

2. Use algebra to find the set of values of x for which

$$|3x^2 - 19x + 20| < 2x + 2$$

(6)



3.

$$y = \sqrt{8 + e^x}, \quad x \in \mathbb{R}$$

Find the series expansion for y in ascending powers of x , up to and including the term in x^2 , giving each coefficient in its simplest form.

(8)



4. (a) Use de Moivre's theorem to show that

$$\cos 6\theta = 32\cos^6 \theta - 48\cos^4 \theta + 18\cos^2 \theta - 1 \quad (5)$$

(b) Hence solve for $0 \leq \theta \leq \frac{\pi}{2}$

$$64\cos^6 \theta - 96\cos^4 \theta + 36\cos^2 \theta - 3 = 0$$

giving your answers as exact multiples of π . (5)



6. The transformation T from the z -plane, where $z = x + iy$, to the w -plane, where $w = u + iv$, is given by

$$w = \frac{4(1-i)z - 8i}{2(-1+i)z - i}, \quad z \neq \frac{1}{4} - \frac{1}{4}i$$

The transformation T maps the points on the line l with equation $y = x$ in the z -plane to a circle C in the w -plane.

- (a) Show that

$$w = \frac{ax^2 + bxi + c}{16x^2 + 1}$$

where a , b and c are real constants to be found.

(6)

- (b) Hence show that the circle C has equation

$$(u - 3)^2 + v^2 = k^2$$

where k is a constant to be found.

(4)



Question 6 continued

Lined area for writing the answer to Question 6.



Question 7 continued

Handwritten response area for Question 7, consisting of 25 horizontal lines.



8.

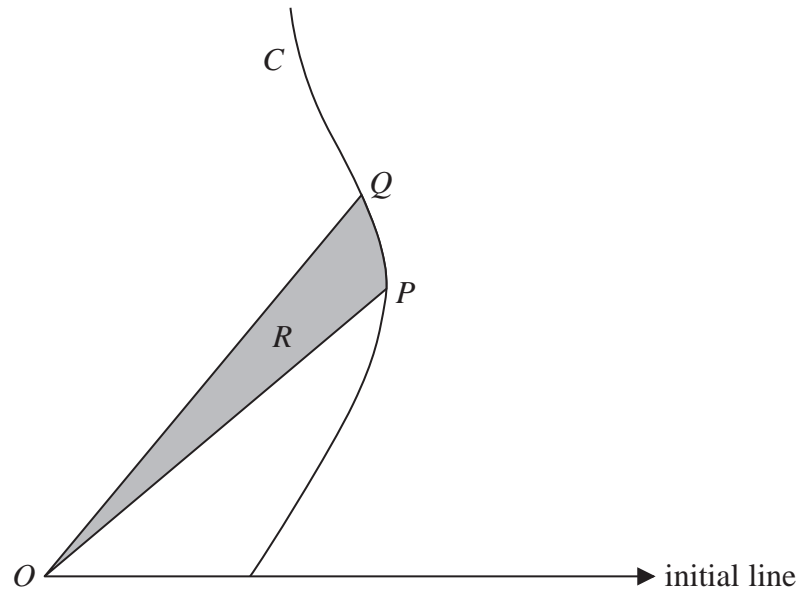


Figure 1

Figure 1 shows a sketch of part of the curve C with polar equation

$$r = 1 + \tan \theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The tangent to the curve C at the point P is perpendicular to the initial line.

(a) Find the polar coordinates of the point P .

(5)

The point Q lies on the curve C , where $\theta = \frac{\pi}{3}$

The shaded region R is bounded by OP , OQ and the curve C , as shown in Figure 1

(b) Find the exact area of R , giving your answer in the form

$$\frac{1}{2} (\ln p + \sqrt{q} + r)$$

where p , q and r are integers to be found.

(7)



