General Certificate of Education
June 2007
Advanced Level Examination

MATHEMATICS **MPC4**
Unit Pure Core 4

Monday 18 June 2007 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Find the remainder when $2x^2 + x - 3$ is divided by $2x + 1$. *(2 marks)*

(b) Simplify the algebraic fraction $\dfrac{2x^2 + x - 3}{x^2 - 1}$. *(3 marks)*

2 (a) (i) Find the binomial expansion of $(1 + x)^{-1}$ up to the term in x^3. *(2 marks)*

(ii) Hence, or otherwise, obtain the binomial expansion of $\dfrac{1}{1 + 3x}$ up to the term in x^3. *(2 marks)*

(b) Express $\dfrac{1 + 4x}{(1 + x)(1 + 3x)}$ in partial fractions. *(3 marks)*

(c) (i) Find the binomial expansion of $\dfrac{1 + 4x}{(1 + x)(1 + 3x)}$ up to the term in x^3. *(3 marks)*

(ii) Find the range of values of x for which the binomial expansion of $\dfrac{1 + 4x}{(1 + x)(1 + 3x)}$ is valid. *(2 marks)*

3 (a) Express $4\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 360°$, giving your value for α to the nearest 0.1°. *(3 marks)*

(b) Hence solve the equation $4\cos x + 3\sin x = 2$ in the interval $0° < x < 360°$, giving all solutions to the nearest 0.1°. *(4 marks)*

(c) Write down the minimum value of $4\cos x + 3\sin x$ and find the value of x in the interval $0° < x < 360°$ at which this minimum value occurs. *(3 marks)*

4 A biologist is researching the growth of a certain species of hamster. She proposes that the length, x cm, of a hamster t days after its birth is given by

$$x = 15 - 12\mathrm{e}^{-\frac{t}{14}}$$

(a) Use this model to find:

(i) the length of a hamster when it is born; *(1 mark)*

(ii) the length of a hamster after 14 days, giving your answer to three significant figures. *(2 marks)*

(b) (i) Show that the time for a hamster to grow to 10 cm in length is given by $t = 14\ln\left(\frac{a}{b}\right)$, where a and b are integers. *(3 marks)*

(ii) Find this time to the nearest day. *(1 mark)*

(c) (i) Show that

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{14}(15 - x)$$ *(3 marks)*

(ii) Find the rate of growth of the hamster, in cm per day, when its length is 8 cm. *(1 mark)*

5 The point $P(1, a)$, where $a > 0$, lies on the curve $y + 4x = 5x^2y^2$.

(a) Show that $a = 1$. *(2 marks)*

(b) Find the gradient of the curve at P. *(7 marks)*

(c) Find an equation of the tangent to the curve at P. *(1 mark)*

Turn over for the next question

6 A curve is given by the parametric equations

$$x = \cos\theta \qquad y = \sin 2\theta$$

(a) (i) Find $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}$. *(2 marks)*

(ii) Find the gradient of the curve at the point where $\theta = \dfrac{\pi}{6}$. *(2 marks)*

(b) Show that the cartesian equation of the curve can be written as

$$y^2 = kx^2(1 - x^2)$$

where k is an integer. *(4 marks)*

7 The lines l_1 and l_2 have equations $\mathbf{r} = \begin{bmatrix} 8 \\ 6 \\ -9 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -3 \\ -1 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} -4 \\ 0 \\ 11 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ respectively.

(a) Show that l_1 and l_2 are perpendicular. *(2 marks)*

(b) Show that l_1 and l_2 intersect and find the coordinates of the point of intersection, P. *(5 marks)*

(c) The point $A(-4, 0, 11)$ lies on l_2. The point B on l_1 is such that $AP = BP$.

Find the length of AB. *(4 marks)*

8 (a) Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{1+2y}}{x^2}$$

given that $y = 4$ when $x = 1$. *(6 marks)*

(b) Show that the solution can be written as $y = \dfrac{1}{2}\left(15 - \dfrac{8}{x} + \dfrac{1}{x^2}\right)$. *(2 marks)*

END OF QUESTIONS