General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MFP3

Unit Further Pure 3

Thursday 14 June 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \sqrt{(2x)} + \sqrt{y}$$

and $$y(2) = 9$$

Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\mathrm{f}(x_r, y_r)$ and $k_2 = h\mathrm{f}(x_r + h, y_r + k_1)$ and $h = 0.25$, to obtain an approximation to $y(2.25)$, giving your answer to two decimal places. *(5 marks)*

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2 (a) Write down the expansion of $\sin 2x$ in ascending powers of x up to and including the term in x^5. *(1 mark)*

(b) Show that, for some value of k,

$$\lim_{x \to 0} \left[\frac{2x - \sin 2x}{x^2 \ln(1 + kx)} \right] = 16$$

and state this value of k. *(4 marks)*

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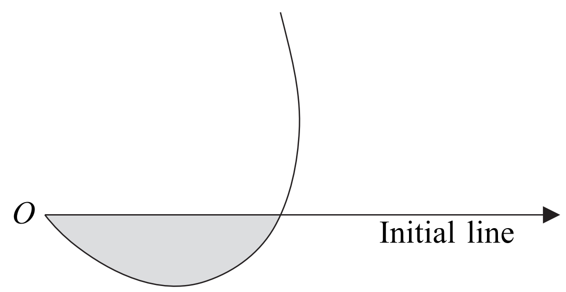
3 The diagram shows a sketch of a curve C, the pole O and the initial line.



The polar equation of C is

$$r = 2\sqrt{1 + \tan\theta}, \qquad -\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{4}$$

Show that the area of the shaded region, bounded by the curve C and the initial line, is $\dfrac{\pi}{2} - \ln 2$. *(4 marks)*

4 (a) By using an integrating factor, find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{4}{2x+1}y = 4(2x+1)^5$$

giving your answer in the form $y = \mathrm{f}(x)$. *(7 marks)*

(b) The gradient of a curve at any point (x, y) on the curve is given by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4(2x+1)^5 - \frac{4}{2x+1}y$$

The point whose x-coordinate is zero is a stationary point of the curve. Using your answer to part **(a)**, find the equation of the curve. *(3 marks)*

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5 (a) Find $\int x^2 \mathrm{e}^{-x}\,\mathrm{d}x$. *(4 marks)*

(b) Hence evaluate $\int_0^{\infty} x^2 \mathrm{e}^{-x}\,\mathrm{d}x$, showing the limiting process used. *(3 marks)*

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6 It is given that $y = \ln(1 + \sin x)$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(2 marks)*

(b) Show that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\mathrm{e}^{-y}$. *(3 marks)*

(c) Express $\frac{\mathrm{d}^4y}{\mathrm{d}x^4}$ in terms of $\frac{\mathrm{d}y}{\mathrm{d}x}$ and e^{-y}. *(3 marks)*

(d) Hence, by using Maclaurin's theorem, find the first four non-zero terms in the expansion, in ascending powers of x, of $\ln(1 + \sin x)$. *(3 marks)*

7 (a) Show that the substitution $x = e^t$ transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 3 + 20\sin(\ln x)$$

into $$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 3 + 20\sin t$$ *(7 marks)*

(b) Find the general solution of the differential equation

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 3 + 20\sin t$$ *(11 marks)*

(c) Write down the general solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 3 + 20\sin(\ln x)$$ *(1 mark)*

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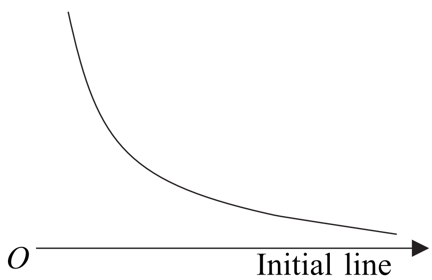
8 (a) A curve has cartesian equation $xy = 8$. Show that the polar equation of the curve is $r^2 = 16\operatorname{cosec} 2\theta$. *(3 marks)*

(b) The diagram shows a sketch of the curve, C, whose polar equation is

$$r^2 = 16\operatorname{cosec} 2\theta, \quad 0 < \theta < \frac{\pi}{2}$$



(i) Find the polar coordinates of the point N which lies on the curve C and is closest to the pole O. *(2 marks)*

(ii) The circle whose polar equation is $r = 4\sqrt{2}$ intersects the curve C at the points P and Q. Find, in an exact form, the polar coordinates of P and Q. *(4 marks)*

(iii) The obtuse angle PNQ is α radians. Find the value of α, giving your answer to three significant figures. *(5 marks)*