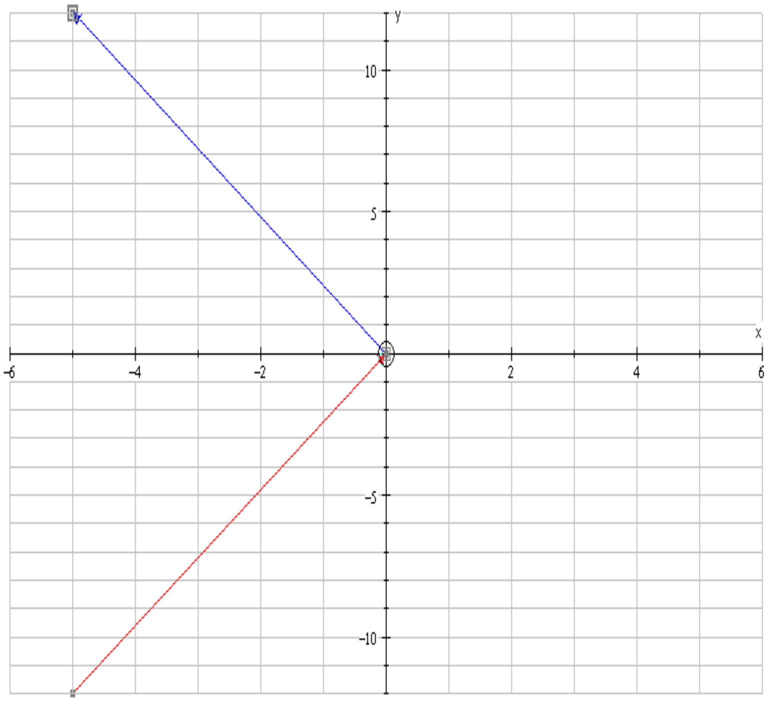
Further Pure Mathematics FP1 (6667)

Mock paper mark scheme

Question number		Scheme	Marks
1.	(a)	$\mathbf{R}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	B1
	(b)	$\mathbf{RS} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	B1 B1
	(c)	Rotation, 180° about O, or π about O	B1 B1
		Or enlargement, scale factor –1	
			(5 marks)
2.		Parabola, or $y^2 = 4ax$ seen	B1
		$a = 3$ or $a = -3$ seen	B1
		$y^2 = -12x$	B1
			(3 marks)
3.	(a)	$1+2\sqrt{3}\mathrm{i}-3+1+\sqrt{3}\mathrm{i} \ = \ -1+3\sqrt{3}\mathrm{i}$	M1 A1 A1
			(3)
		$\dfrac{(1+\sqrt{3}\mathrm{i})(2+\sqrt{3}\mathrm{i})}{(2-\sqrt{3}\mathrm{i})(2+\sqrt{3}\mathrm{i})} = \dfrac{-1+3\sqrt{3}\,\mathrm{i}}{7}$	M1 A1 A1
			(3)
			(6 marks)
4.	(a)	$\mathrm{f}(2) = -1$, $\mathrm{f}(3) = 3$; and so $\alpha = 2 + \dfrac{1}{1+3} = 2.25$	B1 B1;
			M1 A1(4)
	(b)	$\mathrm{f}'(x) = 3x^2 - 8x + 5$	M1
		$\mathrm{f}'(2.5) = 3.75$	A1
		$\mathrm{f}(2.5) = 0.125$	B1
	(b)	$\therefore u_1 = 2.5 - \dfrac{.125}{3.75} = 2.47$	M1 A1 (5)
			(9 marks)

Question number		Scheme	Marks
5.	(a)	Determinant $5ab$	B1
		$\mathbf{X}^{-1} = \frac{1}{5ab}\begin{pmatrix} 3b & -2b \\ a & a \end{pmatrix}$	M1 A1 (3)
	(b)	$\mathbf{Z} = \mathbf{Y}\mathbf{X}^{-1}$	M1
		$= \frac{1}{5ab}\begin{pmatrix} 10ab & -5ab \\ 5ab & 0 \end{pmatrix} \qquad = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$	M1 A1 ft A1 cao (4)
			(7 marks)
6.	(a)	$\sum 2r^3 - 6r$	M1
		$2\frac{n^2}{4}(n+1)^2 - 6\frac{n}{2}(n+1)$	A1
		$= \frac{n}{2}(n+1)\left[n(n+1)-6\right]$	M1
		$= \frac{n}{2}(n+1)\left[n^2+n-6\right]$	
		$= \frac{n}{2}(n+1)(n+3)(n-2) \quad (*)$	A1 cos (4)
	(b)	f(50) – f(9) = 3243600 – 3780	M1
		= 3239820	A1 (2)
			(6 marks)
7.	(a)	Solve quadratic to obtain $z = -5 \pm 12\mathrm{i}$	M1 A1 A1 (3)
	(b)	$\lvert z_1 \rvert = \lvert z_2 \rvert = 13$	B1, B1
		arg $z_1 =$ 1.97 and arg $z_2 = -$ 1.97	M1 A1 A1 (5)

Question number	Scheme	Marks
(c)		B1 B1 (2)
(d)	$\lvert\pm 24\mathrm{i}\rvert = 24$	M1 A1 (2)
		(12 marks)
8. (a)	$c^2 = 9$	B1 (1)
(b)	$y = \frac{9}{x} \Rightarrow \frac{dy}{dx} = -\frac{9}{x^2}$	M1
	Gradient of curve and of tangent is $-\frac{1}{t^2}$	A1
	Gradient of normal is $-\frac{1}{\text{gradient of tangent}}$	M1
	Equation is $y - \frac{3}{t} = t^2(x - 3t)$ giving printed answer	M1 A1 cso (5)
(c)	When $t = 2$, $y = 4x + 1.5 - 24$	M1 A1
	$\therefore \frac{9}{x} = 4x + 1.5 - 24$	M1
	Attempt to solve e.g. $4x^2 - 22.5x - 9 = 0$ and formula	M1 A1;
	or factorise $x = -\frac{3}{8}$; $y = -24$	M1 A1 (7)
		(13 marks)

Question number	Scheme	Marks
9. (a)	$n=1, \quad u_1 = 3+2(1-1) = 3$, so result true for n = 1	B1
	Assume true for k	
	Then $u_{k+1} = 3(3^k + 2(3^{k-1} - 1)) + 4$	M1
	So $u_{k+1} = 3^{k+1} + 2(3^k) - 6 + 4$	M1
	$u_{k+1} = 3^{k+1} + 2(3^k) - 2 \;=\; 3^{k+1} + 2(3^k - 1)$, so result true for	A1
	k+1, so by induction the result is true for all positive integers	A1 (5)
(b) (i)	$\mathbf{A}^1 = \begin{pmatrix} 4 & 0 \\ 3\times 3 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 9 & 1 \end{pmatrix}$ so result true for n = 1	B1
	Assume true for k	
	Then $\mathbf{A}^{k+1} = \begin{pmatrix} 4 & 0 \\ 9 & 1 \end{pmatrix}\begin{pmatrix} 4^k & 0 \\ 3(4^k - 1) & 1 \end{pmatrix}$	B1
	$= \begin{pmatrix} 4^{k+1} & 0 \\ 9.4^k + 3.4^k - 3 & 1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 4^{k+1} & 0 \\ 3.4^k(3+1) - 3 & 1 \end{pmatrix} = \begin{pmatrix} 4^{k+1} & 0 \\ 3.4^{k+1} - 3 & 1 \end{pmatrix}$	M1 A1
	so result true for k + 1	M1
	So by induction the result is true for all positive integers	A1 (7)
(ii)	For n = –1, $\mathbf{A}^{-1} = \begin{pmatrix} \frac{1}{4} & 0 \\ \frac{-9}{4} & 1 \end{pmatrix}$	M1
	This is the correct inverse of **A**, so result is valid	A1 (2)
		(14 marks)