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**MATHEMATICS**

**9709/32**

Paper 3 Pure Mathematics

**March 2017**

MARK SCHEME

Maximum Mark: 75

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**Published**

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## **Mark Scheme Notes**

Marks are of the following three types:

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol  $\nabla$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
  - Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking  $g$  equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

- AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
- AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
- CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
- CWO Correct Working Only – often written by a ‘fortuitous’ answer
- ISW Ignore Subsequent Working
- SOI Seen or implied
- SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

### **Penalties**

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Remove logarithm and obtain $1 + 2^x = e^2$	<b>B1</b>
	Use correct method to solve an equation of the form $2^x = a$ , where $a > 0$	<b>M1</b>
	Obtain answer $x = 2.676$	<b>A1</b>
	<b>Total:</b>	<b>3</b>

Question	Answer	Marks
2	<i>EITHER:</i>	
	State or imply non-modular inequality $(x - 4)^2 < (2(3x + 1))^2$ , or corresponding quadratic equation, or pair of linear equations $x - 4 = \pm 2(3x + 1)$	<b>(B1)</b>
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for $x$	<b>M1</b>
	Obtain critical values $x = -\frac{6}{5}$ and $x = \frac{2}{7}$	<b>A1</b>
	State final answer $x < -\frac{6}{5}$ , $x > \frac{2}{7}$	<b>A1)</b>
	<i>OR:</i>	
	Obtain critical value $x = -\frac{6}{5}$ from a graphical method, or by inspection, or by solving a linear equation or inequality	<b>(B1)</b>
	Obtain critical value $x = \frac{2}{7}$ similarly	<b>B2</b>
	State final answer $x < -\frac{6}{5}$ , $x > \frac{2}{7}$	<b>B1)</b>
<b>Total:</b>	<b>4</b>	

Question	Answer	Marks
3(i)	Sketch a relevant graph, e.g. $y = e^{-\frac{1}{2}x}$	<b>B1</b>
	Sketch a second relevant graph, e.g. $y = 4 - x^2$ , and justify the given statement	<b>B1</b>
	<b>Total:</b>	<b>2</b>
3(ii)	Calculate the value of a relevant expression or values of a pair of expressions at $x = -1$ and $x = -1.5$	<b>M1</b>
	complete the argument correctly with correct calculated values	<b>A1</b>
	<b>Total:</b>	<b>2</b>

Question	Answer	Marks
3(iii)	Use the iterative formula correctly at least once	<b>M1</b>
	Obtain final answer – 1.41	<b>A1</b>
	Show sufficient iterations to 4 d.p. to justify – 1.41 to 2 d.p., or show there is a sign change in the interval (– 1.415, – 1.405)	<b>A1</b>
	<b>Total:</b>	<b>3</b>

Question	Answer	Marks
4(i)	State $R = 17$	<b>B1</b>
	Use trig formula to find $\alpha$	<b>M1</b>
	Obtain $\alpha = 61.93^\circ$ with no errors seen	<b>A1</b>
	<b>Total:</b>	<b>3</b>
4(ii)	Evaluate $\cos^{-1}(4/17)$ to at least 1d.p. ( $76.39^\circ$ to 2 d.p.)	<b>B1</b>
	Use a correct method to find a value of $x$ in the interval $0^\circ < x < 180^\circ$	<b>M1</b>
	Obtain answer, e.g. $x = 7.2^\circ$	<b>A1</b>
	Obtain second answer, e.g. $x = 110.8^\circ$ and no others	<b>A1</b>
	[Ignore answers outside the given interval.]	
	[Treat answers in radians as a misread.]	
	<b>Total:</b>	<b>4</b>

Question	Answer	Marks
5	Use product rule	<b>M1</b>
	Obtain correct derivative in any form	<b>A1</b>
	Equate derivative to zero, use Pythagoras and obtain a quadratic equation in $\tan x$	<b>M1</b>
	Obtain $\tan^2 x - a \tan x + 1 = 0$ , or equivalent	<b>A1</b>
	Use the condition for a quadratic to have only one root	<b>M1</b>
	Obtain answer $a = 2$	<b>A1</b>
	Obtain answer $x = \frac{1}{4}\pi$	<b>A1</b>
	<b>Total:</b>	<b>7</b>

Question	Answer	Marks
6(i)	Verify that the point with position vector $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ lies in the plane	<b>B1</b>
	<i>EITHER:</i>	
	Find a second point on $l$ and substitute its coordinates in the equation of $p$	<b>(M1</b>
	Verify that the second point, e.g. $(3, 1, -2)$ , lies in the plane	<b>A1)</b>
	<i>OR:</i>	
	Expand scalar product of a normal to $p$ and the direction vector of $l$	<b>(M1</b>
	Verify scalar product is zero	<b>A1)</b>
	<b>Total:</b>	<b>3</b>

Question	Answer	Marks
6(ii)	<i>EITHER:</i>	
	Use scalar product to obtain a relevant equation in $a$ , $b$ and $c$ , e.g. $2a - b + c = 0$	<b>(B1)</b>
	Obtain a second relevant equation, e.g. $3a + b - 5c = 0$ , and solve for one ratio e.g. $a : b$	<b>M1</b>
	Obtain $a : b : c = 4 : 13 : 5$ , or equivalent	<b>A1</b>
	Substitute $(3, -1, 2)$ and the values of $a$ , $b$ and $c$ in the general equation and find $d$	<b>M1</b>
	Obtain answer $4x + 13y + 5z = 9$ , or equivalent	<b>(A1)</b>
	<i>OR1:</i>	
	Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} - \mathbf{j} + \mathbf{k}) \times (3\mathbf{i} + \mathbf{j} - 5\mathbf{k})$	<b>(M1)</b>
	Obtain two correct components	<b>A1</b>
	Obtain correct answer, e.g. $4\mathbf{i} + 13\mathbf{j} + 5\mathbf{k}$	<b>A1</b>
	Substitute $(3, -1, 2)$ in $4x + 13y + 5z = d$ , or equivalent, and find $d$	<b>M1</b>
	Obtain answer $4x + 13y + 5z = 9$ , or equivalent	<b>(A1)</b>
	<i>OR2:</i>	
	Using the relevant point and relevant vectors form a 2-parameter equation for the plane	<b>(M1)</b>
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(3\mathbf{i} + \mathbf{j} - 5\mathbf{k})$	<b>A1</b>
	State three correct equations in $x$ , $y$ , $z$ , $\lambda$ and $\mu$	<b>A1</b>
	Eliminate $\lambda$ and $\mu$	<b>M1</b>
	Obtain answer $4x + 13y + 5z = 9$ , or equivalent	<b>(A1)</b>
	<i>OR3:</i>	
	Using the relevant point and relevant vectors form a determinant equation for the plane	<b>(M1)</b>
State a correct equation, e.g. $\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & -1 & 1 \\ 3 & 1 & -5 \end{vmatrix} = 0$	<b>A1</b>	
Attempt to expand the determinant	<b>M1</b>	
Obtain or imply two correct cofactors	<b>A1</b>	

Question	Answer	Marks
	Obtain answer $4x + 13y + 5z = 9$ , or equivalent	A1)
	<b>Total:</b>	<b>5</b>



Question	Answer	Marks
7(i)	State or imply $\frac{dV}{dt} = 2\frac{dh}{dt}$	<b>B1</b>
	State or imply $\frac{dV}{dt} = 1 - 0.2\sqrt{h}$	<b>B1</b>
	Obtain the given answer correctly	<b>B1</b>
	<b>Total:</b>	<b>3</b>
7(ii)	State or imply $du = -\frac{1}{2\sqrt{h}} dh$ , or equivalent	<b>B1</b>
	Substitute for $h$ and $dh$ throughout	<b>M1</b>
	Obtain $T = \int_3^5 \frac{20(5-u)}{u} du$ , or equivalent	<b>A1</b>
	Integrate and obtain terms $100 \ln u - 20u$ , or equivalent	<b>A1</b>
	Substitute limits $u = 3$ and $u = 5$ correctly	<b>M1</b>
	Obtain answer 11.1, with no errors seen	<b>A1</b>
	<b>Total:</b>	<b>6</b>

Question	Answer	Marks
8(i)	Substitute $z = -1 + i$ and attempt expansions of the $z^2$ and $z^4$ terms	M1
	Use $i^2 = -1$ at least once	M1
	Complete the verification correctly	A1
	<b>Total:</b>	<b>3</b>
8(ii)	State second root $z = -1 - i$	B1
	Carry out a complete method for finding a quadratic factor with zeros $-1 + i$ and $-1 - i$	M1
	Obtain $z^2 + 2z + 2$ , or equivalent	A1
	Attempt division of $p(z)$ by $z^2 + 2z + 2$ and reach a partial quotient $z^2 + kz$	M1
	Obtain quadratic factor $z^2 - 2z + 5$	A1
	Solve 3-term quadratic and use $i^2 = -1$	M1
	Obtain roots $1 + 2i$ and $1 - 2i$	A1
<b>Total:</b>	<b>7</b>	

Question	Answer	Marks
9(i)	State or imply the form $\frac{A}{2+x} + \frac{Bx+C}{4+x^2}$	B1
	Use a relevant method to determine a constant	M1
	Obtain one of the values $A = -2, B = 1, C = 4$	A1
	Obtain a second value	A1
	Obtain the third value	A1
	<b>Total:</b>	<b>5</b>

Question	Answer	Marks
9(ii)	Use correct method to obtain the first two terms of the expansion of $(1 + \frac{1}{2}x)^{-1}$ , $(2 + x)^{-1}$ , $(1 + \frac{1}{4}x^2)^{-1}$ or $(4 + x^2)^{-1}$	<b>M1</b>
	Obtain correct unsimplified expansions up to the term in $x^2$ of each partial fraction	<b>A1<sup>✓</sup> + A1<sup>✓</sup></b>
	Multiply out up to the term in $x^2$ by $Bx + C$ , where $BC \neq 0$	<b>M1</b>
	Obtain final answer $\frac{3}{4}x - \frac{1}{2}x^2$	<b>A1</b>
	[Symbolic binomial coefficients, e.g. ${}_{-1}C_2$ , are not sufficient for the first M1. The f.t. is on $A, B, C$ .]	
	[In the case of an attempt to expand $x(6 - x)(2 + x)^{-1}(4 + x^2)^{-1}$ , give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]	
	<b>Total:</b>	<b>5</b>

Question	Answer	Marks
10(i)	State or imply derivative is $2\frac{\ln x}{x}$	<b>B1</b>
	State or imply gradient of the normal at $x = e$ is $-\frac{1}{2}e$ , or equivalent	<b>B1</b>
	Carry out a complete method for finding the $x$ -coordinate of $Q$	<b>M1</b>
	Obtain answer $x = e + \frac{2}{e}$ , or exact equivalent	<b>A1</b>
	<b>Total:</b>	<b>4</b>
10(ii)	Justify the given statement by integration or by differentiation	<b>B1</b>
	<b>Total:</b>	<b>1</b>
10(iii)	Integrate by parts and reach $ax(\ln x)^2 + b \int x \cdot \frac{\ln x}{x} dx$	<b>M1*</b>
	Complete the integration and obtain $x(\ln x)^2 - 2x \ln x + 2x$ , or equivalent	<b>A1</b>
	Use limits $x = 1$ and $x = e$ correctly, having integrated twice	<b>DM1</b>
	Obtain exact value $e - 2$	<b>A1</b>
	Use $x$ -coordinate of $Q$ found in part (i) and obtain final answer $e - 2 + \frac{1}{e}$	<b>B1<sup>✓</sup></b>
	<b>Total:</b>	<b>5</b>