





2. Three forces  $\mathbf{F}_1 = (3\mathbf{i} - \mathbf{j} + \mathbf{k})\text{N}$ ,  $\mathbf{F}_2 = (2\mathbf{i} - \mathbf{k})\text{N}$ , and  $\mathbf{F}_3$  act on a rigid body.

The force  $\mathbf{F}_1$  acts through the point with position vector  $(\mathbf{i} + 2\mathbf{j} + \mathbf{k})\text{m}$ , the force  $\mathbf{F}_2$  acts through the point with position vector  $(\mathbf{i} - 2\mathbf{j})\text{m}$  and the force  $\mathbf{F}_3$  acts through the point with position vector  $(\mathbf{i} + \mathbf{j} + \mathbf{k})\text{m}$ .

Given that the system  $\mathbf{F}_1$ ,  $\mathbf{F}_2$  and  $\mathbf{F}_3$  reduces to a couple  $\mathbf{G}$ ,

- (a) find  $\mathbf{G}$ . (6)

The line of action of  $\mathbf{F}_3$  is changed so that the system  $\mathbf{F}_1$ ,  $\mathbf{F}_2$  and  $\mathbf{F}_3$  now reduces to a couple  $(6\mathbf{i} + 8\mathbf{j} + 2\mathbf{k})\text{N m}$ .

- (b) Find an equation of the new line of action of  $\mathbf{F}_3$ , giving your answer in the form  $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ , where  $\mathbf{a}$  and  $\mathbf{b}$  are constant vectors. (5)

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3. A spacecraft is moving in a straight line in deep space. The spacecraft moves by ejecting burnt fuel backwards at a constant speed of  $2000\text{ m s}^{-1}$  relative to the spacecraft. The burnt fuel is ejected at a constant rate of  $c\text{ kg s}^{-1}$ . At time  $t$  seconds the total mass of the spacecraft, including fuel, is  $m\text{ kg}$  and the speed of the spacecraft is  $v\text{ m s}^{-1}$ .

(a) Show that, while the spacecraft is ejecting burnt fuel,

$$m \frac{dv}{dt} = 2000c \tag{7}$$

At time  $t = 0$ , the mass of the spacecraft is  $M_0\text{ kg}$  and the speed of the spacecraft is  $2000\text{ m s}^{-1}$ . When  $t = 50$ , the spacecraft is still ejecting burnt fuel and its speed is  $6000\text{ m s}^{-1}$ .

(b) Find  $c$  in terms of  $M_0$ . (7)

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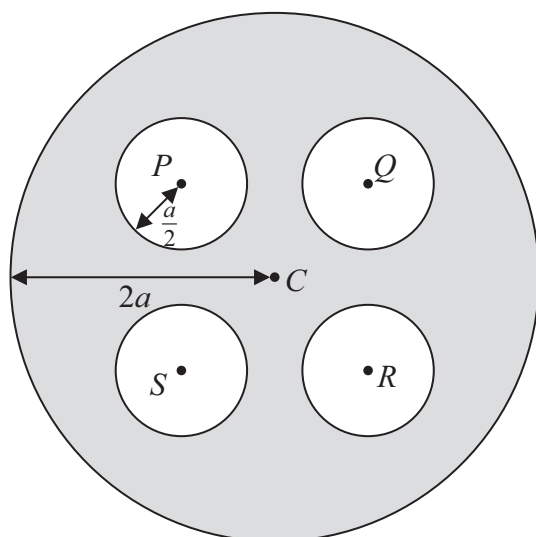


Figure 1

A uniform circular lamina has radius  $2a$  and centre  $C$ . The points  $P$ ,  $Q$ ,  $R$  and  $S$  on the lamina are the vertices of a square with centre  $C$  and  $CP = a$ . Four circular discs, each of radius  $\frac{a}{2}$ , with centres  $P$ ,  $Q$ ,  $R$  and  $S$ , are removed from the lamina. The remaining lamina forms a template  $T$ , as shown in Figure 1.

The radius of gyration of  $T$  about an axis through  $C$ , perpendicular to  $T$ , is  $k$ .

- (a) Show that  $k^2 = \frac{55a^2}{24}$  (7)

The template  $T$  is free to rotate in a vertical plane about a fixed smooth horizontal axis which is perpendicular to  $T$  and passes through a point on its outer rim.

- (b) Write down an equation of rotational motion for  $T$  and deduce that the period of small oscillations of  $T$  about its stable equilibrium position is

$$2\pi \sqrt{\left(\frac{151a}{48g}\right)} \quad \text{(8)}$$

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