FP1 Mark Schemes from old P4, P5, P6 and FP1, FP2, FP3 papers (back to June 2002)

Please note that the following pages contain mark schemes for questions from past papers which were not written at an AS standard and may be less accessible than those you will find on future AS FP1 papers from Edexcel. Some questions would certainly worth more marks at AS level.

The standard of the mark schemes is variable, depending on what we still have – many are scanned, some are handwritten and some are typed.

The questions are available on a separate document, originally sent with this one.

Question number	Scheme	Marks
1. (a)	$w = \dfrac{22+4i}{6-8i} \times \dfrac{6+8i}{6+8i}$	M1
	$= \dfrac{100+200i}{100} = 1+2i$	A1, A1
	A1 for w correct as $\dfrac{100+200i}{100}$ or for 1 or for 2i	
	final A1 for 1+ 2i only.	(3)
	OR $22+4i = (a+bi)(6-8i)$ with	M1
	$6a+8b = 22$, $6b-8a = 4$	A1 + A1 (3)
	$\rightarrow a = 1,\ b = 2$	
(b)	$\arg z = \arctan \frac{4}{22}$ OR $\tan(\arg z) = \frac{4}{22}$	M1
	$\arg z = 0.18$ only	A1 (2)

[P4 January 2002 Qn 1]

Question number	Scheme	Marks
2. (a)	$\sum r^2 - \sum 1$	M1
	$\frac{1}{6}n(n+1)(2n+1) - n$	A1 B1
	correct completion to $\frac{1}{6}n(n-1)(2n+5)$ ⊛	M1 A1 (5)
(b)	As this answer is a +ve integer, $n(n-1)(2n+5)$ is exactly divisible by 6 (or equivalent argument)	M1 A1 (2)

[P4 January 2002 Qn 3]

3.	(a)	$f'(x) = 3x^2 + 1, (>0)$ or $\underline{\text{no solutions of } f'(x) = 0}$	M1, $\underline{\text{A1}}$
		No turning points, so $f(x)$ only crosses x-axis once	
		Hence α is only root of $f(x) = 0$	A1 csa (3)
	(b)	Using $\alpha - \dfrac{f(\alpha)}{f'(\alpha)}$ with $\alpha = 1.2 \rightarrow \underline{1.21}$ only	M1 $\underline{\text{A1}}$ (2)
	(c)	$f(1.205) = -0.045 < 0$, $f(1.215) = 0.0086 > 0$	M1
		α lies in interval $(1.205, 1.215)$ and is 1.21 to 3s.f.	A1 (2)

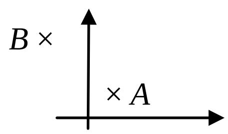
[P4 January 2002 Qn 4]

4.	(i)	Other root is $2 - i$	B1 (1)
	(ii)	$(2 + i)^2 + b(2 + i) + c = 0$ [or equivalent]	M1
		Imaginary parts $b = -4$	B1
		Real parts $c + 3 + 2b = 0$, $\underline{c = 5}$	M1$\underline{\text{A1}}$ (4)

[*P4 January 2002 Qn 5]

5.	$\Sigma 6r^2 - \Sigma 6 = n(n + 1)(2n + 1)$, $-\ 6n$	M1, A1
	$= n(2n^2 + 3n - 5)$	M1
	$= n(n - 1)(2n - 5)$ (*)	A1
		(4 marks)

[P4 June 2002 Qn 1]

6.	(*a*)	$\lvert w\rvert = \sqrt{50}$ (or equivalent)	B1 (1)
	(*b*)		B1 (1)
	(*c*)	$\lvert\overrightarrow{OA}\rvert = 5$	B1
		$\overrightarrow{BA} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ $\quad \lvert\overrightarrow{BA}\rvert = 5$, $\therefore$ isosceles	M1, A1
		$5^2 + 5^2 = (\sqrt{50})^2$, $\therefore$ right-angled (or gradient method)	M1, A1 (5)
	(*d*)	$\arg\left(\dfrac{z}{w}\right) = \arg z - \arg w$	M1
		$= (-)\angle AOB = \dfrac{\pi}{4}$	M1, A1 (3)
			(10 marks)

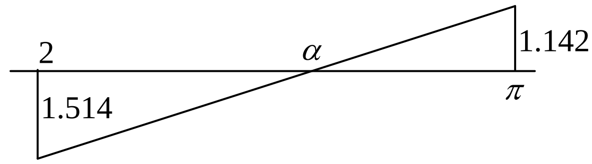
[P4 June 2002 Qn 5]

7.	$\dfrac{dy}{dx} = -\dfrac{4}{x^2}$; at $x = 2p$ $\dfrac{dy}{dx} = -\dfrac{1}{p^2}$	M1, A1
	Equation of tangent at P, $y - \dfrac{2}{p} = -\dfrac{1}{p^2}(x - 2p)$	M1
	$(y = -\dfrac{1}{p^2}x + \dfrac{4}{p}$, $\quad p^2y + x = 4p$ etc)	
	At Q $\quad q^2y + x + 4q$ $\quad$ Two correct equations in any form	A1
	$(p^2 - q^2)y = 4(p - q)$	M1
	$y = \dfrac{4}{p+q}$ $\quad$ (✱)	A1
	$x = 4p - \dfrac{4p^2}{p+q} = \dfrac{4pq}{p+q}$ $\quad$ (✱)	M1, A1 (8)

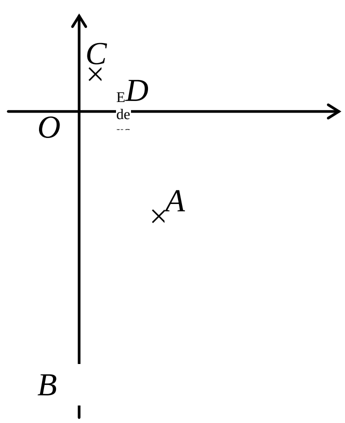
[*P5 June 2002 Qn 7]

8.	(*a*)	For $n = 1$ $\quad 2^5 + 5^2 = 57$, which is divisible by 3	M1, A1
		Assume true for $n = k$ $\quad (k+1)$th term is $2^{3k+5} + 5^{k+2}$	B1
		$(k+1)$th term $\pm$ kth term $= 2^{3k+5} + 5^{k+2} \pm 2^{3k+2} + 5^{k+1}$	M1
		$= 2^{3k+2}(2^3 \pm 1) + 5^{k+1}(5 \pm 1)$	M1, A1
		$= 6(2^{3k+2} + 5^{k+1}) + 3.\, 2^{3k+2}$ or $= 4(2^{3k+2} + 5^{k+1}) + 3.\, 2^{3k+2}$	M1
		which is divisible by 3 $\Rightarrow$ $(k+1)$th term is divisible by 3	A1
		Thus by induction true for all n $\qquad$ cso	B1 **(9)**
	(*b*)	For $n = 1$ $\quad$ RHS $= \begin{pmatrix} -2 & -1 \\ 9 & 4 \end{pmatrix}$	B1
		Assume true for $n = k$	
		$\begin{pmatrix} -2 & -1 \\ 9 & 4 \end{pmatrix}^{k+1} = \begin{pmatrix} -2 & -1 \\ 9 & 4 \end{pmatrix}\begin{pmatrix} 1-3k & -k \\ 9k & 3k+1 \end{pmatrix} = \begin{pmatrix} -2-3k & 2k-3k-1 \\ 9+9k & -9k+12k+4 \end{pmatrix}$	M1 A3/2/1/0 (-1 each error)
		$= \begin{pmatrix} 1-3(k+1) & -(k+1) \\ 9(k+1) & 3(k+1)+1 \end{pmatrix}$	B1
		$\therefore$ If true for k then true for $k+1$ $\quad \therefore$ by induction true for all n	B1 **(7)**
			(16 marks)

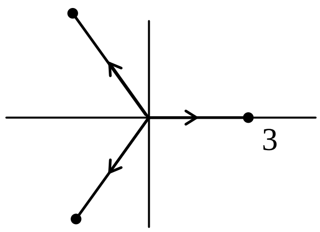
[P6 June 2002 Qn 6]

9.	f(2) = −1.514	B1
	f(π) = 1.142 	B1
	$\dfrac{\pi-\alpha}{\alpha-2}=\dfrac{1.142}{1.514}$	M1
	$\pi \times 1.514 + 2 \times 1.142 = (1.142 + 1.514)\alpha$	
	$\alpha = 2.65$	A1 **(4)**

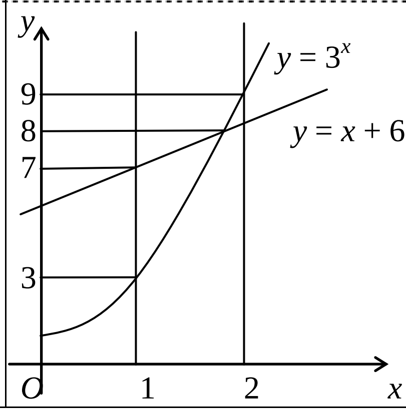
[*P4 January 2003 Qn 4]

10. (*a*)	$z^2 = (3 - 3\text{i})(3 - 3\text{i}) = -18\text{i}$	M1 A1 **(2)**
(*b*)	$\dfrac{1}{z}=\dfrac{(3+3\text{i})}{(3-3\text{i})(3+3\text{i})}=\dfrac{3+3\text{i}}{18}=\dfrac{1+\text{i}}{6}$	M1 A1 **(2)**
(*c*)	$\lvert z \rvert = \sqrt{(9 + 9)} = \sqrt{18} = 3\sqrt{2}$	
	$\lvert z \rvert = 18$ two correct	M1
	$\left\lvert\dfrac{1}{z}\right\rvert=\sqrt{\dfrac{1}{18}}=\dfrac{1}{3\sqrt{2}}=\dfrac{\sqrt{2}}{6}$ all three correct	A1 **(2)**
(*d*)	 two correct	B1
	four correct	B1 **(2)**
(*e*)	$\dfrac{OB}{OD}=18,\quad \dfrac{OA}{OC}=\dfrac{3\sqrt{2}}{\sqrt{2}/6}=18$	M1 A1
	$\angle AOB = \angle COD = 45 \quad \therefore$ similar	B1 **(3)**
		(11 marks)

[P4 January 2003 Qn 6]

11. (*a*)	$x^3 - 27 = (x-3)(x^2 + 3x + 9)$	M1	
	(x = 3 is one root). Others satisfy $(x^2 + 3x + 9) = 0$ (*)	A1	(2)
(*b*)	Roots are $x = 3$	B1	
	and $x = \dfrac{-3 \pm \sqrt{9-36}}{2}$	M1	
	$= -\dfrac{3}{2} + \dfrac{3\sqrt{3}}{2}\mathrm{i},\ -\dfrac{3}{2} - \dfrac{3\sqrt{3}}{2}\mathrm{i}$	A1	(3)
(*c*)	 3 and one other root in correct quad	B1	
	Root in complex conjugate posn.	B1 ft	(2)
		(7 marks)	

[#P4 June 2003 Qn 3]

12. (*a*)	f(1) = –4 f(2) = 1	M1	
	Change of sign (and continuity) implies $\alpha \in (1, 2)$	A1	(2)
(*b*)	f(1.5) = –2.3… $\Rightarrow 1.5 < \alpha < 2$	B1	
	f(1.75) = –0.9… $\Rightarrow 1.75 < \alpha < 2$		
	f(1.875) = –0.03… $\Rightarrow 1.875 < \alpha < 2$	B1	(2)
	NB Exact answer is 1.8789…		
Alt to (*a*)	 Two graphs with single point of intersection (x > 0)	M1	
	Two calculations at both x = 1 and x = 2	A1	

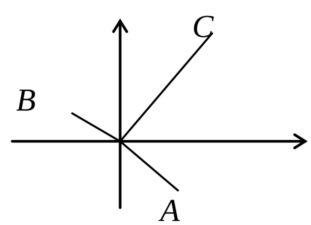
[*P4 June 2003 Qn 4]

13.	(*a*)	$\frac{4+3\mathrm{i}}{2+4\mathrm{i}} = \frac{(4+3\mathrm{i})(2-4\mathrm{i})}{20} = \frac{20-10\mathrm{i}}{20}$ $(= 1 - \tfrac{1}{2}\mathrm{i})$	M1
		$\lvert z \rvert = \sqrt{1^2 + (-\tfrac{1}{2})^2}$, $= \frac{\sqrt{5}}{2}$ awrt 1.12, accept exact equivalents	M1, A1 (3)
	(*b*)	$\frac{(a+3\mathrm{i})(2-a\mathrm{i})}{(2+a\mathrm{i})(2-a\mathrm{i})} = \frac{5a+(6-a^2)\mathrm{i}}{4+a^2}$ accept in (*a*) if clearly applied to (*b*)	M1
		$(\tan\tfrac{\pi}{4} =)\ 1 = \frac{6-a^2}{5a}$ obtaining quadratic or equivalent	M1 A1
		$a^2 + 5a - 6 = (a+6)(a-1)$	
		$a = -6, 1$	M1 A1
		Reject $a = -6$, wrong quadrant/$-\tfrac{3\pi}{4}$, $\Rightarrow$ one value	A1 (6)
			(9 marks)
Alt.	(*a*)	$\lvert 4+3\mathrm{i} \rvert = 5$, $\lvert 2+4\mathrm{i} \rvert = \sqrt{20}$	M1
		$\lvert z \rvert = \frac{5}{\sqrt{20}}$ $\left(= \frac{\sqrt{5}}{2}\right)$	M1 A1 (3)
	(*b*)	$\arg z = \arg(a+3\mathrm{i}) - \arg(2+a\mathrm{i})$	
		$\frac{\pi}{4} = \arctan\frac{3}{a} - \arctan\frac{a}{2}$	M1
		$1 = \dfrac{\frac{3}{a} - \frac{a}{2}}{1 + \frac{3}{2}}$	M1 A1 (3)
		leading to $a^2 + 5a - 6 = 0$, then as before	

[P4 June 2003 Qn 5]

14.	$f(1) = 3 \times 7 - 1 = 20$; divisible by 4	B1
	$f(k+1) = (2k+3)7^{k+1} - 1$	B1
	Showing that $f(k+1) = f(k) + 4m$ or equivalent	M1 A1
	e.g. $f(k+1) - f(k) = (2k+3)7^{k+1} - 1 - \{(2k+1)7^{k} - 1\}$	
	$= (12k+20)7^{k} = 4(3k+5)7^{k}$	
	If true for $n = k$, then true for $n = k+1$	M1
	Conclusion , with no wrong working seen.	A1

[P6 June 2003 Qn 2]

15. (*a*)	$\lvert z \rvert = 2\sqrt{2}$ $\quad \lvert w \rvert = 2$	M1, A1
	$\therefore \lvert wz^2 \rvert = (2\sqrt{2})^2 \times 2 = 16$	M1, A1
	$\arg z = -\frac{\pi}{4}$ $\quad \arg w = \frac{5\pi}{6}$; $\quad \therefore \ \arg wz^2 = -\frac{\pi}{4} - \frac{\pi}{4} + \frac{5\pi}{6} = \frac{\pi}{3}$, 60°	M1, A1 (6)
ALT	$z^2 = -8\text{i}$; $\therefore z^2w = 8 + 8\sqrt{3}\text{i}$	M1, A1
	$\lvert z^2w \rvert = \sqrt{8^2 + 8^2 \times 3}$	M1
	$= 16$	A1
	$\arg z^2w = \tan^{-1}\sqrt{3}$	M1
	$= \frac{\pi}{3}$	A1
(*b*)	 Points *A* and *B*	B1
	Point *C*	B1ft
	angle $BOC = \frac{5\pi}{6} - \frac{\pi}{3}$	M1
	$= \frac{\pi}{2}$, 90°	A1 (4)
		(10 marks)

[P4 January 2004 Qn 3]

16.	**(a)**	Expand brackets and attempt to use appropriate formulae.	**M1**
		$\sum r^2+6r+5=\frac{n}{6}(n+1)(2n+1)+6\frac{n}{2}(n+1)+5n$	**A1**
		$=\frac{n}{6}[2n^2+3n+1+18n+18+30]$	**M1**
		$=\frac{n}{6}[2n^2+21n+49]=\frac{n}{6}(n+7)(2n+7)$ *	**A1** **(4)**
	(b)	Use S(40) – S(9) = $\frac{40}{6}\times 47\times 87-\frac{9}{6}\times 16\times 25$	**M1**
		$=26660$	**A1** **(2)**

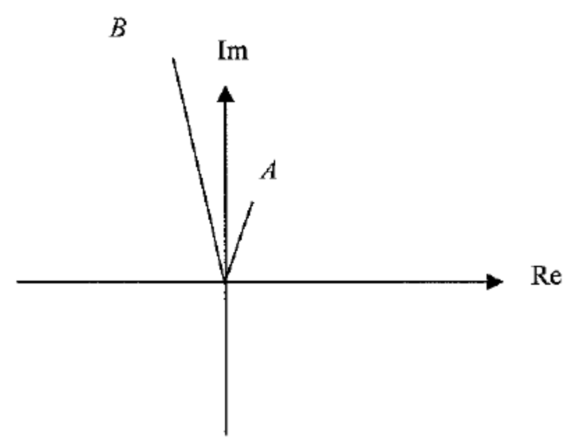
[P4 June 2004 Qn 1]

17.	$\mathrm{f}(1)=-1$ and $\mathrm{f}(2)=2$	**B1**
	$\frac{2}{1}=\frac{2-\alpha}{\alpha-1}\Rightarrow\ \alpha=1\frac{1}{3}$	**B1** **(2)**

[*P4 June 2004 Qn 2]

18.	**(a)**	$z=a+ib\rightarrow(a^2-b^2)+2abi=-16+30i$	**M1**
		Equating imaginary parts $2ab=30$ and thus $ab=15$ *	**A1** **(2)**
	(b)	Also $(a^2-b^2)=-16$	**B1**
		Attempt to solve by valid method involving elimination of unknown	**M1**
		$\therefore z=3+5i$ or $z=-3-5i$	**A1 A1** **(4)**

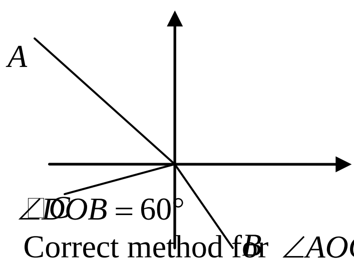
[P4 June 2004 Qn 3]

19.	**(a)**	$w=(1+\sqrt{3}i)(2+2i)$	
		$=(2-2\sqrt{3}),+(2\sqrt{3}+2)\,\mathrm{i}$	**M1** **A1, A1**
			(3)
	(b)	$\arg w = \arctan\left(\dfrac{2\sqrt{3}+2}{2-2\sqrt{3}}\right)$ or adds two args e.g. $60^{\circ}+45^{\circ}$	**M1**
		$=\dfrac{7\pi}{12}$ *or* 105^{o} or 1.83 radians	**A1**
			(2)
	(c)	$\lvert w\rvert=\sqrt{32}=4\sqrt{2}$	**M1 A1**
			(2)
	(d)	 f.t. w in quadrant other than first	**B1** **B1**
			(2)
	(e)	$\lvert AB\rvert^{2}=4+32-16\sqrt{2}\cos 45$ (=20), then square root	**M1**
		AB = $2\sqrt{5}$	**A1**
			(2)
		Or $\quad w-z=1-2\sqrt{3}+i(2+\sqrt{3})$	
		$\therefore AB=\lvert w-z\rvert=\sqrt{(1-2\sqrt{3})^{2}+(2+\sqrt{3})^{2}}$	**M1**
		$=\sqrt{20}=2\sqrt{5}$	**A1c.a.o**
			(2)

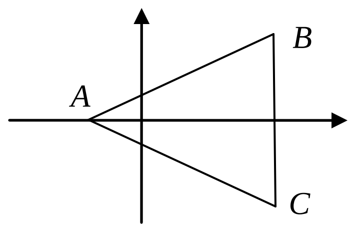
[P4 June 2004 Qn 5]

20.	$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-\frac{c}{t^2}}{c}=-\frac{1}{t^2}$$	M1 A1
	The normal to the curve has gradient t^2.	B1
	The equation of the normal is $y-\frac{c}{t}=t^2(x-ct)$	M1
	The equation may be written $y=t^2x+\frac{c}{t}-ct^3$ *	A1 (5)

[*P5 June 2004 Qn 8]

21.	(a) $\lvert z\rvert^2=\left(-2\sqrt{2}\right)^2+\left(2\sqrt{2}\right)^2=1$, $\lvert w\rvert^2=1^2+\left(\sqrt{3}\right)^2=4$	M1
	$\left\lvert\frac{z}{w}\right\rvert=\frac{\lvert z\rvert}{\lvert w\rvert}=\frac{4}{2}=2$	M1 A1 (3)
	(b) $\frac{z}{w}=\frac{-2\sqrt{2}+2\sqrt{2}\mathrm{i}}{1-\mathrm{i}\sqrt{3}}=\frac{-2\sqrt{2}+2\sqrt{2}\mathrm{i}}{1-\mathrm{i}\sqrt{3}}\times\frac{1+\mathrm{i}\sqrt{3}}{1+\mathrm{i}\sqrt{3}}$	M1
	$=\frac{\sqrt{2}}{4}\left(-1-\sqrt{3}-\mathrm{i}\left(\sqrt{3}-1\right)\right)$	
	$\arctan\frac{\sqrt{3}-1}{\sqrt{3}+1}=15^\circ$	M1
	In correct quadrant $\arg\left(\frac{z}{w}\right)=-165^\circ$ $\qquad -\frac{11\pi}{12}$	A1 (3)
	(c)  A B C ft	B1 B1 B1ft (3)
	(d) $\angle DOB=60^\circ$	B1
	Correct method for $\angle AOC$	M1
	$\angle AOC=45^\circ+15^\circ=60^\circ$	A1 (3)
	(e) ! $AOC=\frac{1}{2}\times4\times2\times\sin60^\circ=2\sqrt{3}$ $\qquad$ awrt 3.46	M1 A1 (2)

[#FP1/P4 January 2005 Qn 8]

22.	(a) $z^3+6z+20=(z+2)(z^2-2z+10)$	
	Long division or any complete method	M1
	$z=\dfrac{2\pm\sqrt{(4-40)}}{2}=1\pm 3\mathrm{i}$	M1 A1 (3)
	(b)  In correct quadrants and on negative x - axis only required	B1ft (1)
	(c) $m_{AB}=\dfrac{3}{3}=1,\quad m_{AC}=-1$ Full method	M1
	$m_{AB}m_{AC}=-1 \Rightarrow$ triangle is right angled	A1 (2)
		(6)
	Alternative to (c)	
	$AB^2+AC^2=18=18=36=BC^2$	
	Result follows by (converse of) Pythagoras, or any complete method.	M1 A1

[#FP1/P4 June 2005 Qn 2]

23.	$\mathrm{f}(1.2)=-0.2937\ \ldots$ $\qquad$ $\mathrm{f}(1.2)$ to 1sf or better	B1
	$\mathrm{f}(1.1)=0.42\ ...,\quad \mathrm{f}(1.15)=-2.05\ \ldots$ $\qquad$ Attempt at $\mathrm{f}(1.1)$, $\mathrm{f}(1.15)$	M1
	$\alpha\approx 1.2$	A1 (3)

[*FP1/P4 June 2005 Qn 4]

24.	(a).	$\frac{\pi}{2} + \arctan\frac{4}{6}$ or $\pi - \arctan\frac{6}{4}$ or equiv. in degrees	M1
		$\arg z = \underline{2.159}$	A1 c.a.o. (2)
	(b)	$\lvert w \rvert = \sqrt{20} \Rightarrow \sqrt{20} = \frac{A}{\sqrt{5}}$ Full method for A using $\lvert w \rvert = \sqrt{20}$	M1
		$\Rightarrow \underline{A = 10}$	A1
		$w = \frac{A}{2-i} \times \left(\frac{2+i}{2+i}\right)$, $\underline{w = 4 + 2i}$	M1, A1 (4)
	(c)	$\arg\left(\frac{w}{z}\right) = \arg w - \arg z = \arctan\left(\frac{2}{4}\right) - (a)$	M1
		$= 0.463\ldots - 2.159$ 2dp or better	A1 ft (a)
		$= -1.695\ldots$ awrt $\underline{-1.70}$	A1 (3)
	ALT (c)	$\frac{w}{z} = -0.0769\ldots \quad -0.6153\ldots i$ Attempt $w \div z$ and use arctan	M1 (9)
		$\Rightarrow \arg\left(\frac{w}{z}\right) = -\left[\pi - \arctan\left(\frac{0.6153\ldots}{0.0769\ldots}\right)\right]$ Expression (2dp)	A1
		$=$ awrt $\underline{-1.70}$	A1 (3)

[FP1/P4 June 2005 Qn 5]

25.	(a)	$2y\frac{dy}{dx} = 4a \quad \frac{dy}{dx} = \frac{4a}{2y} = \frac{1}{p}$	B1
		$y - 2ap = \frac{1}{p}\left(x - ap^2\right), \qquad py = x + ap^2 \qquad (*)$	M1,A1 (3)
	(b)	At Q, parameter $= 4p \qquad 4py = x + 16ap^2$	M1 A1 (2)

[*FP2/P5 June 2005 Qn 5]

26.	(a) $\frac{6x+10}{x+3} = 6 - \frac{8}{x+3}$	B1 (1)
	(b) $u_1 = 5.2 > 5$	B1
	If result true for $n = k$, i.e. $u_k > 5$, $u_{k+1} = 6 - \frac{8}{u_k + 3}$	
	If $u_k > 5$, then $\frac{8}{u_k + 3} < 1$ so $u_{k+1} > 5$	M1A1
	Hence result is true for $n = k + 1$ Conclusion and no wrong working seen	A1 (4)
		[5]

[FP3/P6 June 2005 Qn 1]

Question number	Scheme	Marks
27.	$\sum_{r=1}^{n}(r-1)(r+2) = \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r - \left(\sum_{r=1}^{n}\right)2$	M1
	$= \frac{1}{6}n(n+1)(2n+1) + \frac{1}{2}n(n+1), -2n$	A1, A1
	$= \frac{1}{6}n\left(2n^2 + 6n - 8\right)$ M: Use factor n and use common denom. (e.g.3, 6, 12)	M1
	$= \frac{1}{3}n\left(n^2 + 3n - 4\right) = \frac{1}{3}(n-1)n(n+4)$ M: Attempt complete factorisation (*)	M1 A1 cso (6)
		Total 6 marks

[FP1/P4 January 2006 Qn 1]

28.	(a) $z+2\text{i}=\text{i}z+\lambda$ $\qquad (1-\text{i})z=\lambda-2\text{i}, \qquad z=\frac{\lambda-2\text{i}}{1-\text{i}}$	M1, A1
	$z=\frac{\lambda-2\text{i}}{1-\text{i}}\times\frac{1+\text{i}}{1+\text{i}}, \quad =\frac{1}{2}(.........)$	M1, A1
	$=\left(\frac{\lambda}{2}+1\right)+\left(\frac{\lambda}{2}-1\right)\text{i}$ (*)	A1 cso (5)
	(b) $\frac{\frac{\lambda}{2}-1}{\frac{\lambda}{2}+1}=\frac{1}{2}, \quad \lambda=6$ $\qquad$ 2nd M: Solving $\frac{\frac{\lambda}{2}-1}{\frac{\lambda}{2}+1}=k$ (constant k)	M1, M1 A1 (3)
	(c) $z=4+2\text{i}, \quad |z|^2=4^2+2^2=20$ M: Subs. λ value and attempt $|z|$ or $|z|^2$	M1 A1 (2)
		Total 10 marks

[FP1/P4 January 2006 Qn 3]

29.	(a) $\text{f}(1.8)=19.6686\ ...\ -20=-0.3313\ \ldots$	awrt ±0.33	B1
	$\text{f}(2)=20.6424\ ...\ =0.6424\ \ldots$	awrt ±0.64	B1
	$\frac{\alpha-1.8}{\text{"}0.33\text{"}}=\frac{2-\alpha}{\text{"}0.64\text{"}}$	or equivalent	M1
	$\alpha\approx1.87$	cao	A1 (4)
	(b) 112 (min) (1 hr 52 min)		B1 (1) **(5)**

[#*FP1/P4 January 2006 Qn 5]

30.	(a) Correct method for finding $\frac{\mathrm{d}y}{\mathrm{d}x}$ $\left[\frac{1}{P}\right]$	M1
	Gradient of normal $= -\ p$	A1
	Equation of normal: $y - 2ap = (-p)(x - ap^2)$	M1
	$y + \ px = \ 2ap \ + ap^3$ AG	A1* (4)
	(b) Using both equations and eliminating x or y	M1
	$(p-q)x = 2a(p-q) + a(p^3 - q^3)$ may be unsimplified	A1
	$x = 2a + a\,(p^2 + pq + q^2)$	A1
	Finding the other coordinate	M1
	$y = -\,apq(\ p + q)$	A1 (5)

[*FP2/P5 Jan 2006 Qn 9]

31.	Complete method for finding image: e.g. $\begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ 2x+1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$	M1
	The image is the point $(2, -1)$	A1 (2)

[*FP3/P6 Jan 2006 Qn 3]

32.	When $n = 1$, LHS $= 1(2)^1 = 2$; RHS $= 2\{1 + 0\} = 2 \Rightarrow$ true for $n = 1$	B1
	Suppose true for $n = k$, then	
	$\sum_{1}^{k+1} r2^r = 2\{1 + (k-1)2^k + (k+1)2^{k+1}\}$	M1
		A1
	$= 2 + k2^{k+1} + k2^{k+1}$	
	$= 2(1 + k2^{k+1})$	M1
	$= 2[1 + \{(k+1) - 1\}2^{k+1}]$	A1(cso)
		(5)
	So, if true for $n = k$ then true for $n = k + 1$, but true for $n = 1$, $\therefore$ true, by induction, for all values of $n \in Z^+$.	

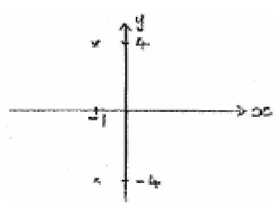
[*FP3/P6 January 2006 Qn 5]

33 .	(a) $2z + \mathrm{i}w = -1$		
	$\mathrm{i}z - \mathrm{i}w = 3\mathrm{i} - 3$		
	Adding $2z + \mathrm{i}z = -4 + 3\mathrm{i}$ Eliminating either variable	M1	
	$z = \dfrac{-4+3\mathrm{i}}{2+\mathrm{i}}$	A1	
	$z = \dfrac{-4+3\mathrm{i}}{2+\mathrm{i}} \times \dfrac{2-\mathrm{i}}{2-\mathrm{i}}$	M1	
	$= \dfrac{-8+3+4\mathrm{i}+6\mathrm{i}}{5}$		
	$= -1 + 2\mathrm{i}$	A1	(4)
	(b) $\arg z = \pi - \underline{\arctan 2}$	<u>M1</u>	
	≈ 2.03 cao	A1	(2)
			(6 marks)

[FP1 June 2006 Qn 1]

34.	(a)	$\text{f}(0.24) \approx -0.058,\ \text{f}(0.28) = 0.089$ accept 1sf	M1
		Change of sign (and continuity) $\Rightarrow\ \alpha \in (0.24, 0.28)$	A1 (2)
	(b)	$\text{f}(0.26) \approx 0.017 \quad (\Rightarrow \quad \alpha \in (0.24, 0.26))$ accept 1sf	M1
		$\text{f}(0.25) \approx -0.020 \quad (\Rightarrow \quad \alpha \in (0.25, 0.26))$	
		$\text{f}(0.255) \approx -0.001 \quad \Rightarrow \quad \alpha \in (0.255, 0.26)$	M1 A1 (3)
			(5)

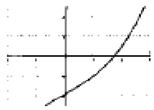
[*FP1 June 2006 Qn 6]

Question Number	Scheme	Marks
35.	(a) Method for finding z : $z = \dfrac{-2 \pm \sqrt{4-68}}{2}$, $= \dfrac{-2 \pm \sqrt{64}\, i}{2}$	M1, A1
	[Completing the square: $(z+1)^2 + 16 = 0$, $z = -1 \pm \sqrt{16}\, i$ M1,A1	A1 (3)
	$z = -1 \pm 4i$ $(a = -1,\ b = \pm 4)$	
	(b) 	B1 √ (1)
		[4]

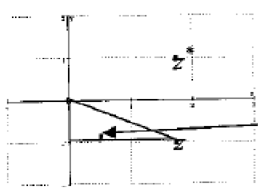
[FP1 Jan 2007 Qn 1]

36.	(a) $\frac{z_2}{z_1} = \frac{1+pi}{5+3i} \cdot \frac{(5-3i))}{(5-3i)}$	M1
	$= \frac{5+5pi-3i+3p}{(34)}$ [Multiply out and attempt use of $i^2 = -1$]	M1
	$= \frac{5+3p}{34} + \frac{5p-3}{34}i$ or $\frac{5+3p}{34} - \frac{3-3p}{34}i$	A1 (3)
	(b) For $\frac{z_2}{z_1} = c + id$ using $\frac{d}{c} = \tan\frac{\pi}{4}$:	M1
	$[\ 5p - 3 = 5 + 3p\ ]$ $\Rightarrow p = 4$	A1 (2)
		[5]
	Notes: In (a) if $\frac{z_1}{z_2}$ used treat as MR. Can score (a)M1M1A0 (b)M1A0 $\left[(a)\frac{5+3p}{1+p^2} + \frac{3-5p}{1+p^2}i \quad (b)\ -\frac{1}{4}\right]$ Allow A1 if answer "all over" 34, real and imag. collected up) $1 + pi = (a+ib)(5+3i)$: M1 compare real and imag. is first M mark If denominator in (a) incorrect, both marks in (b) still available In (b), if use $\arg z_2 - \arg z_1 = \frac{\pi}{4}$: M1 for $\arctan p - \arctan\frac{3}{5} = \frac{\pi}{4}$ $[\arctan p = \frac{\pi}{4} + 0.5404.... = 1.3258]$ Allow A1 for p = 4 without further work or for that shown in brackets, i.e. assume values retained on calculator (no penalty because it looks as though not exact)	

[FP1 Jan 2007 Qn 3]

37.	(a) $f'(x) = 3x^2 + 8$ $\quad 3x^2 + 8 = 0.......$ or $3x^2 + 8 > 0.......$ Correct derivative and, e.g., 'no turning points' or 'increasing function'.	M1 A1	
	 Simple sketch, (increasing, crossing positive x-axis) (or, if the M1 A1 has been scored, a <u>reason</u> such as 'crosses x-axis only once').	B1	(3)
	(b) Calculate $f(1)$ and $f(2)$ (<u>Values</u> must be seen) $f(1) = -10$, $f(2) = 5$, Sign change, $\therefore$ Root	M1 A1	(2)
	(c) $x_1 = 2 - \dfrac{f(2)}{f'(2)}$, $\quad = 2 - \dfrac{5}{20}$ $\quad (= 1.75)$	M1, A1	
	$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$, $\quad \left(= 1.75 - \dfrac{0.359375}{17.1875}\right) = 1.729$ (ONLY) (α)	M1, A1	(4)
	(d) Calculate $f(\alpha - 0.0005)$ and $f(\alpha + 0.0005)$ (or a 'tighter' interval that gives a sign change). $f(1.7285) = -0.0077...$ and $f(1.7295) = 0.0092...$, $\therefore$ Accurate to 3 d.p.	M1 A1	 (2)
			11
	(a) M: Differentiate and consider sign of $f'(x)$, or equate $f'(x)$ to zero. <u>Alternative</u>: M1: Attempt to rearrange as $x^3 - 19 = -8x$ or $x^3 = 19 - 8x$ (condone sign slips), and to sketch a cubic graph and a straight line graph. A1: Correct graphs (shape correct and intercepts 'in the right place'). B1: Comment such as "one intersection, therefore one root"). (c) 1st A1 can be implied by an answer of 1.729, provided N.R. has been used. <u>Answer only</u>: No marks. The Newton-Raphson method must be seen. (d) For A1, correct <u>values</u> of f(1.7285) and f(1.7295) must be seen, together with a conclusion. If only 1 s.f. is given in the values, allow rounded (e.g. -0.008) <u>or</u> truncated (e.g. -0.007) values.		

[FP1 June 2007 Qn 4]

38.	(a) $z^* = \sqrt{3} + \mathrm{i}$	B1	
	$\dfrac{z}{z^*} = \dfrac{(\sqrt{3}-\mathrm{i})(\sqrt{3}-\mathrm{i})}{(\sqrt{3}+\mathrm{i})(\sqrt{3}-\mathrm{i})} = \dfrac{3-2\sqrt{3}\mathrm{i}-1}{3+1}, \quad = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\mathrm{i}$ (*)	M1, A1cso	(3)
	(b) $\left\|\dfrac{z}{z^*}\right\| = \sqrt{\left(\dfrac{1}{2}\right)^2 + \left(\dfrac{\pm\sqrt{3}}{2}\right)^2}, \quad = 1$ $\left[\text{Or: } \left\|\dfrac{z}{z^*}\right\| = \dfrac{\|z\|}{\|z^*\|} = \dfrac{\sqrt{3+1}}{\sqrt{3+1}}, \quad = 1\right]$	M1, A1	(2)
	(c) $\arg(w) = \arctan\left(\pm\dfrac{\text{imag}(w)}{\text{real}(w)}\right)$ or $\arg(w) = \arctan\left(\pm\dfrac{\text{real}(w)}{\text{imag}(w)}\right)$, where w is z or z^* or $\dfrac{z}{z^*}$	M1	
	$\arg\left(\dfrac{z}{z^*}\right) = \arctan\left(\dfrac{-\sqrt{3}/2}{1/2}\right) \quad = -\dfrac{\pi}{3}$	A1	
	$\arctan\left(\dfrac{-1}{\sqrt{3}}\right) = -\dfrac{\pi}{6}$ and $\arctan\left(\dfrac{1}{\sqrt{3}}\right) = \dfrac{\pi}{6}$ (Ignore interchanged z and z^*)	A1	
	$\arg z - \arg z^* = -\dfrac{\pi}{6} - \dfrac{\pi}{6} = -\dfrac{\pi}{3} = \arg\left(\dfrac{z}{z^*}\right)$	A1	(4)
	(d)  z and z^* (Correct quadrants, approx. symmetrical)	B1	
	$\dfrac{z}{z^*}$ (Strictly <u>inside</u> the triangle shown here)	B1	(2)
	(e) $(x-(\sqrt{3}-\mathrm{i}))(x-(\sqrt{3}+\mathrm{i}))$ Or: Use sum of roots $\left(=\dfrac{-b}{a}\right)$ and product of roots $\left(=\dfrac{c}{a}\right)$.	M1	
	$x^2 - 2\sqrt{3}x + 4$	A1	(2) **13**
	(a) M: Multiplying both numerator and denominator by $\sqrt{3}-\mathrm{i}$, and multiplying out brackets with <u>some</u> use of $\mathrm{i}^2 = -1$. (b) Answer 1 with no working scores both marks. (c) Allow work in degrees: $-60°$, $-30°$ and $30°$ Allow arg between 0 and 2π: $\dfrac{5\pi}{3}$, $\dfrac{11\pi}{6}$ and $\dfrac{\pi}{6}$ (or 300°, 330° and 30°). Decimals: Allow marks for awrt -1.05 (A1), -0.524 and 0.524 (A1), but then A0 for final mark. (Similarly for 5.24 (A1), 5.76 and 0.524 (A1)). (d) Condone wrong labelling (or lack of labelling), if the intention is clear.		

[FP1 June 2007 Qn 6]

39.	(a) Gradient of $PQ = \dfrac{2ap - 2aq}{ap^2 - aq^2} = \dfrac{2}{p+q}$ Can be implied	B1
	Use of any correct method or formula to obtain an equation of PQ in any form.	M1
	Leading to $(p+q)y = 2(x + apq)$ ✱	A1 **(3)**
	(b) Gradient of normal at P is $-p$. Given or implied at any stage	B1
	Obtaining any correct form for normal at either point. Allow if just written down.	M1 A1
	$y + px = 2ap + ap^3$	
	$y + qx = 2aq + aq^3$	
	Using both normal equations and eliminating x or y. Allow in any unsimplified form.	M1
	$(p-q)x = 2a(p-q) + a(p^3 - q^3)$ Any correct form for x or y	A1
	Leading to $x = a(p^2 + q^2 + pq + 2)$ ✱ cso	A1
	$y = -apq(p+q)$ ✱ cso	A1 **(7)**

[*FP2 June 2007 Qn8]

40.	$n=1$: $\quad 1^2 = \frac{1}{3}\times 1\times 1\times 3$	B1
	(Hence result is true for $n=1$.)	
	$\sum_{r=1}^{k+1}(2r-1)^2 = \sum_{r=1}^{k}(2r-1)^2 + (2k+1)^2$	
	$=\frac{1}{3}k(2k-1)(2k+1)+(2k+1)^2$, by induction hypothesis	M1
	$=\frac{1}{3}(2k+1)(2k^2-k+6k+3)$	
	$=\frac{1}{3}(2k+1)(2k^2+5k+3)$	
	$=\frac{1}{3}(2k+1)(2k+3)(k+1)$	M1 A1
	$=\frac{1}{3}(k+1)[2(k+1)-1][2(k+1)+1]$	
	(Hence, if result is true for $n=k$, then it is true for $n=k+1$.)	
	By Mathematical Induction, above implies the result is true for all $n \in \mathbb{Z}^+$. ∗	
	cso	A1 **(5)**
		(5 marks)

[FP3 June 2007 Qn5]

41.	(a) $\mathrm{f}(k+1)-\mathrm{f}(k)=3^{4k+4}+2^{4k+6}-3^{4k}-2^{4k+2}$		
	$=3^{4k}\left(3^4-1\right)+2^{4k+2}\left(2^4-1\right)$	M1	
	$=3^{4k}\times 80+2^{4k+2}\times 15$ can be implied	A1	
	$=3^{4k-1}\times 240+2^{4k+2}\times 15=15\left(16\times 3^{4k-1}+2^{4k+2}\right)$	M1	
	Hence $15 \mid \mathrm{f}(k+1)-\mathrm{f}(k)$ ∗ cso	A1	**(4)**
	Note: $\mathrm{f}(k+1)-\mathrm{f}(k)$ is divisible by 240 and other appropriate multiples of 15 lead to the required result.		
	(b) $n=1$: $\mathrm{f}(1)=3^4+2^6=145=5\times 29 \Rightarrow 5 \mid \mathrm{f}(1)$	B1	
	(Hence result is true for $n=1$.)		
	From (a) $\mathrm{f}(k+1)-\mathrm{f}(k)=15\lambda$, say. By induction hypothesis $\mathrm{f}(k)=5\mu$, say.		
	$\mathrm{f}(k+1)=\mathrm{f}(k)+15\lambda=5(\mu+3\lambda) \Rightarrow 5 \mid \mathrm{f}(k+1)$	M1	
	(Hence, if result is true for $n=k$, then it is true for $n=k+1$.)		
	By Mathematical Induction, above implies the result is true for all $n\in\mathbb{Z}^+$. ∗		
	Accept equivalent arguments cso	A1	**(3)**
		(7 marks)	

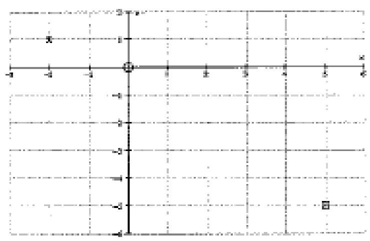
[*FP3 June 2007 Qn6]

42.	Use (2x+1) as factor to give $f(x) = (2x+1)(x^2 - 6x + 10)$ Attempt to solve quadratic to give $x = \frac{6 \pm \sqrt{(36-40)}}{2}$ Two complex roots are $= 3 \pm \mathrm{i}$	M1 A1 M1 A1 M1 A1 (6) **[6]**
	Notes: First M if method results in quadratic expression with 3 terms (even with remainder). Second M for use of correct formula on their quadratic. Third M for using i from negative discriminant.	

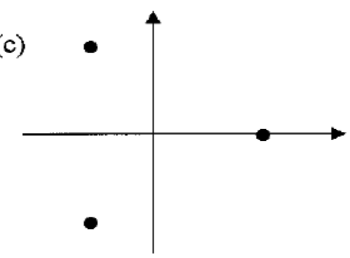
[FP1 January 2008 Qn 2]

43.	$\mathrm{f}(0.7) = -0.195\,028\,497$, $\mathrm{f}(x)0.8 = 0.297\,206\,781$ 3 dp or better Using $\frac{0.8-\alpha}{\alpha-0.7} = \frac{\mathrm{f}(0.8)}{-\mathrm{f}(0.7)}$ to obtain $\alpha = \frac{-0.8\,\mathrm{f}(0.7) + 0.7\,\mathrm{f}(0.8)}{\mathrm{f}(0.8) - \mathrm{f}(0.7)}$ $\alpha = 0.739\,620\,991$ 3dp or better	B1, B1 M1 A1 (4)

[*FP1 January 2008 Qn 4]

44.	(i) Multiply top and bottom by conjugate to give $\frac{-2-i}{5}$	M1 A1
	(ii) Expand and simplify using $i^2 = -1$ to give $3-4\mathrm{i}$	M1 A1 (4)
(b)	$z^2 - z = 5 - 5i,\quad \lvert z^2 - z \rvert = 5\sqrt{2}$ *	M1A1 (2)
(c)	$\arg(z^2 - z) = -\frac{\pi}{4}$ or -45^0 or $7\pi/4$ or 315^0 or -0.7853…or 5.497…	M1 A1 (2)
(d)	 one mark for each point	B1, B1 ft (2)
		[10]
	Notes: (a) -2-*i* or 2+*i* OK for method. Attempt to expand required. (b) square root required for method (c) 2 for correct answer only, tan required for method. 2dp or better. (d) Position of points not clear but both quadrants correct first B1 only.	

[FP1 January 2008 Qn 6]

45.	(a) 4	B1	(1)
	(b) $(x-4)(x^2+4x+16)$	M1 A1	
	$x=\frac{-4\pm\sqrt{16-64}}{2}$, $\quad x=-2\pm2\sqrt{3}\,\mathrm{i}$ (or equiv. surd for $2\sqrt{3}$)	M1, A1	(4)
	(c)  Root on + ve real axis, one other in correct quad.	B1	
	Third root in conjugate complex position	B1ft	(2)
			7
	M1 in part (b) needs(x-"their 4") times quadratic($x^2+ax+..$)or times(x^2+16)		
	M1 needs solution of three term quadratic		
	So (x^2+16) special case, results in B1M1A0M0A0B0B1 possibly		
	Alternative scheme for (b)		
	$(a+ib)^3=64$, so $a^3+3a^2ib+3a(ib)^2+(ib)^3=64$ and equate real, imaginary parts	M1	
	so $a^3-3ab^2=64$ and $3a^2b\ -b^3=0$	A1	
	Solve to obtain $a=-2,\ b=\sqrt{12}$	M1A1	
	Alternative ii		
	$(x-4)(x-a-ib)(x-a+ib)=0$ expand and compare coefficients	M1	
	two of the equations $-2a-4=0,\ 8a+a^2+b^2=0,\ 4(a^2+b^2)=64$	A1	
	Solve to obtain $a=-2,\ b=\sqrt{12}$	M1A1	
	(c)Allow vectors, line segments or points in Argand diagram.		
	Extra points plotted in part (c) – lose last B mark		
	Part (c) answers are independent of part (b)		

[FP1 June 2008 Qn 1]

46.	(a) $z = \dfrac{(a+2\mathrm{i})(a+\mathrm{i})}{(a-\mathrm{i})(a+\mathrm{i})} = \dfrac{a^2+3a\mathrm{i}-2}{a^2+1}$	M1 A1	
	$\dfrac{a^2-2}{a^2+1} = \dfrac{1}{2}$, $2a^2-4 = a^2+1$ $a=\sqrt{5}$ (presence of $-\sqrt{5}$ also is A0)	M1, A1	(4)
	(b) Evaluating their " $\dfrac{3a}{a^2+1}$ ", or " $3a$ " $\left(\dfrac{\sqrt{5}}{2} \text{ or } 3\sqrt{5}\right)$ (ft errors in part a)	B1ft	
	$\tan\theta = \dfrac{3a}{a^2-2}$ $\left(=\dfrac{3\sqrt{5}}{3}\right)$, $\arg z = 1.15$ (accept answers which round to 1.15)	M1, A1	(3)
			7
	(b) B mark is treated here as a method mark The M1 is for tan (argz) = Imaginary part / real part answer in degrees is A0		
	<u>Alternative method</u>:		
	(a) $\left(\dfrac{1}{2}+\mathrm{i}y\right)(a-\mathrm{i}) = a+2\mathrm{i} \Rightarrow \dfrac{1}{2}a+y=a$ and $ay-\dfrac{1}{2}=2$	M1 A1	
	$y=\dfrac{1}{2}a$ and $ay=\dfrac{5}{2} \Rightarrow \dfrac{1}{2}a^2=\dfrac{5}{2} \Rightarrow a=\sqrt{5}$	M1 A1	(4)
	(b) $y=\dfrac{\sqrt{5}}{2}$ (May be seen in part (a))	B1ft	
	$\tan\theta=\sqrt{5}$ $\arg z = 1.15$	M1 A1	(3)
	<u>Further Alternative method in (b)</u>		
	Use $\arg(a+2\mathrm{i})-\arg(a-\mathrm{i})$	B1	
	$= 0.7297-(-0.4205) = 1.15$	M1A1	(3)

[FP1 June 2008 Qn 3]

47.	(a)	$\begin{pmatrix} k & -2 \\ 1-k & k \end{pmatrix}\begin{pmatrix} t \\ 2t \end{pmatrix} = \begin{pmatrix} t(k-4) \\ t(1+k) \end{pmatrix}$ $t(1+k) = 2t(k-4)$ $k = 9$	M1 dM1 A1 (3)
	(b)	det $\mathbf{A} = k^2 + 2(1-k)$ (Must be seen in part (b)) $= (k-1)^2 + 1$, which is always positive $\mathbf{A}$ is non-singular	M1 M1 A1cso (3)
	(c)	$\mathbf{A}^{-1} = \dfrac{1}{k^2-2k+2}\begin{pmatrix} k & 2 \\ k-1 & k \end{pmatrix}$	M1 A1 (2)
	(d)	$k = 3, \quad \mathbf{A}^{-1} = \frac{1}{5}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$ $\mathbf{Ap} = \mathbf{q} \Rightarrow \mathbf{p} = \mathbf{A}^{-1}\mathbf{q} \qquad \mathbf{p} = \frac{1}{5}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 6 \\ -1 \end{pmatrix}$ Alt. $\begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \Rightarrow 3x - 2y = 4, -2x + 3y = -3$ B1 M1 A1 for solving two sim. eqns. in x and y to give $x = 1.2$, $y = -0.2$ (o.e.)	B1 M1 A1 (3) **(11)**
		(b) 2nd M: Alternative is to use quadratic formula on the quadratic equation, or to use the discriminant, with a <u>comment</u> about 'no real roots', or 'can't equal zero', or a comment about the condition for singularity. $\left(x = \dfrac{2 \pm \sqrt{4-8}}{2}\right)$ A1 Conclusion. (c) M: Need $\dfrac{1}{\text{their det A}}$, k's unchanged and attempt to change sign for either -2 (leaving as top right) or $1-k$ (leaving as bottom left). (d) M: Requires an attempt to multiply the matrices.	

[FP3 June 2008 Qn 5]