & BC perpendicular	A1	2	convincingly proved plus statement SC B1 for –1/(their grad AB) or statement that $m_1m_2=-1$ for perpendicular lines if M0 scored
(b)(i)	$M\ (0,6)$	B2	2	B1 + B1 each coordinate correct
(ii)	$(AB^2=)\quad (3-1)^2+(7-3)^2$ $(BC^2=)\quad (3+1)^2+(7-9)^2$	M1		either expression correct, simplified or unsimplified
	$AB^2=2^2+4^2$ or $BC^2=4^2+2^2$ or $\sqrt{20}$ found as a length	A1		Must see either AB^2 =.., or BC^2 =...,
	$AB^2=BC^2 \Rightarrow AB=BC$ or $AB=\sqrt{20}$ and $BC=\sqrt{20}$	A1	3	
(iii)	grad $BM=\dfrac{7-6}{3-0}$ or –1/(grad AC) attempted	M1		ft their M coordinates
	$=\dfrac{1}{3}$	A1		correct gradient of line of symmetry
	BM has equation $y=\dfrac{1}{3}x+6$	A1	3	CSO, any correct form
	Total		**12**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$\frac{dy}{dt} = \frac{4t^3}{8} - 4t + 4$	M1 A1 A1	3	one term correct another term correct all correct (no + c etc) unsimplified
(ii)	$\frac{d^2y}{dt^2} = \frac{12t^2}{8} - 4$	M1 A1	 2	ft one term "correct" correct unsimplified (penalise inclusion of $+c$ once only in question)
(b)	$t = 2$; $\frac{dy}{dt} = 4 - 8 + 4$ $\frac{dy}{dt} = 0 \Rightarrow$ stationary value $t = 2$; $\frac{d^2y}{dt^2} = 6 - 4 = 2$ $\Rightarrow y$ has MINIMUM value	M1 A1 M1 A1	 4	Substitute $t = 2$ into their $\frac{dy}{dt}$ CSO; shown = 0 plus statement Sub $t = 2$ into their $\frac{d^2y}{dt^2}$ CSO
(c)(i)	$t = 1$; $\frac{dy}{dt} = \frac{1}{2} - 4 + 4$ $= \frac{1}{2}$	M1 A1	 2	Substitute $t = 1$ into their $\frac{dy}{dt}$ OE; CSO NMS full marks if $\frac{dy}{dt}$ correct
(ii)	$\frac{dy}{dt} > 0 \Rightarrow$ (depth is) INCREASING	E1√	1	allow decreasing if states that their $\frac{dy}{dt} < 0$ **Reason must be given not just the word increasing or decreasing**
	Total		**12**	
4(a)	$\sqrt{50} = 5\sqrt{2}$; $\sqrt{18} = 3\sqrt{2}$; $\sqrt{8} = 2\sqrt{2}$ At least two of these correct $\frac{5\sqrt{2} + 3\sqrt{2}}{2\sqrt{2}}$ *Answer* $= 4$	M1 A1 A1	 3	or $\times\frac{\sqrt{8}}{\sqrt{8}}$ or $\left(\times\frac{\sqrt{2}}{\sqrt{2}}\right)$ or $\sqrt{\frac{25}{4}} + \sqrt{\frac{9}{4}}$ any **correct** expression all in terms of $\sqrt{2}$ or with denominator of 8, 4 or 2 simplifying numerator eg $\frac{\sqrt{400} + \sqrt{144}}{8}$ CSO
(b)	$\frac{(2\sqrt{7} - 1)(2\sqrt{7} - 5)}{(2\sqrt{7} + 5)(2\sqrt{7} - 5)}$ *numerator* = $4 \times 7 - 2\sqrt{7} - 10\sqrt{7} + 5$ *denominator* = 3 *Answer* $= 11 - 4\sqrt{7}$	M1 m1 B1 A1	 4	OE expanding numerator (condone one error or omission) (seen as denominator)
	Total		**7**	

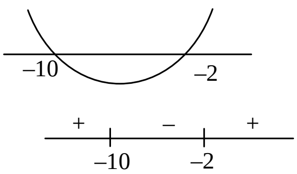
MPC1 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$x^2 - 8x + 15 + 2$	B1		Terms in *x* must be collected, PI
	their $(x-4)^2 \quad (+k)$	M1		ft $(x-p)^2$ for their quadratic
	$=(x-4)^2+1$	A1	3	ISW for stating $p=-4$ if correct expression seen
(b)(i)		M1		$\cup$ shape in any quadrant (generous)
		A1		correct with min at (4, 1) stated or 4 and 1 marked on axes condone within first quadrant only
		B1	3	crosses *y*-axis at (0, 17) stated or 17 marked on *y*-axis
(ii)	$y = k$	M1		$y =$ constant
	$y = 1$	A1	2	Condone $y = 0x + 1$
(c)	Translation (*not shift, move etc*)	E1		and no other transformation
	with vector $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$	M1		One component correct or ft either their *p* or *q*
		A1	3	CSO; condone 4 across, 1 up; or two separate vectors etc
	Total		**11**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=24x-19-6x^2$	M1 A1		2 terms correct all correct (no + c etc)
	when $x=2,\ \frac{\mathrm{d}y}{\mathrm{d}x}=48-19-24$	m1		
	$\Rightarrow$ gradient $=5$	A1	4	CSO
(ii)	grad of normal $=-\frac{1}{5}$	B1$\surd$		ft their answer from (a)(i)
	$y+6=\left(\mathit{their}-\frac{1}{5}\right)(x-2)$ or $y=\left(\mathit{their}-\frac{1}{5}\right)x+c$ and c evaluated using $x=2$ and $y=-6$	M1		ft grad of their normal using **correct** coordinates BUT must not be tangent condone omission of brackets
	$x+5y+28=0$	A1	3	CSO; condone all on one side in different order
(b)(i)	$\frac{12}{3}x^3-\frac{19}{2}x^2-\frac{2}{4}x^4$	M1 A1 A1		one term correct another term correct all correct (ignore $+c$ or limits)
	$=32-38-8$	m1		$\mathrm{F}(2)$ attempted
	$=-14$	A1	5	CSO; withhold A1 if changed to +14 here
(ii)	Area $\Delta=\frac{1}{2}\times2\times6=6$	B1		condone -6
	Shaded region area $=14-6$	M1		difference of $\pm\left\|\int\right\|\ \pm\left\|\Delta\right\|$
	$=8$	A1	3	CSO
	Total		**15**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$x=\pm 2$ or $y=\pm 6$ or $(x-2)^2+(y+6)^2$ $C(2,-6)$	M1 A1	 2	 correct
(ii)	$(r^2=)\,4+36-15$ $\Rightarrow r=5$	M1 A1	 2	(RHS =) *their* $(-2)^2$ + *their* $(6)^2 - 15$ Not ± 5 *or* $\sqrt{25}$
(b)	explaining why $\lvert y_C \rvert > r$; $6>5$ full convincing argument, but must have correct y_C and r	E1 E1	 2	Comparison of y_C and r , eg $-6 + 5 = -1$ or indicated on diagram Eg "highest point is at $y = -1$" scores E2 E1: showing no real solutions when $y=0$ +E1 stating centre or any point below x-axis
(c)(i)	$(PC^2=)\quad (5-2)^2+(k+6)^2$ $=9+k^2+12k+36$ $PC^2=k^2+12k+45$	 M1 A1	 2	ft their C coords and attempt to multiply out **AG** CSO (must see $PC^2=$ at least once)
(ii)	$\left.\begin{array}{l}PC>r\Rightarrow PC^2>25\\ \Rightarrow k^2+12k+20>0\end{array}\right\}$	B1	1	**AG** Condone $\left.\begin{array}{l}k^2+12k+45>25\\ \Rightarrow k^2+12k+20>0\end{array}\right\}$
(iii)	$(k+2)(k+10)$ $k=-2,\ k=-10$ are critical values Use of sketch or sign diagram:  $\Rightarrow\ k>-2,\ k<-10$ Condone $k>-2$ OR $k<-10$ for full marks but not AND instead of OR Take final line as their answer	M1 A1 M1 A1	 4	**Correct** factors or **correct** use of formula May score M1, A1 for correct critical values seen as part of incorrect final answer with or without working. If previous A1 earned, sign diagram or sketch must be correct for M1, otherwise M1 may be earned for an attempt at the sketch or sign diagram using their critical values. $k\geqslant -2,\ k\leqslant -10$ loses final A mark *Answer only* of $k>-2,\ k>-10$ *etc* scores M1, A1, M0 since the critical values are evident. *Answer only* of $k>2,\ k<-10$ *etc* scores M0, M0 since the critical values are not both correct.
	Total		**13**	
	TOTAL		**75**	

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XMCA2

Q	Solution	Marks	Total	Comments
1(a)	$x = -\frac{3}{2}$	B1		Seeing $-\frac{3}{2}$ OE
	p(−1.5)= $2(-1.5)^4 + 3(-1.5)^3 - 8(-1.5)^2 - 14(-1.5) - 3$	M1		Attempting to evaluate p(−1.5 or p(1.5)
	p(−1.5) = 10.125−10.125−18+21−3 = 0] so (2x + 3) is a factor of p(x)]	A1	3	CSO Need both the arithme to show '= 0' and the conclusion.
(b)(i)	$x^3-4x-1=0 \Rightarrow x(x^2-4)-1=0 \Rightarrow x^2-4=\frac{1}{x}$	M1		Dividing throughout by x OE
	$x^2 = \frac{1}{x}+4 \Rightarrow x = \sqrt{\frac{1}{x}+4}$ (since x>0)	A1	2	CSO
(ii)	$x_2 = 2.1213$ $x_3 = 2.1146$ $x_4 = 2.1149$	B1 B1 B1	3	AWRT 2.121 AWRT 2.1146 CAO
	Total		8	
2(a)	$\frac{5+x}{(1-x)(2+x)} = \frac{A}{1-x}+\frac{B}{2+x}$ $\Rightarrow$ 5 + x = A(2 + x) + B (1 − x)	M1		Either multiplication by denominator or cover up rule attempted.
	Substitute x = 1 ; Substitute x = −2	m1		Either use (any) two values o to find A and B or equate coefficients to form and atten to solve A−B=1 and 2A+B=5
	A = 2 , B = 1	A1	3	
(b)(i)	$(1-x)^{-1} = 1+(-1)(-x)+px^2$	M1		$p \neq 0$
	$= 1+x+x^2...$	A1	2	
(ii)	$2^{-1}\left[1+\frac{x}{2}\right]^{-1} = \frac{1}{2}\left[1+(-1)\left(\frac{x}{2}\right)+\frac{(-1)(-2)}{2!}\left(\frac{x}{2}\right)^2+...\right]$	M1		$[1+(-1)\left(\frac{x}{2}\right)+kx^2]$
		A1		Correct expn of $\left(1+\frac{x}{2}\right)^{-1}$
	$\frac{5+x}{(1-x)(2+x)}$ = $2(1-x)^{-1} + (2+x)^{-1}$	M1		Using (a) with powers '−1'. P
	$= 2(1+x+x^2...) + \frac{1}{2}\left(1-\frac{x}{2}+\frac{x^2}{4}+...\right)$	m1		Dep on prev 3Ms
	= 2.5 + 1.75x + 2.125x^2 +…	A1F	5	Ft only on wrong integer valu for A and B, ie simplified (A+1/2B)+(A−1/4B)x+(A+1/8B [Award equivalent marks for other valid methods.]
	Total		10	

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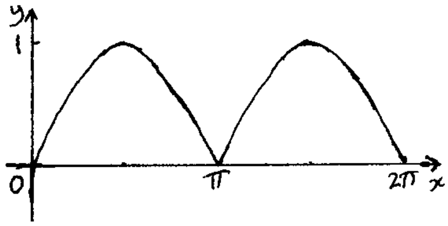
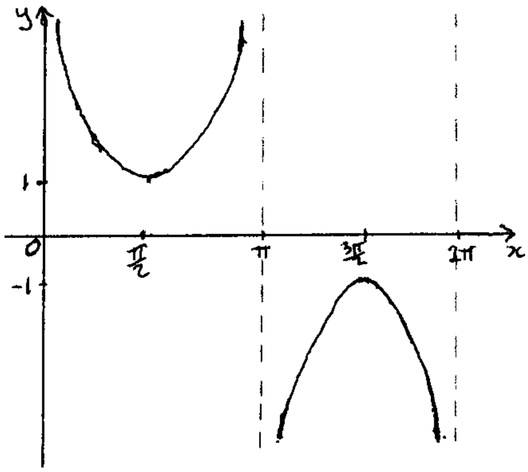
XMCA2 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)		M1 A1	2	Modulus graph Correct shape including cusp at (π, 0). Ignore any part of graph beyond $0 \le x \le 2\pi$.
(ii)	$k = 1$	B1	1	
(b)		M1 A1	2	Two branch curve, genera shape correct. Min at (α, 1) Max at (β, 1 with α roughly halfway between 0 and π, and β roughly halfway between π and 2π and curve asymptotic to $x = 0$, $x = \pi$ and $x = 2\pi$.
	Total		5	
4(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x+2)3\mathrm{e}^{3x} - \mathrm{e}^{3x}(1)}{(x+2)^2}$	B1 M1 A1	3	$(\mathrm{e}^{3x})' = 3\mathrm{e}^{3x}$ Quotient rule OE
(b)	When $x = 0$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6e^0 - e^0}{2^2} = \frac{5}{4}$ $A\left(0, \frac{1}{2}\right)$ Equation of tangent at A: $y - \frac{1}{2} = \frac{5}{4}(x - 0)$	M1 A1F B1 A1	4	Attempt to find dy/dx at x= ACF
	Total		7	

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XMCA2 (cont)

Q	Solution	Marks	Total	Comments
5	$V = \pi\int_0^1 \cos(x^2)\,\mathrm{d}x$	M1 A1		$\int \cos(x^2)\,\mathrm{d}x$ Correct limits. (Condone k or missing π until the final mark)

	Applying Simpson's rule to $\int_0^1 \cos(x^2)\,\mathrm{d}x$			
	x 0 0.25 0.5 0.75 1	B1		PI
	Y=y^2 1 0.9980(47) 0.9689(12) 0.8459(24) 0.5403(02) [πY vals. 3.1415(9) 3.1354(5) 3.0439(2) 2.6575(5) 1.6974(0)]	B1		PI
	$\frac{0.25}{3}\times\{Y(0)+Y(1)+4[Y(0.25)+Y(0.75)]+2Y(0.5)\}$	M1		Use of Simpson's rule
	$V=\pi\times\frac{10.8539....}{12}$ So V = 2.8416 (to 4 d.p.)	A1	6	CAO
	Total		6	

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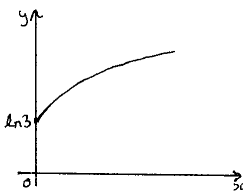
XMCA2 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)		B2,1,0	2	B2 correct sketch-no part c curve in 2nd,3rd or 4th quadrants and 'ln3' (B1 for general shape in 1^{s} quadrant, ignore other quadrants; ln3 not requirec
(ii)	Range of f: f(x) ≥ ln3	M1 A1	2	≥ln3 or >ln3 or f≥ln3 Allow y for f(x).
(b)(i)	$y = \mathrm{f}^{-1}(x) \Rightarrow \mathrm{f}(y) = x$ $\Rightarrow \ln(2y+3) = x$ $\Rightarrow 2y+3 = \mathrm{e}^x$ $\mathrm{f}^{-1}(x) = \frac{\mathrm{e}^x - 3}{2}$	 M1 m1 A1	 3	 $x \Leftrightarrow y$ at any stage Use of $\ln m = N \Rightarrow m = \mathrm{e}^N$ ACF-Accept y in place of $\mathrm{f}^{-1}(x)$
(ii)	Domain of f^{-1} is: $x \geq \ln 3$	B1F	1	ft on (a)(ii) for RHS
(c)	$\frac{\mathrm{d}}{\mathrm{d}x}[(\ln(2x+3)] = \frac{1}{(2x+3)}\times 2$	M1 A1	 2	1/(2x+3)
(d)(i)	P, the pt of intersection of y = f(x) and $y = \mathrm{f}^{-1}(x)$, must lie on the line $y = x$; so P has coordinates (α, α). f(α) = α ln(2α+3) = α $\Rightarrow$ 2α + 3 = e$^{\alpha}$	M1; M1 A1	 3	 OE eg $\mathrm{f}^{-1}(\alpha) = \alpha$ A.G. CSO

(ii)	$\frac{d}{dx}\left[f^{-1}(x)\right]=\frac{1}{2}e^{x}$	B1F		$\frac{e^{\alpha}-3}{2}=\alpha \Rightarrow e^{\alpha}=2\alpha+3$
	Product of gradients = $\frac{e^{x}}{2x+3}$ At $P(\alpha, \alpha)$, the product of the gradients is $\frac{e^{\alpha}}{2\alpha+3}=\frac{2\alpha+3}{2\alpha+3}=1$	B1	2	AG CSO
	Total		15	

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XMCA2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\frac{dy}{dx}=x\,e^{x}+e^{x}$.	M1 A1		M1 Product rule OE.
	At stationary point(s) $e^{x}(x+1)=0$	m1		
	$e^{x}>0$	E1		OE eg accept $e^{x}\neq 0$
	Only one value of x for st. pt. Curve has exactly one st pt	A1		CSO with conclusion.
	Stationary point is $(-1, -e^{-1})$	A1	6	
(b)	Stationary point is $(-1, k-e^{-1})$	B1F		Or E1 for $y=x\,e^{x}$ to $y=x\,e^{x}+k$ is a vertical translation of k units.
	St. pt is on x-axis, so $k=e^{-1}$.	B1	2	
	Total		8	
8	$\int \frac{1}{y}\,dy=\int \frac{\cos x}{6+\sin x}\,dx$	M1		Separating variables with intention to then integrate.
	$\ln y=\ln(6+\sin x)$ $(+c)$	A1 A1		A1 for each side. Condone missing '+c'
	$\ln 2=\ln 6+c$	m1		Substituting $x=0$, $y=2$ to find c
	$\ln y=\ln(6+\sin x)+\ln 2-\ln 6$			
	so $y=\frac{1}{3}(6+\sin x)$	A1	5	Correct simplified form not involving logs
	Total		5	
9(a)	$y=e^{2x}\rightarrow e^{-2x}\rightarrow 6e^{-2x}$.			
	Reflection; in the y-axis	M1;A1		
	Stretch,	M1		M1 'Stretch' with either (I) or (II).
	(I) parallel to y-axis, (II) scale factor 6.	A1	4	
				For correct alternatives to the stretch after writing $y=e^{-2x+\ln 6}$ award B1 for 'translation in x-dirn.' and B1 for the correct vector (OE) noting order of transformations.
(b)(i)	Area of rectangle/shaded region below x-axis $=3k$	B1		

	Area of shaded region above x-axis $= \int_0^k 6e^{-2x}\,dx$	B1		
	$= \left[-3e^{-2x}\right]_0^k = -3e^{-2k} - (-3)$	M1 A1		F(k) − F(0) following an integration. ACF
	Total area of shaded region $= 3k - 3e^{-2k} + 3 = 4$ $3k-1-3e^{-2k} = 0 \Rightarrow (3k-1)e^{2k} - 3 = 0$.	M1 A1	6	AG CSO
(ii)	Let $f(k) = (3k-1)e^{2k} - 3$ $f(0.6) = 0.8e^{1.2} - 3 = -0.3(4..) < 0$ $f(0.7) = 1.1e^{1.4} - 3 = 1.(46..) > 0$	M1		Both f(0.6) and f(0.7) [or better] attempted
	Since change of sign (and f continuous), $0.6 < k < 0.7$	A1	2	AG Note: Must see the explicit reference to 0.6 and 0.7 otherwise A0
	Total		12	

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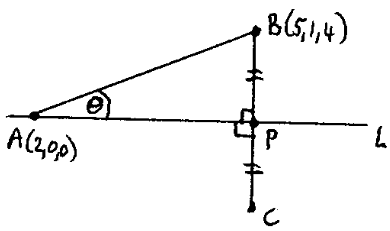
XMCA2 (cont)

Q	Solution	Marks	Total	Comments
10(a)	$\overrightarrow{AB} = \begin{bmatrix}5\\1\\4\end{bmatrix} - \begin{bmatrix}2\\0\\0\end{bmatrix} = \begin{bmatrix}3\\1\\4\end{bmatrix}$	M1 A1		M1 for ± ($\overrightarrow{OB} - \overrightarrow{OA}$) OE for $\overrightarrow{BA}$
	Line AB: $r = \begin{bmatrix}2\\0\\0\end{bmatrix} + \lambda\begin{bmatrix}3\\1\\4\end{bmatrix}$	B1F	3	OE Ft on $\overrightarrow{AB}$
(b)	$\begin{bmatrix}3\\1\\4\end{bmatrix} \bullet \begin{bmatrix}1\\2\\1\end{bmatrix} = 3+2+4 = 9$	M1		$\pm\overrightarrow{AB} \bullet$ direction vector of l evaluated
	$\sqrt{3^2+1^2+4^2} = \sqrt{26}$; $\sqrt{1^2+2^2+1^2} = \sqrt{6}$	B1F		Either; Ft on either of c's vectors
	$\sqrt{26}\sqrt{6}\cos\theta = 9$ $\cos\theta = \frac{9}{\sqrt{26}\sqrt{6}} = \frac{9}{\sqrt{2}\sqrt{13}\sqrt{2}\sqrt{3}}$	M1		Use of $\lvert a \rvert \lvert b \rvert \cos\theta = a \cdot b$
	$= \cos\theta = \frac{9}{2\sqrt{13}\sqrt{3}} = \frac{9}{2\sqrt{39}}$	A1	4	AG CSO
(c)(i)				

	$\overrightarrow{BP} = \begin{bmatrix} 2+p \\ 2p \\ p \end{bmatrix} - \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} p-3 \\ 2p-1 \\ p-4 \end{bmatrix}$	M1 A1		Condone one slip
	$\overrightarrow{BP} \bullet \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 0$; $6p = 9 \Rightarrow p = 1.5$	M1 A1		"$\pm\ \overrightarrow{BP} \bullet$ direction vector of $l = 0$". Condone one slip
	P (3.5, 3, 1.5) is mid point of *BC*	A1	5	
(ii)	$\frac{x_C+5}{2} = 3.5$ $\frac{y_C+1}{2} = 3$ $\frac{z_C+4}{2} = 1.5$	M1		
	$\Rightarrow$ *C* (2, 5, −1)	A1	2	Condone written as a column vector. Award equivalent marks for alternative valid methods.
	Total		14	

XMCA2 (cont)

Q	Solution	Marks	Total	Comments
11(a)	$\sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x$	M1		
	$= [2\sin x\cos x]\cos x + [1-2\sin^2 x]\sin x$	B1;B1		B1 for each […]. Accept alternative correct forms for cos2*x*
	$= 2\sin x(1-\sin^2 x) + (1-2\sin^2 x)\sin x$	m1		All in terms of sin *x*
	$= 2\sin x - 2\sin^3 x - \sin x - 2\sin^3 x$			
	$\sin 3x = 3\sin x - 4\sin^3 x$.	A1	5	CSO
(b)	$2\sin 3x = 1 - \cos 2x$			
	$2(3\sin x - 4\sin^3 x) = 1 - \cos 2x$	M1		Using (a)
	$2(3\sin x - 4\sin^3 x) = 1 - (1-2\sin^2 x)$	M1 A1		Equation in sin *x*
	$2\sin x\,(3 - \sin x - 4\sin^2 x) = 0$			
	$[2\sin x = 0]$ $(3 - 4\sin x)(1 + \sin x) = 0$	m1		Factorising/solving quadratic in sin
	$\sin x = 0$; $x = 180°$	B1		
	$\sin x = 0.75$; $x = 48.6°,\ 131.4°$	A1		Ignore solns outside $0°<x<360°$ throughout
	$\sin x = -1$; $x = 270°$	A1	7	
	Total		12	
12(a)(i)	$u = x$ and $\frac{dv}{dx} = \sec^2 x$	M1		Attempt to use parts formula in the 'correct direction'
	$\frac{du}{dx} = 1$ and $v = \tan x$	A1		PI
	….. $= x\tan x - \int \tan x\, dx$	A1		
	$= x\tan x - \ln(\sec x)\ (+c)$	A1	4	OE CSO (Condone absence of +*c*)
(ii)	$\int x\tan^2 x\, dx = \int x(\sec^2 x - 1)\, dx$	M1		Use of identity $1 + \tan^2 x = \sec^2 x$

	$= [x\tan x - \ln(\sec x)] - \frac{1}{2}x^2$ (+ c)	A1F	2	[…] ft on (a)(i)
(b)	$x = 2\sin\theta$, $dx = 2\cos\theta\, d\theta$	M1		"$dx = f(\theta)\, d\theta$" OE
	$\int \sqrt{4-x^2}\, dx = \int \sqrt{4(1-\sin^2\theta)}\, 2\cos\theta\, d\theta$	m1 A1		Eliminating all x's
	$= \int 4\cos^2\theta\, d\theta = \int 2(\cos 2\theta + 1)\, d\theta$	m1		Use of $\cos 2\theta$ to integrate $\cos^2\theta$.
	$= \sin 2\theta + 2\theta$ (+ c) $= 2\sin\theta\sqrt{1-\sin^2\theta} + 2\theta$ (+ c)	A1F		Ft a slip
	$= x\sqrt{\left(1-\frac{x^2}{4}\right)} + 2\sin^{-1}\left(\frac{x}{2}\right)$ (+ c)	A1	6	ACF (accept unsimplified)
	Total		12	

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XMCA2 (cont)

Q	Solution	Marks	Total	Comments
13	$x = 3t + t^3$ $\quad y = 8 - 3t^2$ $\frac{dx}{dt} = 3 + 3t^2$ $\quad \frac{dy}{dt} = -6t$	M1		Both attempted and at least one correct.
	$\frac{dy}{dx} = \frac{-6t}{3+3t^2}$	M1 A1		Chain rule.
	At $P(-4, 5)$, $t = -1$	B1		
	At $P(-4, 5)$, $\frac{dy}{dx} = \frac{6}{3+3} = 1$ Gradient of normal at P is -1 Eqn of normal at P: $y - 5 = -1(x+4)$ $y + x = 1$	M1 A1		ACF
	Normal cuts curve C when $8 - 3t^2 + 3t + t^3 = 1$ $\Rightarrow t^3 - 3t^2 + 3t + 7 = 0$ $\Rightarrow (t+1)(t^2 - 4t + 7) = 0$ (*)	M1 A1 m1		
	$(t^2 - 4t + 7) = 0$ has no real solutions since $(-4)^2 < 4(1)(7)$. $t = -1$ is only real solution of (*) so normal only cuts C at P, where $t = -1$ ie the normal does not cut C again.	M1 E1	11	
	Total		11	