



# A-LEVEL

# Mathematics

Further Pure 2 – MFP2

Mark scheme

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6360  
June 2015

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Version/Stage: 1.0 Final

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from [aqa.org.uk](http://aqa.org.uk)

## Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

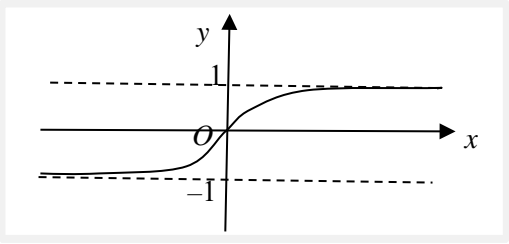
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

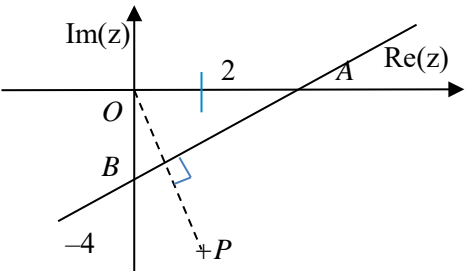
**Otherwise we require evidence of a correct method for any marks to be awarded.**

Q1	Solution	Mark	Total	Comment
	<p><b>(a)</b> <math>r+1 = A(r+2) + B</math> or</p> $1 = \frac{A(r+2)}{r+1} + \frac{B}{r+1}$ <p>either <math>A=1</math> or <math>B=-1</math></p> $\frac{1}{(r+2)r!} = \frac{1}{(r+1)!} - \frac{1}{(r+2)!}$ <p><b>(b)</b> <math>\frac{1}{2!} - \frac{1}{3!} + \frac{1}{3!} - \frac{1}{4!} + \dots</math></p> $\frac{1}{(n+1)!} - \frac{1}{(n+2)!}$ <p>Sum = <math>\frac{1}{2} - \frac{1}{(n+2)!}</math></p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>3</b></p> <p><b>2</b></p> <p><b>5</b></p>	<p><b>OE</b> with factorials removed</p> <p><b>correctly</b> obtained</p> <p>allow if seen in part <b>(b)</b></p> <p>use of their result from part <b>(a)</b> at least twice</p> <p>must simplify 2! and must have scored at least <b>M1 A1</b> in part <b>(a)</b></p>
	<b>Total</b>		<b>5</b>	
	<p><b>(a) Alternative Method</b> Substituting two values of <math>r</math> to obtain two correct equations in <math>A</math> and <math>B</math> with factorials evaluated correctly</p> $r=0 \Rightarrow \frac{1}{2} = A + \frac{B}{2} ; r=1 \Rightarrow \frac{1}{3} = \frac{A}{2} + \frac{B}{6}$ <p>earns <b>M1</b> then <b>A1, A1</b> as in main scheme</p> <p><b>NMS</b> <math>\frac{1}{(r+1)!} - \frac{1}{(r+2)!}</math> earns <b>3 marks</b>.</p> <p><b>However</b>, using an <b>incorrect</b> expression resulting from poor algebra such as <math>1 = A(r+2)! + B(r+1)!</math> with candidate often fluking <math>A=1, B=-1</math> scores <b>M0</b> ie zero marks which you should denote as <span style="border: 1px solid black; padding: 2px;">FIW</span></p> <p>These candidates can then score a maximum of <b>M1</b> in part <b>(b)</b>.</p>			<p><b>(b) ISW</b> for incorrect simplification after correct answer seen</p>

Q2	Solution	Mark	Total	Comment
<p>(a)</p>	 <p>Graph roughly correct through <math>O</math></p> <p>Correct behaviour as <math>x \rightarrow \pm\infty</math> &amp; grad at <math>O</math></p> <p>Asymptotes have equations <math>y = 1</math> &amp; <math>y = -1</math></p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p>	<p><b>3</b></p>	<p>condone infinite gradient at <math>O</math> for <b>M1</b></p> <p>must state equations</p> <p><b>both</b> correct <b>ACF</b> or <b>correct</b> squares of these expressions seen</p> <p>attempt to combine their squared terms with correct single denominator</p> <p><b>AG</b> valid proof convincingly shown to equal 1 <b>including LHS</b> seen</p> <p>correct use of identity from part <b>(b)</b></p> <p>forming quadratic in <math>\tanh x</math></p> <p>obtained from correct quadratic</p> <p>FT a value of <math>k</math> provided <math> k  &lt; 1</math></p> <p>both solutions correct and no others any equivalent form involving <math>\ln</math></p>
	<p><b>(b)</b></p> $\operatorname{sech} x = \frac{2}{e^x + e^{-x}} ; \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ $(\operatorname{sech}^2 x + \tanh^2 x) = \frac{2^2 + (e^x - e^{-x})^2}{(e^x + e^{-x})^2}$ $\operatorname{sech}^2 x + \tanh^2 x = \frac{e^{2x} + 2 + e^{-2x}}{e^{2x} + 2 + e^{-2x}} = 1$	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>3</b></p>	
	<p><b>(c)</b></p> $6(1 - \tanh^2 x) = 4 + \tanh x$ $6 \tanh^2 x + \tanh x - 2 = 0$ $\tanh x = \frac{1}{2}, \quad \tanh x = -\frac{2}{3}$ $\tanh x = k \Rightarrow x = \frac{1}{2} \ln \left( \frac{1+k}{1-k} \right)$ $x = \frac{1}{2} \ln 3, \quad x = \frac{1}{2} \ln \frac{1}{5}$	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1F</b></p> <p><b>A1</b></p>	<p><b>5</b></p>	
	<p style="text-align: right;"><b>Total</b></p>		<p><b>11</b></p>	
<p><b>(a)</b></p> <p><b>(b)</b></p>	<p>Actual asymptotes need not be shown, but if asymptotes are drawn then curve should not cross them for <b>A1</b>. Gradient should not be infinite at <math>O</math> for <b>A1</b>.</p> <p>Condone trailing equal signs up to final line provided “<math>\operatorname{sech}^2 x + \tanh^2 x =</math>” is seen on previous line for <b>A1</b></p> <p>Denominator may be <math>e^{4x} + 4e^{2x} + 6 + e^{4x} + 4e^{-2x} + e^{-4x}</math> etc for <b>M1</b> and <b>A1</b></p> <p>Accept <math>\operatorname{sech}^2 x + \tanh^2 x = \frac{(e^x + e^{-x})^2}{(e^x + e^{-x})^2} = 1</math> for <b>A1</b></p> <p><b>Alternative :</b> <math>\left( \frac{1}{\cosh^2 x} + \frac{\sinh^2 x}{\cosh^2 x} \right) = \frac{1 + \left( \frac{1}{2}(e^x - e^{-x}) \right)^2}{\left( \frac{1}{2}(e^x + e^{-x}) \right)^2}</math> scores <b>B1 M1</b></p> <p>and then <b>A1</b> for <math>\operatorname{sech}^2 x + \tanh^2 x = \frac{\frac{1}{4}e^{2x} + \frac{1}{4}e^{-2x} + \frac{1}{2}}{\frac{1}{4}e^{2x} + \frac{1}{2} + \frac{1}{4}e^{-2x}} = 1</math>, (all like terms combined in any order).</p>			

Q3	Solution	Mark	Total	Comment
(a)	$\frac{dx}{dt} = 1 - \frac{1}{t^2}$	B1		OE eg $\frac{t(2t) - (t^2 + 1)}{t^2}$ ACF
	$\frac{dy}{dt} = \frac{2}{t}$	B1		
	$\left( \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 \right) = 1 - \frac{2}{t^2} + \frac{1}{t^4} + \frac{4}{t^2}$	M1		squaring and adding their expressions and attempting to multiply out
	$1 + \frac{2}{t^2} + \frac{1}{t^4} = \left( 1 + \frac{1}{t^2} \right)^2$	A1	4	AG be convinced
	(b)	$2\pi \int_1^2 (2\ln t) \left( 1 + \frac{1}{t^2} \right) dt$	B1	
		M1		integration by parts - clear attempt to integrate $1 + \frac{1}{t^2}$ and differentiate $2\ln t$
$(2\pi) \left\{ (2\ln t) \left( t - \frac{1}{t} \right) - \int \frac{2}{t} \left( t - \frac{1}{t} \right) (dt) \right\}$		A1		correct (may omit limits, $2\pi$ and $dt$ )
$2\pi \left[ (2\ln t) \left( t - \frac{1}{t} \right) - \left( 2t + \frac{2}{t} \right) \right]$ $= 2\pi(3\ln 2 - 5 + 4)$ $= \pi(6\ln 2 - 2)$		A1	5	correct including $2\pi$ (no limits required)
	<b>Total</b>		<b>9</b>	
(b)	<p>May have two separate integrals and attempt both using integration by parts for M1</p> <p>Must see <math>(2\pi) \left\{ 2t \ln t - \int 2(dt) - \left( 2t^{-1} \ln t - \int 2t^{-2}(dt) \right) \right\}</math> (may omit limits, <math>2\pi</math> and <math>dt</math>) for first A1</p> <p>and <math>2\pi \left[ (2t \ln t - 2t) - (2t^{-1} \ln t + 2t^{-1}) \right]</math> for second A1</p> <p>Condone poor use of brackets if later recovered.</p>			

Q4	Solution	Mark	Total	Comment
<p><b>(a)</b></p> <p><b>(b)</b></p>	$f(k+1) = 2^{4k+7} + 3^{3k+4}$ <p>convincingly showing <math>2^{4k+7} = 16 \times 2^{4k+3}</math></p> $f(k+1) - 16f(k)$ $= (81 - 16 \times 3) \times 3^{3k}$ $= 33 \times 3^{3k}$	<p><b>M1</b></p>		
		<p><b>E1</b></p>		<p>must see <math>16 = 2^4</math> OE</p>
		<p><b>A1</b></p>	<p><b>3</b></p>	
	<p><i>Assume</i> <math>f(k)</math> is a multiple of 11 (*)</p> $f(k+1) = 16f(k) + 33 \times 3^{3k}$ $= 11M + 11N = 11(M + N)$ <p>Therefore <math>f(k+1)</math> is a multiple of 11</p> <p>Since <math>f(1)</math> is multiple of 11 then <math>f(2), f(3), \dots</math> are multiples of 11 by induction (or is a multiple of 11 for all integers <math>n \geq 1</math>)</p>	<p><b>B1</b></p>		<p><math>f(1) = 209 = 11 \times 19</math> or <math>209 \div 11 = 19</math> etc therefore true when <math>n=1</math></p>
	<p><b>M1</b></p>		<p>attempt at <math>f(k+1) = \dots</math> using their result from part (a)</p>	
	<p><b>A1</b></p>		<p>where <math>M</math> and <math>N</math> are integers</p>	
	<p><b>E1</b></p>	<p><b>4</b></p>	<p>must earn previous 3 marks and have (*) before <b>E1</b> can be awarded</p>	
	<b>Total</b>		<b>7</b>	
<p><b>(a)</b></p> <p><b>(b)</b></p>	<p>It is possible to score <b>M1 E0 A1</b></p> <p>Withhold <b>E1</b> for conclusion such as “a multiple of 11 for all <math>n \geq 1</math>” or poor notation, etc</p>			

Q5	Solution	Mark	Total	Comment
<p><b>(a)</b></p>  <p>Straight line Through midpoint of <math>OP</math>, <math>P</math> correct Perpendicular to <math>OP</math>, <math>P</math> correct</p>	<p><b>(b)(i)</b></p> $(x-2)^2 + (y+4)^2 = x^2 + y^2$ $2y - x + 5 = 0$ <p><math>A(5,0)</math> &amp; <math>B(0,-2.5)</math></p> $C\left(\frac{5}{2}, -\frac{5}{4}\right) \Rightarrow \text{complex num} = \frac{5}{2} - \frac{5}{4}i$	<p><b>M1</b> <b>A1</b> <b>A1</b></p>	<p><b>3</b></p>	<p>Ignore the line <math>OP</math> drawn in full or circles drawn as part of construction for locus <math>L</math>.</p> <p><math>P</math> represents <math>2 - 4i</math></p>
	<p><b>(ii)</b> <i>either</i> <math>\alpha = \frac{5}{2} - \frac{5}{4}i</math> <i>or</i> <math>k = \frac{5\sqrt{5}}{4}</math></p> $\left  z - \frac{5}{2} + \frac{5}{4}i \right  = \frac{5\sqrt{5}}{4}$	<p><b>M1</b> <b>A1</b></p>	<p><b>4</b> <b>2</b></p>	<p>may have <math>5 + 0i</math> and <math>0 - 2.5i</math></p> <p>allow statement with correct value for centre or radius of circle</p> <p>must have exact surd form</p>
<b>Total</b>			<b>9</b>	
<p><b>(a)</b></p>	<p>Withhold the final <b>A1</b> (if 3 marks earned) if the straight line does not go beyond the <math>\text{Re}(z)</math> axis and negative <math>\text{Im}(z)</math> axis. The two <b>A1</b> marks can be awarded independently.</p>			
<p><b>(b)(i)</b></p>	<p><b>Alternative 1:</b> <math>\text{grad } OP = -2 \Rightarrow \text{grad } L = 0.5</math> <b>M1</b>; <math>y + 2 = \frac{1}{2}(x - 1)</math> <b>OE A1</b> then <b>A1, A1</b> as per scheme</p> <p><b>Alternative 2:</b> substituting <math>z = x</math> (or <math>a</math>) then <math>z = iy</math> (or <math>ib</math>) into given locus equation</p> <p>Both <math>(x - 2)^2 + 4^2 = x^2</math> <b>and</b> <math>2^2 + (y + 4)^2 = y^2</math> <b>M1</b>; <math>4 - 4x + 16 = 0</math> <b>and</b> <math>4 + 8y + 16 = 0</math> <b>OE for A1</b> then <b>A1, A1</b> as per scheme.</p>			



Q6	Solution	Mark	Total	Comment
<p>(a)</p> <p>(b)</p>	$\sqrt{5+4x-x^2} + \frac{(x-2)\frac{1}{2}(4-2x)}{\sqrt{5+4x-x^2}}$ $(+)\frac{9 \times \frac{1}{3}}{\sqrt{1-\left(\frac{x-2}{3}\right)^2}}$ $\frac{5+4x-x^2}{\sqrt{5+4x-x^2}}$ $\left(\frac{dy}{dx} = \right) 2\sqrt{5+4x-x^2}$ $\frac{1}{k} \left\{ (x-2)\sqrt{5+4x-x^2} + 9\sin^{-1}\left(\frac{x-2}{3}\right) \right\}$ $\frac{1}{\text{"their" } k} \left[ \frac{3}{2}\sqrt{\frac{27}{4}} + 9\sin^{-1}\frac{1}{2} \right]$ $= \frac{9}{8}\sqrt{3} + \frac{3}{4}\pi$	<p>M1</p> <p>A1</p> <p>B1</p> <p>A1</p> <p>A1cso</p> <p>M1</p> <p>m1</p> <p>A1 cso</p>	<p>5</p> <p>3</p>	<p>product rule ( condone one error)</p> <p>correct unsimplified</p> <p>or <math>\frac{9}{\sqrt{3^2-(x-2)^2}}</math> correct unsimplified</p> <p>last two terms above combined correctly</p> <p><math>k = 2</math></p> <p>ft "their" <math>k</math></p> <p>correct sub of limits (simplified at least this far)</p> <p>must have earned <b>5 marks</b> in part(a) to be awarded this mark</p>
	<b>Total</b>		<b>8</b>	
(a)	Second A1 ; may combine all three terms correctly and obtain $\frac{10+8x-2x^2}{\sqrt{5+4x-x^2}}$			

Q7	Solution	Mark	Total	Comment
(a)	$\alpha\beta + \beta\gamma + \gamma\alpha = 0$	B1	2	May use $\gamma$ instead of $\beta$ throughout (b)(i)  Clear attempt to eliminate either $\alpha$ or $\beta$ from “their” equations correct  all 3 roots clearly stated  or substituting correct root into equation  correctly substituting “their” $\alpha^2 = -2i$ and “their” $\alpha^3 = -2 - 2i$  may use any letter instead of $y$  sub their $z$ into cubic equation removing denominators correctly correct and $(y-1)^3$ expanded correctly  sum of new roots = 3  M1 for either of the other two formulae correct in terms of $\alpha\beta\gamma$ , $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha + \beta + \gamma$  may use any letter instead of $y$
	$\alpha\beta\gamma = -\frac{4}{27}$	B1		
(b)(i)	$\alpha\beta + \alpha\beta + \beta^2 = 0 ; \alpha\beta^2 = -\frac{4}{27}$	B1		
	$\alpha^3 = -\frac{1}{27}$ or $\beta^3 = \frac{8}{27}$	M1 A1		
	either $\alpha = -\frac{1}{3}$ or $\beta = \frac{2}{3}$ $\alpha = -\frac{1}{3}, \beta = \frac{2}{3}, \gamma = \frac{2}{3}$	A1 A1		
(ii)	$\left(\sum \alpha = 1 = -\frac{k}{27} \Rightarrow\right) k = -27$	B1	1	
(c)(i)	$\alpha^2 = -2i$	B1	2	
	$\alpha^3 = -2 - 2i$	B1		
(ii)	$27(-2 - 2i) - 2ik + 4 = 0$	M1	2	
	$k = -27 + 25i$	A1		
(d)	$y = \frac{1}{z} + 1 \Rightarrow z = \frac{1}{y-1}$	B1	5	
	$\frac{27}{(y-1)^3} - \frac{12}{(y-1)^2} + 4 = 0$	M1		
	$27 - 12(y-1) + 4(y-1)^3 = 0$	A1		
	$27 - 12y + 12 + 4(y^3 - 3y^2 + 3y - 1) = 0$	A1		
	$4y^3 - 12y^2 + 35 = 0$	A1		
	Alternative: $\sum \alpha' = 3 + \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma} = 3$	(B1)		
	$\sum \alpha' \beta' = 3 + \frac{2(\alpha\beta + \beta\gamma + \gamma\alpha) + \alpha + \beta + \gamma}{\alpha\beta\gamma}$	(M1)		
	$= 0$	(A1)		
	$\prod = 1 + \frac{\alpha\beta + \beta\gamma + \gamma\alpha + 1 + \alpha + \beta + \gamma}{\alpha\beta\gamma}$	(A1)		
	$= \frac{-35}{4}$	(A1)		
	$4y^3 - 12y^2 + 35 = 0$	(A1)	(5)	
<b>Total</b>			<b>17</b>	

Q8	Solution	Mark	Total	Comment
(a)(i)	$(\omega^5 =) \cos 2\pi + i \sin 2\pi = 1$ So $\omega$ is a root of $z^5 = 1$	<b>B1</b>	<b>1</b>	must have conclusion plus verification that $\omega^5 = 1$
(ii)	$\omega^2, \omega^3, \omega^4.$	<b>B1</b>	<b>1</b>	<b>OE</b> powers mod 5 ( must not include 1)
(b)(i)	$1 + \omega + \omega^2 + \omega^3 + \omega^4 = \frac{1 - \omega^5}{1 - \omega} = 0$	<b>B1</b>	<b>1</b>	or clear statement that sum of roots (of $z^5 - 1 = 0$ ) is zero
(ii)	$\left(\omega + \frac{1}{\omega}\right)^2 + \left(\omega + \frac{1}{\omega}\right) - 1$ $= \omega^2 + 2 + \frac{1}{\omega^2} + \omega + \frac{1}{\omega} - 1$ $= \frac{1 + \omega + \omega^2 + \omega^3 + \omega^4}{\omega^2} = 0$	<b>M1</b> <b>A1</b>	<b>2</b>	correct expansion <b>AG</b> correctly shown to = 0 do not allow simply multiplying by $\omega^2$
(c)	$\frac{1}{\omega} = \cos \frac{2\pi}{5} - i \sin \frac{2\pi}{5}$ $\Rightarrow \omega + \frac{1}{\omega} = 2 \cos \frac{2\pi}{5}$ Solving quadratic $\left(\omega + \frac{1}{\omega} = \frac{-1 \pm \sqrt{5}}{2}\right)$ Rejecting negative root since $\cos \frac{2\pi}{5} > 0$ Hence $\cos \frac{2\pi}{5} = \frac{\sqrt{5} - 1}{4}$	<b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b>	<b>4</b>	<b>SC1</b> if result merely stated must see both values must see this line for final <b>A1</b>  It is possible to score <b>SC1 M1 A1</b>
<b>Total</b>			<b>9</b>	
(b)(ii)	May replace $\frac{1}{\omega^2}$ by $\omega^3$ and $\frac{1}{\omega}$ by $\omega^4$ and/or 1 by $\omega^5$ in valid proof. <b>Alternative:</b> $1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0 \Rightarrow \frac{1}{\omega^2} + \frac{1}{\omega} + 1 + \omega + \omega^2 = 0$ <b>M1</b> $\left(\omega + \frac{1}{\omega}\right)^2 - 2 + \left(\omega + \frac{1}{\omega}\right) + 1 = 0 \Rightarrow \left(\omega + \frac{1}{\omega}\right)^2 + \left(\omega + \frac{1}{\omega}\right) - 1 = 0$ <b>A1</b>			