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General Certificate of Education (A-level)
June 2013

Mathematics **MFP2**

(Specification 6360)

Further Pure 2

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

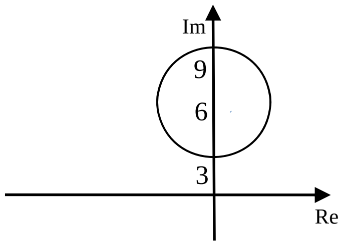
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

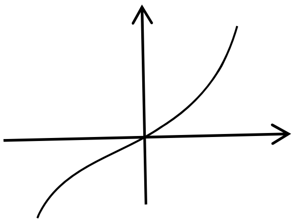
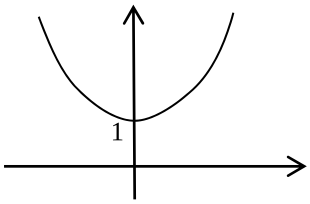
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)				
	Circle	M1		freehand circle
	Centre at 6i	A1		6 marked on Im axis as centre
	Radius 3 & cutting positive Im axis twice	A1	3	radius of 3 clearly indicated with circle in position shown
(b)(i)	(Max $\lvert z \rvert$ is) 9	B1	1	
(ii)	Tangent from O to circle	M1		FT their circle position
	Angle of $\frac{\pi}{6}$ or $\frac{\pi}{3}$ ***correctly*** marked	A1		PI ; condone degrees for first A1
	(Max arg z is) $\frac{2\pi}{3}$	A1cso	3	exactly this
	Total		**7**	

Q	Solution	Marks	Total	Comments
2(a)(i)	$\sinh x$ graph 	M1		 shape - curve through O, in 1st and 3[rd] quadrants
	$\cosh x$ graph 	M1		 shape - curve all above x-axis
	Gradient of $\sinh x > 0$ at origin **and** $\cosh x$ minimum at $(0,1)$	A1	3	
(ii)	$\cosh x = 0$ has ***no*** solutions **and** $\sinh x = -k$ has **one** solution (hence equation has exactly one solution)	E1	1	or $\cosh x > 0$ etc (since $y = -k$ cuts $y = \sinh x$ exactly once)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6\cosh x + 2\cosh x \sinh x$	M1 A1		one term correct all correct - may have $6\cosh x + \sinh 2x$
	$(2)\cosh x(3+\sinh x) = 0$ therefore C has only one stationary point	E1√		putting = 0, factorising **and** concluding statement (may be later)
	$\Rightarrow \sinh x = -3$ $\cosh^2 x = 10$	m1		finding $\sinh x$ from "their" equation
	$y\,(= -18 + 10) = -8$	A1	5	answer must be integer so do not accept calculator approximation rounded to –8
	Total		**9**	

Q	Solution	Marks	Total	Comments
3	$n = 1$, $\frac{3+1}{3-1} = \frac{4}{2} = 2$ ($u_1 = 2$ so formula is) true when $n = 1$	B1		be convinced they have used $u_n = \frac{3n+1}{3n-1}$
	Assume formula is true for $n = k$ (*) $(u_{k+1} =)\quad \dfrac{5\frac{3k+1}{3k-1}-3}{3\frac{3k+1}{3k-1}-1}$	M1		clear attempt at RHS of this formula
		m1		clear attempt to remove "double fraction"
	$(u_{k+1} =)\quad \dfrac{5(3k+1)-3(3k-1)}{3(3k+1)-(3k-1)}$	A1		$\dfrac{6k+8}{6k+4}$
	$u_{k+1} = \dfrac{3k+4}{3k+2}$ *or* $u_{k+1} = \dfrac{3(k+1)+1}{3(k+1)-1}$ Hence formula is true for $n = k+1$ (**)	A1cso		must have "$u_{k+1} =$ " on at least this line
	must have lines (*) & (**) **and** "Result true for $n = 1$ therefore true for $n = 2$, $n = 3$ etc by induction."	E1	6	must also have earned previous 5 marks before E1 is scored
	Total		**6**	
4(a)	$\mathrm{f}(r) - \mathrm{f}(r-1) =$ $r^2(2r^2-1)-(r-1)^2(2(r-1)^2-1)$ $= 2r^4 - r^2 - (r^2 - 2r + 1)(2r^2 - 4r + 1)$	M1		condone one slip here attempt to multiply out "their" $\mathrm{f}(r-1)$
	$= 2r^4 - r^2 - (2r^4 - 8r^3 + 11r^2 - 6r + 1)$	A1		$\mathrm{f}(r)$ & $\mathrm{f}(r-1)$ expanded correctly condone correct unsimplified
	$= 8r^3 - 12r^2 + 6r - 1$ $= (2r-1)^3$	A1cso	3	**AG**
(b)	Attempt to use method of differences $\mathrm{f}(2n) - \mathrm{f}(n)$	M1 m1		$(2n)^2\{2(2n^2)-1\} - n^2(2n^2-1)$
	$\mathrm{f}(2n) - \mathrm{f}(n) = 4n^2(8n^2-1) - n^2(2n^2-1)$	A1		
	$= 30n^4 - 3n^2$ $= 3n^2(10n^2-1)$	A1cso	4	**AG** be convinced
	Total		**7**	

Q	Solution	Marks	Total	Comments
5(a)(i)	$(\alpha\beta\gamma =)\quad -37+36\mathrm{i}$	B1	1	
(ii)	$(\beta\gamma =)\quad (-2+3\mathrm{i})(1+2\mathrm{i}) = -2+3\mathrm{i}-4\mathrm{i}-6$	M1		correct unsimplified but must simplify i^2
	$(-8-\mathrm{i})\ \alpha = -37+36\,\mathrm{i}$			
	$\Rightarrow (8+\mathrm{i})\ \alpha = 37-36\,\mathrm{i}$	A1cso	2	**AG** be convinced
(iii)	$\Rightarrow \alpha = \dfrac{37-36\mathrm{i}}{8+\mathrm{i}} \times \dfrac{8-\mathrm{i}}{8-\mathrm{i}}$	M1		
	$= \dfrac{296-37\mathrm{i}-288\mathrm{i}-36}{65}$	A1		correct unsimplified
	$= \dfrac{260-325\mathrm{i}}{65}$			
	$= 4-5\mathrm{i}$	A1cao	3	**Alternative** $(8+\mathrm{i})\,(m+n\,\mathrm{i}) = 37-36\,\mathrm{i}$ $8m - n = 37;\ m + 8n = -36$ M1 *either* $m = 4$ *or* $n = -5$ A1 $\alpha = 4-5\mathrm{i}$ A1
(b)	$\alpha+\beta+\gamma = -p$			
	$-2+3\mathrm{i} + 1+2\mathrm{i}+4-5\mathrm{i} = 3$			
	$(\Rightarrow p =)\ -3$	B1	1	
(c)	$\alpha\beta+\beta\gamma+\gamma\alpha = q$			$q=\sum\alpha\beta$ and attempt to evaluate three
	$(7+22\mathrm{i}) + (-8-\mathrm{i}) + (14+3\mathrm{i}) = q$	M1		products FT "their" α
	$q = 13+24\mathrm{i}$	A1cao	2	
	Total		**9**	

Q	Solution	Marks	Total	Comments
6(a)	$(5\cosh x - 3\sinh x)$			
	$= \frac{5}{2}\left(e^x + e^{-x}\right) - \frac{3}{2}\left(e^x - e^{-x}\right)$	M1		$\cosh x$ **and** $\sinh x$ correct in terms of e^x
	$= e^x + 4e^{-x}$	A1		may be seen as denominator
	$\frac{1}{5\cosh x - 3\sinh x} = \frac{e^x}{4 + e^{2x}}$	A1cso	3	** must have left hand-side ; $m = 4$
(b)	$u = e^x \Rightarrow du = e^x dx$	M1		or $\frac{du}{dx} = e^x$
	$\Rightarrow \int \frac{1}{4+u^2}(du)$	A1√		FT "their" m from part(a) $\Rightarrow \int \frac{1}{m+u^2}\,du$
	$= \frac{1}{2}\tan^{-1}\frac{u}{2}$	A1√		FT "their" $\frac{1}{\sqrt{m}}\tan^{-1}\frac{u}{\sqrt{m}}$
	$x = 0 \Rightarrow u = 1 \quad x = \ln 2 \Rightarrow u = 2$			
	$\frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}\frac{1}{2}$	A1√		FT "their" $\frac{1}{\sqrt{m}}\left(\tan^{-1}\frac{2}{\sqrt{m}} - \tan^{-1}\frac{1}{\sqrt{m}}\right)$
	$= \frac{\pi}{8} - \frac{1}{2}\tan^{-1}\frac{1}{2}$	A1cso	5	**AG**
	Total		**8**	

Q	Solution	Marks	Total	Comments
7(a)(i)	$\frac{\mathrm{d}}{\mathrm{d}u}\left(2u\sqrt{1+4u^2}\right)=\frac{8u^2}{\sqrt{1+4u^2}}+2\sqrt{1+4u^2}$	M1		M1 for clear use of product rule (condone one error in one term)
		A1		correct unsimplified
	$\frac{\mathrm{d}}{\mathrm{d}u}\left(\sinh^{-1}2u\right)=\frac{2}{\sqrt{1+4u^2}}$	B1		
	$\frac{8u^2+2}{\sqrt{1+4u^2}}=\frac{2(1+4u^2)}{\sqrt{1+4u^2}}=2\sqrt{1+4u^2}$			be convinced – must see this line OE
	$\frac{\mathrm{d}}{\mathrm{d}u}\left(2u\sqrt{1+4u^2}+4\sinh^{-1}2u\right)=4\sqrt{1+4u^2}$	A1cso	4	all working must be correct (not enough to just say k = 4)
(ii)	$\frac{1}{\text{"their"}k}\left[2u\sqrt{1+4u^2}+\sinh^{-1}2u\right]_0^1$	M1		anti differentiation
	$=\frac{\sqrt{5}}{2}+\frac{1}{4}\sinh^{-1}2$	A1√	2	FT "their" k or even use of k
(b)(i)	$y=\frac{1}{2}\cos 4x$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=A\sin 4x$			$\frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin 4x$
	substituted into $\int K\,y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,(\mathrm{d}x)$	M1		clear attempt to use formula for CSA
	$(S=)\ \int_0^{\frac{\pi}{8}}2\pi\times\frac{1}{2}\cos 4x\sqrt{1+4\sin^2 4x}\,\mathrm{d}x$ = printed answer (combining $2\times\frac{1}{2}$)	A1cso	2	**AG** $\frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin 4x$ **and** $2\times\frac{1}{2}$ **and** $\mathrm{d}x$ must be seen to award A1cso
(ii)	$u=\sin 4x\Rightarrow \mathrm{d}u=4\cos 4x\,\mathrm{d}x$	M1		condone $\mathrm{d}u=B\cos 4x\,\mathrm{d}x$ for M1
	$(S=)\ \frac{\pi}{4}\int_0^1\sqrt{1+4u^2}\,(\mathrm{d}u)$	A1		condone limits seen later
		m1		use of their result from (a)(ii) **correctly** FT "their" B
	$(S=)\ \frac{\pi\sqrt{5}}{8}+\frac{\pi}{16}\sinh^{-1}2$	A1cso	4	OE
	Total		**12**	

Q	Solution	Marks	Total	Comments
8(a)(i)	$\cos 4\theta + \mathrm{i}\sin 4\theta = (\cos\theta + \mathrm{i}\sin\theta)^4$	M1		De Moivre & attempt to expand RHS
	$\cos^4\theta + 4\mathrm{i}\cos^3\theta\sin\theta + 6\mathrm{i}^2\cos^2\theta\sin^2\theta$ $+ 4\mathrm{i}^3\cos\theta\sin^3\theta + \mathrm{i}^4\sin^4\theta$	A1		any correct expansion
	Equating "their" real parts	m1		or imaginary parts
	$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$	A1		**AG** be convinced
	$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$	B1	5	correct
(ii)	$\tan 4\theta = \dfrac{\text{“their expression for ” } \sin 4\theta}{\text{“their expression for ” } \cos 4\theta}$	M1		
	Division by $\cos^4\theta$	m1		
	$\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$	A1	3	**AG** be convinced
(b)	$(\tan 4\theta = 1 \Rightarrow)$ $\quad 1 = \dfrac{4t - 4t^3}{1 - 6t^2 + t^4}$	M1		when $\theta = \dfrac{\pi}{16}$
	$1 - 6t^2 + t^4 = 4t - 4t^3$ $\Rightarrow t^4 + 4t^3 - 6t^2 - 4t + 1 = 0$	A1		**AG** be convinced
	$\theta = \dfrac{\pi}{16}$ *satisfies* $\tan 4\theta = 1$ $\Rightarrow \tan\dfrac{\pi}{16}$ is root of quartic equation	E1		both statements required
	(other roots are) $\tan\dfrac{5\pi}{16}, \tan\dfrac{9\pi}{16}, \tan\dfrac{13\pi}{16}$	B1	4	or equivalent tan expressions
(c)	$\sum\alpha = -4$ **and** $\sum\alpha\beta = -6$	B1		watch for minus signs
	$\sum\alpha^2 = \left(\sum\alpha\right)^2 - 2\sum\alpha\beta$	M1		correct formula
	$(= 16 + 12) = 28$	A1cso		
	$\tan\dfrac{9\pi}{16} = -\tan\dfrac{7\pi}{16}$, $\tan\dfrac{13\pi}{16} = -\tan\dfrac{3\pi}{16}$	B1		explicitly seen
	$\tan^2\dfrac{\pi}{16} + \tan^2\dfrac{3\pi}{16} + \tan^2\dfrac{5\pi}{16} + \tan^2\dfrac{7\pi}{16} = 28$	A1cso	5	**AG** must earn previous 4 marks
	Total		**17**	
	TOTAL		**75**	