

Pure Core 4 Past Paper Questions: Mark Scheme

Taken from MAP2, MAP3

Pure 2 June 2001

6	(a)	$\cos 2x = \cos^2 x - \sin^2 x$ $= (\cos x - \sin x)(\cos x + \sin x)$ Hence result	B1 M1 A1	3	Difference of two squares
	(b)	$R = \sqrt{2}, \quad a = 45^\circ$ $\left. \begin{aligned} \sqrt{2} \sin(x + 45) &= \frac{1}{2} \\ x &= 114^\circ \\ x &= 336^\circ \end{aligned} \right\}$	B1B1 M1 A1√ A1√		
Total				8	

Pure 2 January 2002

4	(a)	$\frac{2 \tan x}{1 + \tan^2 x} = \frac{2 \sin x}{\cos x \sec^2 x}$ $= \frac{2 \sin x \cos^2 x}{\cos x}$ $= 2 \sin x \cos x$ $= \sin 2x$	B1 M1 A1 A1	4	Use of $\sec^2 x$ identity reduction to sin/cos only AG
	(b)	$\sin 30^\circ = \frac{1}{2}$ $= \frac{2t}{1+t^2}$ $\left. \begin{aligned} t^2 - 4t + 1 &= 0 \\ t &= \frac{4 \pm \sqrt{16-4}}{2} \\ &= 2 \pm \sqrt{3} \end{aligned} \right\}$ $a = 2, \quad b = -1$	B1 M1 M1 A1 A1		
Total				9	

Q	Solution	Marks	Total	Comments
5 (a)	$\cos \alpha = -\frac{5}{13}$	M1A1	2	$\cos \alpha = \frac{5}{13}$ M0A0 unless from $s^2 + c^2 = 1$ in which case M1A0
(b)	$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$ $= \left(\frac{12}{13} \times \frac{4}{5}\right) - \left(\frac{5}{13} \times \frac{3}{5}\right)$ $= \frac{33}{65}$	M1 A1F A1F	3	use of f.t. $\cos \alpha$ from (a) Non-exact value gets M1A0A0 or possibly M1A1A0
Total			5	

Pure 2 June 2002

8	(a)	Use of an appropriate identity Simplify/cancel to AG	B1 B2	3	
	(b)	$\cos^2 \theta = 2 \sin 2\theta$ $= 4 \sin \theta \cos \theta$ $\cos \theta (\cos \theta - 4 \sin \theta) = 0$ $\cos \theta = 0$ $\theta = 90, 270$ $\tan \theta = \frac{1}{4}$ $\theta = 14^\circ, 194^\circ$	B1 M1 A1A1 A1A1		
Total				9	

Pure 2 January 2003

4	(a)(i)	$L = 2 \sin \theta + 4 \cos \theta$	B1	1	Accept unsimplified
	(ii)	$R \sin(\theta + \alpha) = R(\sin \theta \cos \alpha + \cos \theta \sin \alpha)$ $R \cos \alpha = 2, R \sin \alpha = 4$ $R = \sqrt{20}, \alpha = 1.107$ (AWRT 1.11)	M1 A1F A1FA1F	4	Alternative $2 \sin \theta + 4 \cos \theta$ $= \sqrt{20} \left(\frac{2}{\sqrt{20}} \sin \theta + \frac{4}{\sqrt{20}} \cos \theta \right)$ M1A1F $= \sqrt{20} (\cos \alpha \sin \theta + \sin \alpha \cos \theta)$ A1F $= \sqrt{20} \sin(\theta + \alpha), \alpha = 1.107$ A1F For ft, must be in form $a \sin \theta + b \cos \theta, \alpha$ in radians
	(b)(i)	$L_{\max} = \sqrt{20}$ (4.47)	B1F	1	
	(ii)	Maximum when $\theta + \alpha = \frac{\pi}{2}$ $\theta \approx 0.46$	M1 A1F	2	Or $\theta + \alpha = 90^\circ$ CAO
Total				8	

Pure 2 June 2003

3	(a)	$\tan(45^\circ + \theta) = \frac{\tan 45^\circ + \tan \theta}{1 - \tan 45^\circ \tan \theta}$ $= \frac{1 + \tan \theta}{1 - \tan \theta}$	M1 A1	2	Use of correct formula for $\tan(A + B)$ Replace $\tan 45^\circ = 1$	
		(b)	Put $\theta = 60^\circ : \tan 105^\circ = \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$ $= \frac{(1 + \sqrt{3})^2}{(1 + \sqrt{3})(1 - \sqrt{3})}$ $= \frac{1 + 2\sqrt{3} + 3}{-2}$ $= -2 - \sqrt{3}$		M1 A1 M1 A1F	4
	Total				6	

Pure 2 January 2004

3	(a)	$\beta = \tan^{-1}(2.4) = 1.176^\circ$	B1	1	
	(b)	$10 \sin \theta + 24 \cos \theta \equiv R \sin(\theta + \alpha)$ $= R \sin \theta \cos \alpha + R \cos \theta \sin \alpha$ $R \sin \alpha = 24$ $R \cos \alpha = 10$ $\tan \alpha = 2.4 \quad \therefore \alpha = 1.176^\circ$ $R^2 = 24^2 + 10^2 = 676 \quad R = 26$ $\Rightarrow 26 \sin(\theta + 1.176)$	M1		Any correct attempt at finding R or α
			A1		Correct α (AWRT 1.18)
			A1		Correct R
				3	
	(c)(i)	Maximum value = 26	B1✓	1	On their answer to part (b) (± 26 gets B0) (based on a valid method used in (b))
	(ii)	$\sin(\theta + 1.176) = 1$ $\therefore \theta + 1.176 = \frac{\pi}{2}$ $\theta = 0.395^\circ$	M1		
			A1✓	2	On their value of α (6.68, 13.0,)
Total				7	

Pure 2 June 2004

2(a)	$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \dots (i)$ $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \dots (ii)$ add the two equations (i) & (ii) together $\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cos \beta$	M1			
		A1	2	AG	
(b)(i)	$2 \sin 8x \cos 2x = \sin(8x + 2x) + \sin(8x - 2x)$ $= \sin 10x + \sin 6x$	M1			
		A1	2		
(ii)	$\int 6 \sin 8x \cos 2x \, dx$ $= 3 \int (\sin 10x + \sin 6x) \, dx$ $= 3 \left(\frac{-\cos 10x}{10} - \frac{\cos 6x}{6} \right) + c$ $= -\frac{3}{10} \cos 10x - \frac{1}{2} \cos 6x + c$	M1ft		Use their (i)	
		M1ft		Integration attempted	
		A1ft	3	Any correct form	
Total			7		

Pure 3 June 2001

Q	Solution	Marks	Total	Comments
1 (a)		B1	1	
(b)	$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{8}{8t}$	M1 A1	2	Use of chain rule
(c)	$t=0 \Rightarrow x=1, y=4$ Gradient = $\frac{y-4}{x-1} = \frac{1}{0.5}$ $y=2x+2$	B1 M1 A1	3	Use of (1, 4) OE: e.g. $(y-4)=2(x-1)$
Total			6	

3 (a)	$\frac{dP}{dt}$ is rate of increase of population This is proportional to P ($\Rightarrow \frac{dP}{dt} = kP$)	B1 B1	2	
(b)(i)	$\frac{dP}{P} = k dt$ ($\ln P = kt + c$) $P = (e^{kt+c})$ [$= Ae^{kt}$] $t=0, A=1000$ $t=30, k = \frac{1}{30} \ln 2$	M1 A1 A1 A1	4	
(ii)	$1000e^{\frac{1}{30} \ln 2t} = 5000e^{-0.05t}$ $\frac{1}{30} \ln 2t + 0.05t = \ln 5$ $t=22$	M1 m1 A1 A1	4	Equate populations Take logarithms OE any correct expression
Total			10	

6 (a)	$\begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix} \cdot 4 \begin{bmatrix} 3 \\ 5 \end{bmatrix} (= -5)$ attempted $\cos \theta = \frac{-5}{\sqrt{14}\sqrt{50}}$ $\theta = 100.9^\circ \Rightarrow 79.1^\circ$ line and normal $\Rightarrow 10.9^\circ$ line and plane	M1 m1 B1 A1 B1√	5	$\sqrt{14}$ or $\sqrt{50}$ seen $90^\circ - \angle$ between line and normal
(b)	$\mathbf{AB} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$ Line l_2 is $\mathbf{r} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} + t \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$ $2+3s=3+2t$ $4s=-2+3t$ $s=7, t=10$ $4+3t=-1+5s=34$	B1 B1√ M1 A1√ A1	5	OE: ft on $\begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$ Set up and attempt to solve any two simultaneous equations ft on equations Check: 3 rd equation
Total			10	

7	(a)	$f(x) = \frac{A}{1+2x} + \frac{B}{4-x}$ $= \frac{2}{1+2x} + \frac{1}{4-x}$	B1 M1 A1	3	Any appropriate method
	(b)(i)	$\frac{1}{4-x} = \frac{1}{4} \left(1 - \frac{x}{4}\right)^{-1}$ $= \frac{1}{4} \left[1 + (-1) \left(-\frac{x}{4}\right) + \frac{(-1)(-2)}{2} \left(-\frac{x}{4}\right)^2 \right]$	B1 M1 A1	3	$4 \left(1 - \frac{x}{4}\right)$ AG
	(ii)	$\frac{1}{1+2x} = (1+2x)^{-1}$ $= \left[1 + (-1)(2x) + \frac{(-1)(-2)}{2} (2x)^2 \right]$ $= 1 - 2x + 4x^2$	M1 A1	2	
	(iii)	$f(x) = 2(1 - 2x + 4x^2) + \frac{1}{4} + \frac{x}{16} + \frac{x^2}{64}$ $= \frac{9}{4} - \frac{63}{16}x + \frac{513}{64}x^2$	M1 A1	2	Accept $2.25 - 3.94x + 8.02x^2$
	(iv)	$-4 < x < 4, \quad -\frac{1}{2} < x < \frac{1}{2}$ <p>valid for $-\frac{1}{2} < x < \frac{1}{2}$</p>	B1 B1	2	B2 for $-\frac{1}{2} < x < \frac{1}{2}$ stated
	(c)(i)	$\int f(x) dx = \ln 1+2x - \ln 4-x $	M1 A1	2	$k \ln 1+2x $ $l \ln 4-x $
	(ii)	$\int_0^{0.25} f(x) dx = [0.4055 - 1.3218] - [1.3863]$ $= 0.470$ $\left[\frac{9}{4}x - \frac{63}{16} \cdot \frac{x^2}{2} + \frac{513}{64} \cdot \frac{x^3}{3} \right]_0^{0.25}$ $= 0.481$ <p>Error = 0.011</p>	B1√ M1 A1 A1√	4	ft on $k \ln 1+2x + l \ln 4-x $ ft on difference between integrals
Total				18	

Pure 3 January 2002

2	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{-2}{t^2} \times \frac{1}{2}$ $t = 2 \quad \frac{dy}{dx} = \frac{-1}{4}$ <p>gradient of normal = 4 $y = 4x + c$ $t = 2, x = 7, y = 2$ $y = 4x - 27$</p> <p>Alternative Eliminate t $y = \frac{4}{x-3}, xy = 4 + 3y, x = \frac{4}{y} + 3$ $\frac{dy}{dx} = \frac{-4}{(x-3)^2} \times \frac{dy}{dx} + y = 3 \frac{dy}{dx} \frac{dx}{dy} = \frac{-4}{y^2}$ $t = 2 \quad x = 7 \quad y = 1 \quad \frac{dy}{dx} = \frac{-1}{4}$ gradient of normal = 4 $y = 4x + c$ $y = 4x - 27$</p>	M1 A1 B1F B1F M1 A1 (M1) (A1) (B1ft) (B1ft) (M1) (A1)	6	Use chain rule Substitute $t = 2$ in $\frac{dy}{dx}$ Follow on gradient Use (7,1) and gradient Attempt to differentiate correct expression Follow on $\frac{dy}{dx}$ Follow on gradient
Total			6	

Q	Solution	Marks	Total	Comments
4 (a)(i)	$P = 15000$	B1	1	
	(ii) $11000 = 15000 e^{-2k}$ $-2k = \ln\left(\frac{11}{15}\right)$ $k = 0.155$	M1 m1 A1	3	
(b)	$18000 e^{-0.175t} = 15000 e^{-kt}$	M1	4	OE AWRT 9.1; accept 9.5 Special case – use of trial values of t $t=9$ B2(Max 2/4)
	$1.2 = e^{0.02t}$	A1		
	$\ln 1.2 = 0.02t$	M1		
	$t = 9.1$ year = 2009	A1 (B2)		
Total			8	
7 (a)(i)	$\frac{dh}{dt} = \pm k\sqrt{h}$	M1 A1	2	$\frac{dh}{dt} = \dots \frac{dh}{dt} \propto \sqrt{h} \dots$
	(ii) $\int \frac{dh}{\sqrt{h}} = \int \pm k dt$ $2h^{1/2} = \pm kt + C$ At $t=0, h=1$ $C=2$ $2\sqrt{h} = 2 - kt$	M1 A1 A1	3	
(iii)	At $t=2, h=1/2$ $k = \frac{2 - 2\sqrt{1/2}}{2} = 0.293$	M1 A1	2	Use AG and solve for k
	(b) At $h=0, t = \frac{C}{k} = \frac{2}{0.293}$ $= 6.8$ hours = 6 hrs 50 mins	M1 A1	2	Use $h=0$ and solve for t Accept 410 minutes
Total			9	

Pure 3 June 2002

Q	Solution	Marks	Total	Comments
1 (a)	$\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = 2 \times \frac{-1}{2t}$	M1 A1	2	
	(b) $t=3$ gradient normal = 3 $t=3$ $x=-8$ $y=6$ $y=3x+c$ $y=3x+30$	B1ft B1 M1 A1	4	
Total			6	

<p>2 (a)</p>	$\frac{4-x}{(1-x)(2+x)} = \frac{A}{1-x} + \frac{B}{2+x}$ $4-x = A(2+x) + B(1-x)$	<p>M1</p>		
<p>(b)(i)</p>	$x=1 \quad A=1 \quad x=-2 \quad B=2$ $\frac{1}{2+x} = \frac{1}{2} \left(1 + \frac{x}{2}\right)^{-1}$ $= \frac{1}{2} \left(1 + -1 \times \frac{x}{2} + \frac{-1 \times -2}{2} \left(\frac{x}{2}\right)^2\right)$	<p>M1A1</p>	<p>3</p>	<p>Attempt to find A and B</p>
<p>(ii)</p>	$\frac{1}{1-x} = (1-x)^{-1}$ $= 1 + -1 \times (-x) + \frac{-1 \times -2}{2} (-x)^2$ $= 1 + x + x^2$	<p>M1</p>	<p>3</p>	<p>Use of binomial series $n = -1$ use $\frac{x}{2}$ AG convincingly obtained</p>
	<p>Alternative to part (b) by Maclaurin</p> $f(x) = (2+x)^{-1} \quad f(0) = \frac{1}{2}$ $f'(x) = -(2+x)^{-2} \quad f'(0) = -\frac{1}{4}$ $f''(x) = 2(2+x)^{-3} \quad f''(0) = \frac{2}{8}$ $f(x) = \frac{1}{2} - \frac{1}{4}x + \frac{1}{2} \times \frac{2}{8}x^2$ <p>OR</p> $(2+x)^{-1} = 2^{-1} + (-1) \times 2^{-2}x + \frac{-1 \times -2 \times 2^{-3}}{2!}x^2$	<p>(M1)</p>		<p>Differentiate twice</p>
		<p>(A1)</p>		
		<p>(A1)</p>	<p>2</p>	<p>AG obtained using $x = 0$, in Maclaurin's series</p>
		<p>(M1A1)</p>		<p>use negative powers of 2 all correct</p>
		<p>(A1)</p>		

Q	Solution	Marks	Total	Comments
<p>(c)</p>	$\frac{4-x}{(1-x)(2+x)} = 2 + \frac{x}{2} + \frac{5}{4}x^2$ <p>Alternative to part (c)</p> $(4-x) \left(\frac{1}{2} - \frac{x}{4} + \frac{x^2}{8} \right) (1+x+x^2) = a+bx$ <p>Correct expansion</p>	<p>M1A1</p>	<p>2</p>	<p>Ignore extra terms</p>
	<p style="text-align: right;">Total</p>		<p>10</p>	

3	(a)	$x = 2 \quad y = \pm \frac{5\sqrt{5}}{3} = \pm 3.73$	M1A1	2	allow ± 3.7 , or any correct numerical form		
	(b)	$\frac{d}{dx} \left(\frac{x^2}{9} + \frac{y^2}{25} \right) = \frac{d}{dx} (1)$	M1			attempt implicit differentiation LHS only, with use of chain rule.	
		$\frac{2x}{9} + \frac{2y}{25} \frac{dy}{dx} = 0$	A1			OE correct differentiation	
		$\frac{dy}{dx} = \pm \frac{2}{9} \times \frac{25}{2} \times \frac{3}{5} \times \frac{1}{\sqrt{5}} = \pm 1.5$	M1A1			4	substitute $x = 2$, and values for y . Accept $\pm 1.49\dots$
		Alternative to part (b) $y = 5\sqrt{1 - \frac{x^2}{9}}$ $\frac{dy}{dx} = 5 \times \frac{1}{2} \times -\frac{2}{9}x \left(1 - \frac{x^2}{9} \right)^{-\frac{1}{2}}$ $x = 2; y = \pm 3.73$	(M1) (m1) (A1) (A1)				differentiate a function of form $y = a\sqrt{c + bx^2}$ use chain rule $\frac{dy}{dx} = \pm 1.5$
Total				6			
4	(a)	$\frac{1}{2} m_0 = m_0 e^{-28k}$ $\frac{1}{2} = e^{-28k}$ $\ln \frac{1}{2} = -28k$ $k = 0.024755(256)$	M1 A1 M1 A1	4	Allow any value for m_0 Take lns of an exponential expression AG convincingly obtained NB sign $\frac{-\ln \frac{1}{2}}{28}$ SC – substitute $k = 0.024755$ into e^{-28k} explain why this shows mass is halved max 2/4		
	(b)	$1 = m e^{-100k}$ $m = 11.9 \text{ g}$ Alternative to part (b) $t = -100 \quad m = 1e^{100k} = 11.9$	M1 A1 (M1) (A1)		2	accept 11.89 $\frac{100}{28} = 3.57$ number of half lives $2^{3.57}$	
	Total					6	

Q	Solution	Marks	Total	Comments	
5	(a)	$\int y^2 dy = \int dx$ $\frac{1}{3} y^3 = x + c$	M1 A1A1	4	Separate; attempt to integrate both sides or $\frac{1}{3} y^3 + c = x$ A1A0 2 out of $\frac{1}{3} y^3, x, c$ Accept $3c$ for K . Use of $(1, -1)$ in an expression with a constant. Correct expression connecting y and x . Allow $K = -4$
	(b)	$y = \sqrt[3]{3x + K}$ $-1^3 = 3 \times 1 + K$	A1 M1		
		$y^3 = 3x - 4 \quad y = \sqrt[3]{3x - 4}$	A1		
	Total				

8 (a)	$3 + 4t = 8 - s$	M1		Set up and attempt to solve
	$-2 + 4t = -1 + 3s$			
	$t = 1 \quad s = 1$	m1A1		
	$1 + 3 \times 1 = 2 + 1 \times 2 = 4$	A1		Check third equation
(b)(i)	$(x, y, z) = (7, 2, 4)$	B1ft	5	ft on consistent use of s or t
	$4 \times 1 + 4 \times 11 + 3 \times -16 = 0$	M1		Use scalar product with a direction
	$-1 \times 1 + 3 \times 11 + 2 \times -16 = 0$	A1	2	Both equal zero

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1 (a)	$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{\frac{1}{2}(-\frac{1}{2})x^2}{2}$	M1		
	$= 1 + \frac{1}{2}x - \frac{1}{8}x^2$	A1	2	
(b)(i)	$\sqrt{4+2x} = 2\sqrt{1+\frac{x}{2}}$	B1		(b)(i) Special cases Allow A1F for $2 + \frac{x}{2} + \frac{x^2}{16}$ follow $1 + \frac{1}{2}x + \frac{1}{8}x^2$ or $4\left(1 + \frac{x}{4} - \frac{x^2}{32}\right) = 4 + x - \frac{x^2}{8}$
	$= 2\left(1 + \frac{1}{2}\left(\frac{x}{2}\right) - \frac{1}{8}\left(\frac{x}{2}\right)^2\right)$	M1		
	$= 2 + \frac{x}{2} - \frac{x^2}{16}$	A1	3	
	Alternative using $(a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)a^{n-2}}{2}x^2 \dots$ $(4+2x)^{\frac{1}{2}} = 4^{\frac{1}{2}} + \frac{1}{2}4^{-\frac{1}{2}}2x +$ $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}}{2}(2x)^2$ $= 2 + \frac{1}{2} \cdot \frac{1}{2} \cdot 2x - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{8} 4x^2$ $= 2 + \frac{1}{2}x - \frac{1}{16}x^2$	(M1) (A1) (A1)		M1 – use of $n = \frac{1}{2}; a = 4 \quad x \rightarrow 2x$ A1 – correct A1 – correct simplification
(ii)	$-2 < x < 2$	B1	1	
Total			6	

<p>2 (a)</p>	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ $= -\sin t \times \frac{1}{3\cos t}$ $t = \frac{\pi}{4} \quad \frac{dy}{dx} = -\frac{1}{3}$ <p>Alternative</p> $\frac{x^2}{9} + y^2 = 1 \quad y = \sqrt{1 - \frac{x^2}{9}}$ $\frac{dy}{dx} = \frac{1}{2} \left(1 - \frac{x^2}{9}\right)^{-\frac{1}{2}} \left(-\frac{2x}{9}\right)$ $t = \frac{\pi}{4} \quad x = \frac{3}{\sqrt{2}}$ $\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{1}{2}}} \cdot \left(\frac{-6}{9\sqrt{2}}\right) = -\frac{3}{9} = -\frac{1}{3}$	<p>M1</p> <p>A1</p> <p>B1</p> <p>(M1)</p> <p>(A1)</p> <p>(A1)</p>	<p>3</p> <p>4</p>	<p>Chain rule and derivatives attempted.</p> <p>For B1, substitution of $t = \frac{\pi}{4}$ into expression for $\frac{dy}{dx}$ seen.</p> <p>Allow</p> $-\frac{\sin \frac{\pi}{4}}{3\cos \frac{\pi}{4}} = -\frac{1}{3} \text{ or } -\frac{0.707}{2.121} = -\frac{1}{3}$ <p>AG</p> <p>allow 2.12, 0.71</p> <p>$\sqrt{2}$ OE numerical form Accept 1.4</p>
<p>(b)</p>	$t = \frac{\pi}{4} \quad x = \frac{3}{\sqrt{2}} \quad y = \frac{1}{\sqrt{2}}$ $y - \frac{1}{\sqrt{2}} = -\frac{1}{3} \left(x - \frac{3}{\sqrt{2}}\right)$ $y = -\frac{1}{3}x + \sqrt{2}$ <p>Alternative</p> $x = 3\sin \frac{\pi}{4} \quad y = \cos \frac{\pi}{4}$ $y - \cos \frac{\pi}{4} = -\frac{1}{3} \left(x - 3\sin \frac{\pi}{4}\right)$ $y = -\frac{1}{3}x + \sin \frac{\pi}{4} + \cos \frac{\pi}{4}$ $y = -\frac{1}{3}x + \sqrt{2}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>(M1)</p> <p>(M1)</p> <p>(A1)</p> <p>(A1)</p>	<p>7</p>	<p>Total</p>

Q	Solution	Marks	Total	Comments
3(a)(i)	$\frac{x^2}{x^2-16} = \frac{x^2-16+16}{x^2-16}$ <p>Accepted equivalents</p> $\frac{x^2}{x^2-16} = 1 + \frac{16}{x^2-16}$ $\Rightarrow x^2 = x^2 - 16 + 16$ $= x^2$ $\frac{x^2}{x^2-16} = A + \frac{B}{x^2-16}$ $\Rightarrow x^2 = A(x^2-16) + B$ $x = 4 \Rightarrow B = 16$ $\Rightarrow A = 1$ $\frac{x^2}{x^2-16} = 1 + \frac{A}{x^2-16}$ $\Rightarrow x^2 = x^2 - 16 + A$ $\Rightarrow A = 16$	B1	1	OE eg by division ; AG Use of a particular value of $x, x = 0, 1, 2, \dots$ showing LHS=RHS is B0 (see equivalents)
(ii)	$\frac{16}{x^2-16} = \frac{A}{x-4} + \frac{B}{x+4}$ $16 = A(x+4) + B(x-4)$ $x = 4 \Rightarrow A = 2$ $x = -4 \Rightarrow B = -2$	M1 A1	2	Any equivalent method
(b)	$\int_5^8 \left(1 + \frac{2}{x-4} - \frac{2}{x+4} \right) dx$ $= [x + 2\ln x-4 - 2\ln x+4]_5^8$ $= (8 + 2\ln 4 - 2\ln 12) - (5 + 2\ln 1 - 2\ln 9)$ $= 3 + 2\ln 3$	M1 A1 m1 A1	4	$x + k \ln(x-4) + l \ln(x+4)$ Allow both M1, m1 if $\int dx = x$ is omitted. ft on both A marks on values of A, B . Accept $3 + \ln 9$
Total			7	

7 (a)	$\frac{dx}{dt} = \frac{7}{14\,000x}$	B1	1	AG $\frac{7}{14\,000x}$ seen; ($-7 = 7$ not required)
(b)	$\int_2^3 2000x = \int_0^t 1 dt$ $\left[2000 \frac{x^2}{2} \right]_2^3 = [t]_0^t$ $2000 \left[\frac{9}{2} - \frac{4}{2} \right] = t$ $t = 5000 \text{ (sec)}$ $t = 1.39 \text{ hrs} \Rightarrow 1.23 \text{ pm}$	M1 A1 m1 A1 A1F	5	Attempt separation and integration or $1000x^2 = t + c$ $x = 2, t = 0 \Rightarrow c = 4000$ $x = 3 \Rightarrow t = 5000$ Accept 1 hour 23min (ignore seconds) ft on $0 < t < 20\,000$
Total			6	

Pure 3 June 2003

2	(a)	$\frac{dx}{dt} = 3, \frac{dy}{dt} = \frac{-1}{t^2}$	M1	2	Use chain rule
		$\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = \frac{-1}{3t^2}$	A1		
	(b)	$t = 1, \frac{dy}{dx} = \frac{-1}{3}$	B1F	4	Follow on gradient Use (2, 1) and gradient ft on gradient Accept $y - 1 = 3(x - 2)$
		Gradient of normal = 3	B1F		
		$y = 3x + c$	M1		
	$t = 1, x = 2, y = 1$	A1F			
	$y = 3x - 5$				
Total				6	

4	(a)	$A = 1000$	B1	1	Or ln used. $c = \sqrt[60]{\frac{12000}{A}}$, their A AG convincingly obtained SC: $1000 \times 1.0423^{60} = 12011$ (12.011 or AWRT seen or 12.011×1000 SC1) Attempt to take logs (log or ln seen) Correct use of log laws OE: $t = \frac{\log N - 3}{0.01798}$ etc. ft their A condone more figures if penalised in 3(b)
	(b)	$c^{60} = \frac{12000}{A}$	M1		
		$60 \log c = \log 12$ or $c = \sqrt[60]{12}$	m1	3	
		$c = 1.04228\dots$	A1		
	(c)(i)	$\log N = \log A c^t$	M1		
		$\log N = \log A + t \log c$	m1		
	$t = \frac{\log N - \log A}{\log c}$	A1F	3		
	(ii) $t = 167$ minutes	B1	1		
Total				8	

Pure 3 January 2004

Q	Solution	Marks	Total	Comments
1(a)(i)	$\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = 6 \cdot \frac{1}{6t}$	M1 A1	2	
(ii)	$t = \frac{1}{2}$ gradient = 2	B1✓	1	ft only on $\frac{dy}{dx} = f(t)$
(b)(i)	$t = \frac{y}{6} \quad x = 3\left(\frac{y}{6}\right)^2 = \left[\frac{y^2}{12}\right]$	M1A1	2	Accept $\frac{3y^2}{36}$ Use of tangent $y = 2x + \frac{3}{2}$ or $x = \frac{y}{2} - \frac{3}{4}$ instead of curve: no marks
(ii)	$\frac{dx}{dy} = \frac{2y}{12}$ $t = \frac{1}{2} \quad y = 3$	M1 B1		Alternative: $\frac{dx}{dy} = \frac{2y}{12}$ M1 $\frac{y}{6} = t$ B1
	$\frac{dx}{dy} = \frac{6}{12} \quad \frac{dy}{dx} = \frac{12}{6} = 2$	M1A1	4	$t = \frac{1}{2}; \frac{dx}{dy} = \frac{1}{2}$ M1 $\frac{dy}{dx} = 2$ A1
Total			9	

3 (a)(i)	$t = 0 \quad P = 50$	B1	1	
(ii)	$e^{\frac{-t}{4}} \rightarrow 0 \quad P \rightarrow 100$	B1	1	
(b)	$75 = 100 - 50e^{\frac{-t}{4}} \quad \frac{1}{2} = e^{\frac{-t}{4}}$	M1A1		Allow $\frac{25}{50}$ for $\frac{1}{2}$
	$\ln \frac{1}{2} = \frac{-t}{4} \quad t = 2.8$	M1A1	4	SC trial and improvement 2.8 4/4, 2.77 3/4 else 0
Total			6	

Q	Solution	Marks	Total	Comments
4 (a)	$8 + 3x = A(2 - x) + B(1 + 3x)$	M1	3	Any equivalent method
	$x = 2 \quad 14 = 7B \quad B = 2$	M1		
$x = \frac{-1}{3} \quad 7 = \frac{7}{3}A \quad A = 3$	A1			
(b)	$\frac{1}{1+3x} = (1+3x)^{-1}$		2	Alternative by Maclaurin
	$= 1 + -1(3x) + \frac{-1 \cdot -2}{2}(3x)^2$	M1		
(c)	$\frac{1}{2-x} = \frac{1}{2(1-\frac{x}{2})}$	B1	3	Alternative:
	$= \left(1 + -1\left(\frac{-x}{2}\right) + \frac{-1 \cdot -2}{2}\left(\frac{-x^2}{2}\right) \right)$	M1		
(d)	$\frac{8+3x}{(1+3x)(2-x)}$		3	Answer given, convincingly obtained
	$= 3(1-3x+9x^2) + 2\left(\frac{1}{2} + \frac{x}{4} + \frac{x^2}{8}\right)$	M1M1		
(e)	Valid for $ x < \frac{1}{3}$	B2	2	Alternative:
Total			13	

6 (a)	$\int \frac{dv}{10-5v} = \int dt$	M1	6	Attempt to separate and integrate
	$-\frac{1}{5} \ln(10-5v) = t + c$	M1 A1A1		
(b)	$t = 0 \quad v = 0 \quad c = -\frac{1}{5} \ln 10$	B1✓	3	Find c or use limits
	$t = \frac{1}{5} \ln\left(\frac{10}{10-5v}\right) = \frac{1}{5} \ln\left(\frac{2}{2-v}\right)$	A1		
(b)	$e^{5t} = \frac{2}{2-v}$	M1	3	Alternative:
	$t = 0.5 \quad 2 - v = 2e^{-2.5}$ $v = 1.8358 \quad v = 1.8 \text{ m s}^{-1}$	m1 A1		
Total			9	

Q	Solution	Marks	Total	Comments
7 (a)(i)	$\vec{AB} = \begin{bmatrix} 2 \\ 4 \\ -4 \end{bmatrix}$ $ \vec{AB} = \sqrt{2^2 + 4^2 + 4^2} = 6$	M1A1	2	No marks for \vec{AB} alone
(ii)	M is (4, 1, 0)	B1	1	Accept $\begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$
(b)	$\vec{CM} \cdot \vec{AB} = \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \\ -4 \end{bmatrix}$ $= -8 + 12 - 4 = 0$	M1A1	2	M1 – sensible attempt at $\vec{CM} \cdot \vec{AB}$ Allow \vec{MC} for \vec{CM} $\mp 8 \mp 12 \mp 4 = 0$ must be seen

Pure 3 June 2004

<p>1(a)</p>	$\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = \frac{-1}{2t^2} \cdot \frac{1}{2}$	<p>M1</p>		<p>attempt $\frac{dy}{dt}$ & $\frac{dx}{dt}$; use $\frac{dy}{dt} \cdot \frac{dt}{dx}$ $\left(\frac{dy}{dt} \cdot \frac{dx}{dt} \text{ M0}\right)$</p>
<p>(b)</p>	<p>$t = 1 \quad \frac{dy}{dx} = \frac{-1}{4}$</p> <p>gradient of normal = 4</p> <p>$y = 4x + c \quad t = 1 \quad x = 1 \quad y = \frac{1}{2}$</p> <p>$y = 4x - \frac{7}{2}$</p> <p>Special Cases Eliminate t in part (a) $y = \frac{1}{x+1}; \frac{dy}{dx} = \frac{\pm 1}{(x+1)^2} \text{ M1}$ $= \frac{-1}{(2t)^2} \text{ A1}$ $m_T = -\frac{1}{4} \text{ B1F}$ $\frac{1}{2} = -\frac{1}{4} \times 1 + c; c = \frac{3}{4} \text{ M1}$ $y = -\frac{1}{4}x + \frac{3}{4} \text{ A1F(5/6)}$ Common error $y = \frac{1}{2t} = 2t^{-1}; \frac{dy}{dt} = 2t^{-2}; \frac{dx}{dt} = 2$ $\frac{dy}{dx} = \frac{2t^{-2}}{2} = -t^{-2} \text{ (or } t^{-2}) \text{ M1A0}$ $m_N = -1, +1 \text{ B1F}$ $m_T = +1, -1 \text{ B0F (no ft for just changing sign)}$ $x = 1, y = \frac{1}{2}, \frac{1}{2} = -1 + c \text{ or } \frac{1}{2} = 1 + c$</p>	<p>A1</p> <p>B1F</p> <p>B1F</p> <p>M1</p> <p>A1F</p>	<p>2</p> <p>4</p>	<p>Use chain rule (ISW at this stage)</p> <p>ft $t = 1$ subst in their $\frac{dy}{dx}$</p> <p>Follow on gradient $\frac{-1}{\text{their } -\frac{1}{4}}$</p> <p>Use $(1, \frac{1}{2})$ and gradient</p> <p>$\frac{1}{2} = \text{their } 4 + c; \frac{y - \frac{1}{2}}{x - 1} = \text{their } 4$</p> <p>OE: F on gradient; $y = (\text{their } m_N) x + c$</p> <p>Tangent instead of normal $m_T = \frac{1}{4}$ $\frac{1}{2} = \frac{1}{4} \times 1 + c; c = \frac{1}{4}$ $y = \frac{1}{4}x + \frac{1}{4} \text{ (4/6)}$</p> <p>NB late substitution for t (could be retrospective) B1F B1F but if t's in final answer & no subst'n: either 0/4 or 1/4 if $(1, 1/2)$ and gradient used in linear equation</p>
	<p>M1</p>			

<p>2(a)</p> $(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x + \frac{1}{3}\left(\frac{1}{3}-1\right)\frac{x^2}{2}$ $= 1 + \frac{1}{3}x - \frac{1}{9}x^2$ <p>(b)</p> $(8+4x)^{\frac{1}{3}} = \left(8\left(1+\frac{1}{2}x\right)\right)^{\frac{1}{3}}$ $= 2\left(1 + \frac{1}{3}\frac{1}{2}x - \frac{1}{9}\left(\frac{1}{2}x\right)^2 + \dots\right)$ $= 2 + \frac{1}{3}x - \frac{1}{18}x^2 + \dots$	<p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>Total</p>	<p>2</p> <p>3</p> <p>5</p>	<p>M1 for expression inside bracket</p> <p>SC: $(8+4x)^{\frac{1}{3}}$</p> $= 8^{\frac{1}{3}} + \frac{1}{3}8^{-\frac{2}{3}} \cdot 4x + \frac{1}{3}\left(-\frac{2}{3}\right)8^{-\frac{5}{3}} \frac{(4x)^2}{2}$ <p>[M1 for $8^{\frac{1}{3}}$, $8^{-\frac{2}{3}}$, $8^{-\frac{5}{3}}$</p> <p>M1 for $4x$, $\frac{(4x)^2}{2}$]</p> $= 2 + \frac{1}{3}x - \frac{1}{18}x^2$ <p>Accept recurring decimals or equiv fractions</p>
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<p>3(a)</p> $30 = A(7-2x) + B(x+4)$ $x = -4 \quad 30 = 15A \quad A = 2$ $x = \frac{7}{2} \quad 30 = \frac{15}{2}B \quad B = 4$ <p>(b)</p> $\int_0^3 \frac{2}{x+4} + \frac{4}{7-2x} dx$ $= [2\ln(x+4) - 2\ln(7-2x)]_0^3$ $= 2\ln 7 - 2\ln 1 - 2\ln 4 + 2\ln 7$	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1A1F</p> <p>m1A1F</p> <p>A1</p> <p>Total</p>	<p>3</p> <p>5</p> <p>8</p>	<p>PFs: any valid method</p> <p>for substituting values of x to find A, B</p> <p>M1 for $[c\ln(x+4) + d\ln(7-2x)]$</p> <p>Ignore limits here</p> <p>m1 for $(c\ln 7 + d\ln 1) - (c\ln 4 + d\ln 7)$</p> <p>m1 Use limits right way round.</p> <p>A1 All correct and with $\ln 1 = 0$.</p> <p>A1F for $c\ln 7 - d\ln 7 - c\ln 4$</p> <p>or $-2\ln \frac{4}{49}$ or $-4\ln \frac{2}{7}$</p> <p>or $-1\ln \frac{16}{2401}$ or $1\ln \frac{2401}{16}$</p>
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Q	Solution	Marks	Total	Comments
4(a)	$9(y+2)^2 = 5 + 4(x-1)^2$ $x=2 \quad 9(y+2)^2 = 5 + 4$	M1	3	Substitute $x=2$ $9(y+2)^2 = 5 + 4 \times 3^2$ i.e. $(x+1)^2$
	$y+2 = \pm 1 \quad y = -1, -3$	m1A1		Find two y values. Coords not required $(y+2)^2 = \frac{41}{9}, y+2 = \pm \frac{\sqrt{41}}{3}$ M1A0
(b)	$\frac{d}{dx}(9(y+2)^2) = \frac{d}{dx}(5 + 4(x-1)^2)$	M1	5	Attempt implicit differentiation with use of chain rule: $\frac{dy}{dx}$ attached to y term, not x term
	$18(y+2)\frac{dy}{dx} = 0 + 8(x-1)$	A1A1		
	$(2,-1) \quad (2,-3)$ $\frac{dy}{dx} = \frac{4}{9} \quad \frac{dy}{dx} = -\frac{4}{9}$	m1 A1		Use $x=2$ and candidate's y values OE; CAO
				<p><u>Alternative: explicit differentiation</u></p> $y = \sqrt{\frac{5 + 4(x-1)^2}{9}} - 2$ $\frac{dy}{dx} = \frac{1}{2} \left(\frac{5 + 4(x-1)^2}{9} \right)^{-\frac{1}{2}} \cdot \frac{8}{9} (x-1)$ <p>(M1A2 fully correct; M1A1 if 9 of $\frac{8}{9}$ missing $x=2: \frac{dy}{dx} = \pm \frac{1}{2} (1) \frac{8}{9} = \pm \frac{4}{9}$)</p>
Total			8	

7(a)	$\int \frac{dx}{x} = \int (1-kt) dt$ $\ln x = t - \frac{1}{2} kt^2 + c$ $x = e^{t - \frac{1}{2} kt^2 + c}$ $x = 2000, t = 0 \Rightarrow A = 2000$ $x = Ae^{t - \frac{1}{2} kt^2}, \text{ where } A = e^c$ <p>(if A suddenly appears without justification: A0)</p>	M1		Attempt to separate and integrate. M0 if mixture of x's and t's	
		A1 A1		c required	
		M1		Alternatives	
		M1		(1) $c = \ln 2000$ M1	
		A1	6	$\ln \frac{x}{2000} = t - \frac{1}{2} kt^2$	
				$\frac{x}{2000} = e^{t - \frac{1}{2} kt^2}$	M1
				$x = 2000 e^{t - \frac{1}{2} kt^2}$	A1
				(2) $c = \ln 2000$ M1	
				$x = e^{t - \frac{1}{2} kt^2} + \ln 2000$	M1
				$= e^{t - \frac{1}{2} kt^2} e^{\ln 2000}$	
				$= 2000 e^{t - \frac{1}{2} kt^2}$	A1
				(3) $\int (1-kt) dt$ M1	
				$[\ln x]_{2000}^x = \left[t - \frac{1}{2} kt^2 \right]_0^t$	A1 for $\ln x$ A1 for $t - \frac{1}{2} kt^2$ A1 For both sets of limits
				$\ln x - \ln 2000 = t - \frac{1}{2} kt^2$	
				$\ln \left(\frac{x}{2000} \right) = t - \frac{1}{2} kt^2$	A1
				$x = 2000 e^{t - \frac{1}{2} kt^2}$	AG
				AG convincingly obtained	
(b)	Substituting $t = 12$ $x = 2000$ $12 - \frac{1}{2} k(12)^2 = \ln 1$ $k = \frac{1}{6}$	B1		No simplification required	
		M1		For taking \ln	
		A1	3	OE	
Total			9		

8(a)	$\vec{AB} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$	M1		
	$l_1 \text{ has equation } \mathbf{r} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}.$	A1	2	$\text{OE eg } \mathbf{r} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$
(b)	$\begin{aligned} 3 - \lambda &= 4 + \mu \\ -1 + \lambda &= 1 \\ 2 &= -1 - \mu \end{aligned}$	M1		Set up at least 2 equations and attempt to solve.
	$\lambda = 2 \quad \mu = -3$ <p>Confirm in third equation</p>	A1 A1		
	Intersect at (1, 1, 2)	A1	4	Alternative: showing (1, 1, 2) lies on both lines A2
(c)	$\begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 9 \\ 1 \\ -6 \end{bmatrix}.$	M1		
	is satisfied by $\mu = 5$	A1	2	
(d)	$\vec{CD} \bullet \vec{AB} = 0$	B1		$\vec{CD} \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = 0 \text{ or } \vec{CD} \cdot \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = 0$
	$\left(\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 9 \\ 1 \\ -6 \end{bmatrix} \right) \bullet \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = 0$	M1		not $\vec{CD} \cdot l_1$, unless corrected later
	$(-6 - \lambda)(-1) + (-2 + \lambda) = 0$	m1		
	$\lambda = -2 \quad D \text{ is } (5, -3, 2)$	A1	4	Answer may be in vector form
				<p>Alternative to part(d)</p> $\begin{bmatrix} x-9 \\ y-1 \\ z+6 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = 0 \quad \text{B1}$ $\Rightarrow x - y = 8 \quad \text{M1}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \text{their } \mathbf{r} \text{ from (a)} \quad \text{M1}$ $(5, -3, 2) \quad \text{A1}$
	Total		12	