

GCE

Mathematics

Advanced GCE

Unit 4736: Decision Mathematics 1

Mark Scheme for June 2012

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

© OCR 2012

Any enquiries about publications should be addressed to:

OCR Publications PO Box 5050 Annesley NOTTINGHAM NG15 0DL

Telephone: 0870 770 6622 Facsimile: 01223 552610

E-mail: publications@ocr.org.uk

Annotations and abbreviations

Annotation in scoris	Meaning	
✓and x		
BOD	Benefit of doubt	
FT	Follow through	
ISW	Ignore subsequent working	
M0, M1	Method mark awarded 0, 1	
A0, A1	Accuracy mark awarded 0, 1	
B0, B1	Independent mark awarded 0, 1	
SC	Special case	
٨	Omission sign	
MR	Misread	
Highlighting		
Other abbreviations in mark scheme	Meaning	
M1 dep*	Method mark dependent on a previous mark, indicated by *	
cao	Correct answer only	
oe	Or equivalent	
rot	Rounded or truncated	
soi	Seen or implied	
www	Without wrong working	

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

М

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

В

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

4736 Mark Scheme June 2012

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

	Question	Answer	Marks	Guidance	
1	(i)	A 22 B 60 15 18 20 C D 13 E	M1 A1	Correct graph (exactly this graph, with 10 arcs) (may draw AE so that it does not cross BD) All arcs correctly weighted Condone weights that are ambiguously placed	
1	(")	9 F	[2]	De controller de la con	
1	(ii)	$ \begin{array}{ c c c c c c } \hline 1 & 0 \\ \hline & A \\ \hline & 22 \\ \hline & B \\ \hline \end{array} $		Do not allow boxes interchanged condone extra temp labels so we do not need to ft from (i) explicitly	
		5 46 60 46 C 2 15 15 D 3 18 18 E	M1 A1 B1	Updating from 60 at <i>C</i> (60 as temp and a smaller value as temp or perm) Permanent label 46 at <i>C</i> (seen as perm, not implied) Permanents labels correct at <i>B</i> , <i>D</i> , <i>E</i> and <i>F</i> (seen as perm, not implied) (cao)	
		6 49 49 F	В1	Order of labelling correct at <u>all six</u> vertices, follow through their labels (permanent label, or smallest temporary label if no permanent label shown at a vertex)	
		A-E-F	B1 [5]	cao	
1	(iii)	$3 \times \left(\frac{4000}{200}\right)^2 = 3 \times 400 = 1200$	M1	Correct method, including squaring (may be implied from 1200 or from the final answer being 20, www)	$4000 \div 200 = 20$ only \Rightarrow M0
		or $4000^2 \times 0.000075 = 1200$ or equivalent		May use $\frac{3}{60} = 0.05$ and work in minutes	
		1200 seconds = 20 minutes	A1 [2]	20 (cao) www	

	Questio	n	Answer	Marks	Guidance
2	(i)	(a)	eg	B1	Arcs could cross over, but each vertex must have order 2 and the arcs form a Hamilton cycle
				[1]	
2	(i)	(b)	eg	B1	A simply connected graph with one vertex of order 4, two of order 2 and two of order 1
2	(i)	(c)	eg or	B1	A simply connected graph with one vertex of order 3, three of order 2 and one of order 1

	Questio	n	Answer	Marks	Guidance	
2	(ii)	(a)	C1			
			D2 C2	M1	At least six arcs correct, even with extras	
			C3 S2 C4	A1	cao (arcs may be drawn so that they do not cross)	
			S1 M1	[2]		
2	(ii)	(b)	C1, C4, M1, M2 and D1 in one session	[2] M1	Two sessions, one with C1 and the other with C2,	
4	(11)	(0)	C2, C3, S1, S2 and D2 in the other	IVII	neither has both C1 and C2	
			C2, C3, S1, S2 and D2 in the other	A1	cao	
				[2]	cuo	
2	(ii)	(c)	M1 and D1 would need to be in different sessions and S2 cannot be in the same session as either of these	M1	A reasonable attempt, involving at least two of D1, M1 and S2 Condone 'M1 and D1 are in the same session' for M1 A wholly convincing attempt, explicitly involving all three of D1, M1 and S2	'M1 and D1 in same session so cannot be done by student at same time' is not enough for the A mark
				[2]	Condone 'D1, M1 and S2 would form a cycle'	

	Question	Answer	Marks	Guidance	
3	(i)	$3x + 2y \ge 6$ or $y \ge 3 - 1.5x$ o.e. $x + y \le 4$ or $y \le 4 - x$ o.e. $x \le 3y$ and $y \le 2x$	M1 M1 M1	Check these inputs, tolerance \pm 0.1 on y values $x = 2 \Rightarrow y = 0$, $x = 0 \Rightarrow y = 3$ $x = 3 \Rightarrow y = 1$, $x = 1 \Rightarrow y = 3$ $x = 0 \Rightarrow y = 0$, $x = 3 \Rightarrow y = 1$	For M marks only need boundary lines correct, $(= < \le > \text{ or } \ge)$
		or $y \ge \frac{1}{3}x$ and $y \le 2x$ o.e.	A1 [4]	and $x = 0 \Rightarrow y = 0$, $x = 1 \Rightarrow y = 2$ All <u>inequalities</u> correct, and no extras	Only check inequalities if all three M marks achieved
3	(ii)	$y = \frac{1}{3}x$ meets $y = 4 - x$ at (3, 1)	B1	(3, 1) cao	In (ii), condone ' $x =, y =$
		$y = 2x \text{ meets } y = 4 - x \text{ at } (\frac{4}{3}, \frac{8}{3})$	B1	$(\frac{4}{3}, \frac{8}{3})$ o.e. (cao), in fractions	' instead of coordinates
		$y = \frac{1}{3}x$ meets $y = 3 - 1.5x$ at $(\frac{18}{11}, \frac{6}{11})$	M1	sca <u>using simultaneous equations</u> to find points near both (1.6, 0.5) <u>and</u> (0.9, 1.7) as decimals or fractions (± 0.1 on each coordinate), but not just reading from graph	M1 may be implied from either answer correct as fractions, even if second is wrong or not attempted
		$y = 2x$ meets $y = 3 - 1.5x$ at $(\frac{6}{7}, \frac{12}{7})$	A1	Both $(1\frac{7}{11}, \frac{6}{11})$ o.e. <u>and</u> $(\frac{6}{7}, \frac{12}{7})$ o.e. (cao) as <u>frac</u>	etions etions
			[4]		
3	(iii)	$(3, 1) \Rightarrow P = 7$ $(\frac{4}{3}, \frac{8}{3}) \Rightarrow P = 12$	M1	Calculating $P = x + 4y$ correctly at two of their vertices (vertices may be implied)	
		$(\frac{18}{11}, \frac{6}{11}) \Rightarrow P = 3\frac{9}{11} (3.82) (\frac{42}{11})$ $(\frac{6}{7}, \frac{12}{7}) \Rightarrow P = 7\frac{5}{7} (7.71) (\frac{54}{7})$	A1	All four correct <i>P</i> values shown, as fractions $\underline{\text{or}}$ to ≥ 2 dp (cao)	
		Optimum point is $(\frac{4}{3}, \frac{8}{3})$ with $P = 12$	B1 [3]	$(\frac{4}{3}, \frac{8}{3})$ and 12 (written down) Condone (1.3 to 1.4, 2.6 to 2.7) 11.75 to 12.25	Optimal point in (x, y) form and 12 (within tolerance) written
3	(iv)	(2, 2) with $P = 10$	M1	x = 2, $y = 2$ and 10 (written down, whether identified as optimum point and value or not)	
		The only other integer-valued feasible points are $(1, 2), (2, 1)$ and $(3, 1)$ (giving $P = 9$, 6 and 7, respectively, but the P values need not be seen)	A1 [2]	(2, 2) and 10 identified as optimum and a valid explanation that recognises the four integer-valued points that fall in the feasible region	(2, 2) in (x, y) form, could 'identify' using an arrow or star

	Questio	n	Answer	Marks	Guidance	
4	(i)		3x - 4y + z + s = 12 $6x + 2z + t = 20$ $-10x - 5y + 5z + u = 30$	B1	Correct use of slack variables (s , t and u may be in any order) May scale through (e.g. $-2x - y + z + u = 6$)	Must be +s etc. constraints correct, must have =
			$s \ge 0, t \ge 0, u \ge 0$	B1 [2]	s, t and u non-negative (s, t, u 'positive' \Rightarrow B0) Condone $0 \le s \le 12$ etc.	non-negative integers \Rightarrow B0
4	(ii)		P x y z s t u RHS 1 5 6 -4 0 0 0 0 0 3 -4 1 1 0 0 12 0 6 0 2 0 1 0 20 0 -10 -5 5 0 0 1 30	M1 A1 [2]	Order of rows and columns may be changed 4 × 8 table of values (ignore ratio column if shown) and (their) constraint rows correct Objective row correct Assume blanks in initial tableau mean zero	
4	(iii)		(z is the) only column with a negative in objective row $12 \div 1 = 12$ $20 \div 2 = 10$ $30 \div 5 = 6$	B1	x and y have a positive value in the obj row \Rightarrow B1 Need a negative in obj row therefore z column \Rightarrow B1 But ' z most negative in objective row' \Rightarrow B0 ' z is negative in objective row' \Rightarrow B0 -5, -6, 4 and (z or 4 is) positive \Rightarrow B0 For the constraint rows, calculate the ratios of the entry in the column for RHS \div entry in (their) pivot column and choose the row with the smallest positive ratio	Need to refer to objective (top) row Do not credit if this appears in part (iv)
			6 is the least positive ratio so pivot on 5 (third constraint, as given in the question)	[2]	(allow sight of 12, 10, 6 and 'smallest' as minimum answer) 12, 10, 6 and an arrow pointing at 6 gets B0	

	Question	Answer	Marks	Guidance	
4	(iv)	New pivot row = row $4 \div 5$ New row $1 = \text{row } 1 + 4 \times \text{new pivot row}$ New row $2 = \text{row } 2 - \text{new pivot row}$ New row $3 = \text{row } 3 - 2 \times \text{new pivot row}$ $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	B1 M1 A1 [3]	Correct calculations seen for all four rows, including pivot (accept reasonable abbreviations, eg \div 5, $+$ 4×pr, but not just + 4) Pivot row may be shown as individual calculations Some candidates rewrite initial tableau first Some candidates use an interim tableau with new pivot row followed by the full augmented tableau A new tableau with 4 basis columns (or 3 with P missing) in which P has not decreased and all entries in RHS column are ≥ 0 cao (rows and columns may be in any order, condone omission of P column) Follow through scaled constraints, but not errors, from p	Any column all 0's ⇒ M0 art (i)
4	(v)	Pivot on 10 in x column $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	M1 A1 B1 [3]	A new tableau with 4 basis columns (or 3 with P missing) in which P has not decreased and all entries in RHS column are ≥ 0 cao (rows and columns may be in any order, condone omission of P column) Follow through scaled constraints, but not errors, from part (i) Strict ft their final tableau for non-negative x , y , z and positive P (not P as zero)	Any column all 0's ⇒ M0
4	(vi)	s = 2, $t = 0$, $u = 03x - 4y + z = 10$ which is 2 short of 12 (6x + 2z = 20 and -10x - 5y + 5z = 30)	B1 B1	Strict ft their final tableau for non-negative <i>s</i> , <i>t</i> , <i>u</i> (all three written down, do not imply blank = 0) Showing what (their) positive value(s) of slack variable(s) mean for the constraint(s)	s, t, u all zero (or any of them negative) \Rightarrow B0 for explanation

S (i) Odd vertices: M , N , R , W M		Question	Answer	Marks	Guidance	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	5	(i)	Odd vertices: M, N, R, W	M1		
$RW = \frac{160}{230} NW = \frac{150}{340} NR = \frac{130}{220}$ $Total distance = 1200 + 220 = 1420 \text{ metres}$ $S = \frac{130}{13} 1420, \text{ condone no units (cao)}$ $S = \frac{130}{1420} 1420, \text{ condone no units (cao)}$ $S = \frac{130}{1420} 1420, \text{ condone no units (cao)}$ $S = \frac{1200}{1420} 1420, \text{ condone no units (cao)}$ $S = \frac{120}{1420} 1420, condone no units (cao)$			MN = 70 $MR = 190$ $MW = 90$			
Total distance = $1200 + 220 = 1420$ metres B1 [3] 1420, condone no units (cao) Just repeat $MN = 70$ Total distance = $1200 + 70 = 1270$ metres R and W are still odd so these must be the start and finish vertices M1 A1 Rand W (from 1270 seen) M1 $M - S - W - P - N - V - R - T - M$ $60 + 30 + 170 + 40 + 80 + 50 + 40 + 190$ 660 metres Upper bound M1 $M - S - W - P - N - V$ (not a diagram or table) Tour (cycle) listed (cao) with M at each end 660, condone no units (cao) (not 600 , which uses $WP = 110$, from table) length \leq (their) 660 (but $< \Rightarrow$ B0) Tour which uses $WP = 110$, from table) Tour cycle) listed (cao) with M at each end M and M are an M and M are an M at each end M and M are an M are an M and M are an M are an M and M are an M are an M and M are an M and M are an M are an M are an M are an M and M are an M a				A1	All three pairings seen with correct total weights (cao)	
S (ii) Just repeat $MN = 70$ Total distance = $1200 + 70 = 1270$ metres R and W are still odd so these must be the start and finish vertices [2]						
Solution Just repeat $MN = 70$ Total distance = 1200 + 70 = 1270 metres R and W are still odd so these must be the start and finish vertices Interpretation Interpr			Total distance = $1200 + 220 = 1420$ metres		1420, condone no units (cao)	
Total distance = $1200 + 70 = 1270$ metres R and W are still odd so these must be the start and finish vertices A1	5	(ii)	Just repeat MN – 70	[3]		
				M1	1270, condone no units (cao)	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$						
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			finish vertices			
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	_	(111)				
	5	(111)			`	
Upper bound B1						
Upper bound B1 length \leq (their) 660 (but $\langle \Rightarrow B0 \rangle$) 5 (iv) P N P N P N P N S R B1 Correct tree Allow SV used instead of NV Vertices may be shown in different positions to these T V B1 340 (cao) Vertices may be shown in different positions to these [2] 5 (v) Lower bound = $340 + 30 + 90$ M1 Their $340 + 30 + $ either 90 or 170				D1		
5 (iv) P N P N			Upper bound	B1		
Positions to these Solvy B1 Correct tree Allow SV used instead of NV Nertices may be shown in different positions to these $T = V = T = T = T = T = T = T = T = T = $				[4]		
Weight = 340 [2] 5 (v) Lower bound = 340 + 30 + 90 M1 Their 340 + 30 + either 90 or 170	5	(iv)	or S R	B1		shown in different
[2] 5 (v) Lower bound = 340 + 30 + 90 M1 Their 340 + 30 + either 90 or 170			T V T V	B1	340 (cao)	
5 (v) Lower bound = 340 + 30 + 90 M1 Their 340 + 30 + either 90 or 170			Weight = 340			
			Y 1 1 240 20 00		TTI : 0.40 00 14 00 150	
May be implied from 460 as final answer	5	(v)	Lower bound = $340 + 30 + 90$	MI		
= 460 metres A1 460, condone no units (cao)			= 460 metres	A1		
[2]			- Too metes		roo, condone no unito (cuo)	

4736 Mark Scheme June 2012

	Question	Answer	Marks	Guidance	
6	(i)	A = -4, B = 5, C = 1, X = 0 Y = 1, Z = 5 W = -0.2 X = -0.2 Y = -0.168, Z = 6.72 W = -0.175	B1 M1 A1 M1	Initial values for A , B , C and X Y = 1, $Z = 5W = -0.2$ (exact, not rounded) Updating Y , Z , $W(showing new values, even if numerically wrong) W = -0.175 (-7/40) (exact, not rounded) www$	Note: if initial value of $Z = 0$ then first value of W occurs when Z first becomes non-zero and the next value in the first undetermined.
6	(ii)	Output -0.2 $A = -4, B = 5, C = 1, X = 1$	B1 [6]	-0.2 stated (as final <i>X</i> or as output)	is the first update
6	(II)	A = -4, $B = 3$, $C = 1$, $X = 1Y = 3$, $Z = 0X$ is then reduced to 0 and the algorithm is as (i) leading to an answer of -0.2	M1 A1	Y = 3, $Z = 0Describing what then happens ('X = 0, then as above', even if part (i) did not lead to -0.2. If values from (i) are written out must get to a second value of W).$	
6	(iii)	A = -4, B = 5, C = 1, X = -1 Y = -9, Z = 16 W = -0.4375, X = -0.4375 Y = -2.04, Z = 9.07 W = -0.213, X = -0.213	M1 A1 M1	Y = -9, $Z = 16W = -0.4375$ (-7/16) (allow -0.437 or -0.438, but not values that round to these) Updating Y , Z and W first time (showing new values, even if numerically wrong)	Note: if initial value of $Z = 0$ see comment above
		Y = -0.256, Z = 6.84 W = -0.176 Root is approximately -0.213	M1 dep	Updating <i>Y</i> , <i>Z</i> and <i>W</i> second time (showing new values, even if numerically wrong) Stopping at -0.213 (or anything that rounds to this)	Dependent on previous M1 Dependent on
			[5]	without wrong working seen	both method marks
6	(iv)	May not converge May not terminate May get stuck in a loop	B1 [1]	Allow 'could take a long time', 'takes a long time' 'does not stop', 'gets stuck in a loop' Any one of these, or similar	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning

Telephone: 01223 553998 Facsimile: 01223 552627

Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations is a Company Limited by Guarantee Registered in England Registered Office; 1 Hills Road, Cambridge, CB1 2EU Registered Company Number: 3484466 OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations) Head office

Telephone: 01223 552552 Facsimile: 01223 552553



