

General Certificate of Education  
January 2006  
Advanced Level Examination



**MATHEMATICS**  
**Unit Mechanics 2B**

**MM2B**

Monday 16 January 2006 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take  $g = 9.8 \text{ m s}^{-2}$ , unless stated otherwise.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

**Advice**

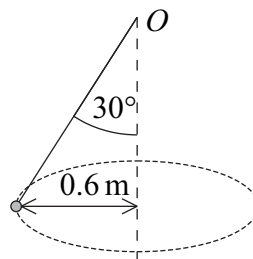
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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- 1 A stone, of mass  $0.4 \text{ kg}$ , is thrown vertically upwards with a speed of  $8 \text{ m s}^{-1}$  from a point at a height of  $6 \text{ metres}$  above ground level.
- (a) Calculate the initial kinetic energy of the stone. *(2 marks)*
- (b) (i) Show that the kinetic energy of the stone when it hits the ground is  $36.3 \text{ J}$ , correct to three significant figures. *(2 marks)*
- (ii) Hence find the speed at which the stone hits the ground. *(3 marks)*
- (iii) State one assumption that you have made. *(1 mark)*
- 2 A particle, of mass  $2 \text{ kg}$ , is attached to one end of a light inextensible string. The other end is fixed to the point  $O$ . The particle is set into motion, so that it describes a horizontal circle of radius  $0.6 \text{ metres}$ , with the string at an angle of  $30^\circ$  to the vertical. The centre of the circle is vertically below  $O$ .



- (a) Show that the tension in the string is  $22.6 \text{ N}$ , correct to three significant figures. *(3 marks)*
- (b) Find the speed of the particle. *(4 marks)*

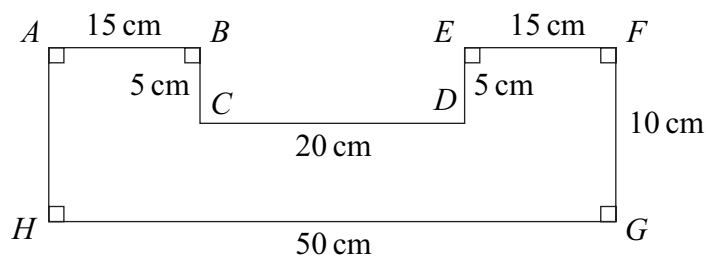
- 3 A particle moves in a straight line and at time  $t$  has velocity  $v$ , where

$$v = 2t - 12e^{-t}, \quad t \geq 0$$

- (a) (i) Find an expression for the acceleration of the particle at time  $t$ . (2 marks)
- (ii) State the range of values of the acceleration of the particle. (3 marks)
- (b) When  $t = 0$ , the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time  $t$ . (4 marks)

- 4 The diagram shows a uniform lamina  $ABCDEFGH$ .



- (a) Explain why the centre of mass is 25 cm from  $AH$ . (1 mark)
- (b) Show that the centre of mass is 4.375 cm from  $HG$ . (4 marks)
- (c) The lamina is freely suspended from  $A$ . Find the angle between  $AB$  and the vertical when the lamina is in equilibrium. (4 marks)
- (d) Explain, briefly, how you have used the fact that the lamina is uniform. (1 mark)

- 5 A particle moves such that at time  $t$  seconds its acceleration is given by

$$(2 \cos t\mathbf{i} - 5 \sin t\mathbf{j}) \text{ m s}^{-2}$$

- (a) The mass of the particle is 6 kg. Find the magnitude of the resultant force on the particle when  $t = 0$ . (3 marks)
- (b) When  $t = 0$ , the velocity of the particle is  $(2\mathbf{i} + 10\mathbf{j}) \text{ m s}^{-1}$ .

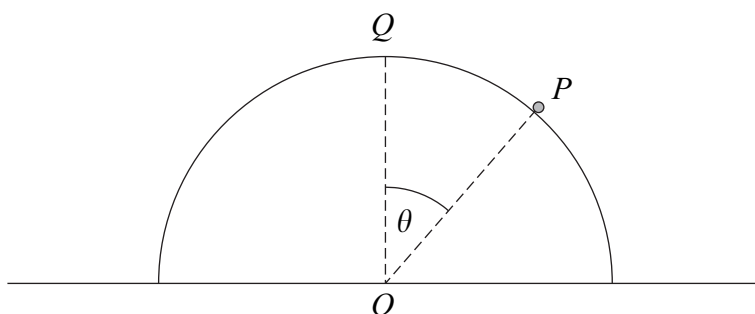
Find an expression for the velocity of the particle at time  $t$ . (5 marks)

- 6 A student is modelling the motion of a small boat as it moves on a lake. When the speed of the boat is  $12 \text{ m s}^{-1}$ , the engine is switched off. At time  $t$  seconds later, it has a velocity of  $v \text{ m s}^{-1}$  and experiences a resistance force of magnitude  $20v$  newtons. The mass of the boat is  $80 \text{ kg}$ .

To set up a simple model for the motion of the boat, the student assumes that the water in the lake is still and that the boat travels in a straight line.

- (a) Explain how these two assumptions allow the student to create a simple model. (2 marks)
- (b) State one other assumption that the student should make. (1 mark)
- (c) (i) Express  $\frac{dv}{dt}$  in terms of  $v$ . (2 marks)
- (ii) Find an expression for  $v$  in terms of  $t$ . (5 marks)

- 7 A particle  $P$ , of mass  $m \text{ kg}$ , is placed at the point  $Q$  on the top of a smooth upturned hemisphere of radius 3 metres and centre  $O$ . The plane face of the hemisphere is fixed to a horizontal table. The particle is set into motion with an initial horizontal velocity of  $2 \text{ m s}^{-1}$ . When the particle is on the surface of the hemisphere, the angle between  $OP$  and  $OQ$  is  $\theta$  and the particle has speed  $v \text{ m s}^{-1}$ .



- (a) Show that  $v^2 = 4 + 6g(1 - \cos \theta)$ . (4 marks)
- (b) Find the value of  $\theta$  when the particle leaves the hemisphere. (5 marks)

8 A particle, of mass 10 kg, is attached to one end of a light elastic string of natural length 0.4 metres and modulus of elasticity 100 N. The other end of the string is fixed to the point  $O$ .

- (a) Find the length of the elastic string when the particle hangs in equilibrium directly below  $O$ . (2 marks)
- (b) The particle is pulled down and held at a point  $P$ , which is 1 metre vertically below  $O$ .

Show that the elastic potential energy of the string when the particle is in this position is 45 J. (2 marks)

- (c) The particle is released from rest at the point  $P$ . In the subsequent motion, the particle has speed  $v \text{ m s}^{-1}$  when it is  $x$  metres **below**  $O$ .
- (i) Show that, while the string is taut,

$$v^2 = 39.6x - 25x^2 - 14.6 \quad (7 \text{ marks})$$

- (ii) Find the value of  $x$  when the particle comes to rest for the first time after being released, given that the string is still taut. (3 marks)

**END OF QUESTIONS**

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