



A-LEVEL

Mathematics

Pure Core 2 – MPC2

Mark scheme

6360
June 2015

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
	(Area of sector $\Rightarrow \frac{1}{2}r^2\theta$)	M1		$\frac{1}{2}r^2\theta$ seen, or used, for the sector area
	$\frac{1}{2}(5^2)\theta = 15 \quad \left(\theta = \frac{15}{12.5}\right)$	A1		A correct equation in θ or in $r\theta$ eg $2.5r\theta = 15$
	(Perimeter of sector $\Rightarrow 5 + 5 + 5\theta$)	M1		$r + r + r\theta$ seen, or used, for the perimeter
	$= 10 + 5 \times \frac{6}{5} = 16 \text{ (cm)}$	A1	4	16
	Total		4	

Q2	Solution	Mark	Total	Comment
(a)	$\frac{AC}{\sin 48^\circ} = \frac{20}{\sin 72^\circ}$	M1		Correct use of sine rule with AC being the only unknown
	$AC = \frac{20 \sin 48^\circ}{\sin 72^\circ} \quad (= \frac{14.86\dots}{0.951\dots})$	A1		Correct expression for AC . PI by 15.62(7774..)
	$= 15.62(7774..) = 15.6 \text{ (cm to 3 sf)}$	A1	3	AG Need some intermediate evaluation between $\frac{20 \sin 48^\circ}{\sin 72^\circ}$ and 15.6
(b)	Angle $ACB = 60^\circ$	B1		Either $ACB = 60^\circ$ stated or used or seen on diagram or $AB = AWRT 18.2$
	$(AM^2) = 10^2 + (15.6)^2 - 2 \times 10 \times 15.6 \times \cos C$ $= 10^2 + (15.6)^2 - 156$	M1 m1		RHS of relevant cosine rule used correctly $10^2 + (15.6)^2 - 156$ OE; accept evaluation to, 187 to 188 incl., as evidence
	$AM = 13.7 \text{ (cm to 3 sf)}$	A1	4	Condone more accurate answer
	Total		7	
(b)	Allow use of 15.6 or better for AC			
(b)	Altn using perpendicular from A to BC Either $ACB = 60^\circ$ stated or used or seen on diagram or $AB = AWRT 18.2$ (B1) $(AM^2) = (15.6 \sin 60^\circ)^2 + (10 - 15.6 \cos 60^\circ)^2$ OR $(AM^2) = (18.2 \sin 48^\circ)^2 + (18.2 \cos 48^\circ - 10)^2$ (M1) $= (13.5)^2 + (2.2)^2$ (m1) Correct evaluations to at least 1dp accept evaluation to, 187 to 188 incl., as evidence. $AM = 13.7 \text{ (cm to 3 sf)}$ (A1) Condone more accurate answer			

Q3	Solution	Mark	Total	Comment
(a)	(3rd term=) $ar^2 = 48(0.6)^2$ $= 17.28$	M1 A1	2	ar^{3-1} stated or used OE fraction eg 432/25. NMS 17.28 OE scores 2 marks unless FIW.
(b)	$\{S_{\infty} =\} \frac{a}{1-r} = \frac{48}{1-0.6}$ $\{S_{\infty} =\} 120$	M1 A1	2	$\frac{a}{1-r}$ <u>used</u> with $a = 48$ and $r = 0.6$ OE Correct exact value for S_{∞} . NMS 120 scores 2 marks unless FIW.
(c)	$\sum_{n=4}^{\infty} u_n = S_{\infty} - \sum_{n=1}^3 u_n$ $\sum_{n=1}^3 u_n = (48+28.8 + c's (a))$ $\sum_{n=4}^{\infty} u_n = 120 - 94.08 = 25.92$ <p>Altn.</p> $\sum_{n=4}^{\infty} u_n = \frac{u_4}{1-r}$ $u_4 = 17.28 \times 0.6 = 10.368$ $\sum_{n=4}^{\infty} u_n = \frac{10.368}{1-0.6} = 25.92$	M1 A1F A1 (M1) (A1F) (A1)	3	OE eg RHS = $S_{\infty} - (a + ar + ar^2)$ OE eg $\sum_{n=1}^3 u_n = \frac{48(1-0.6^3)}{1-0.6}$ (=94.08) PI 25.92 OE exact value Ft on c's (a)×0.6. PI by $\sum_{n=4}^{\infty} u_n =$ correct evaluation of $1.5 \times c's(a)$ 25.92 OE exact value
	Total		7	

Q4	Solution	Mark	Total	Comment
(a)	$\frac{2}{x^2} = 2x^{-2}$	B1		PI by its derivative as $-4x^{-3}$ or $4x^{-3}$
	$\frac{d^2y}{dx^2} = -4x^{-3} - \frac{1}{4}$	M1 A1	3	Differentiating one term correctly. ACF
(b)(i)	$\frac{2}{x^2} - \frac{x}{4} = 0$	M1		
	$(x_M =) 2$	A1	2	NMS 2/2 for correct answer.
(b)(ii)	(At M) $\frac{d^2y}{dx^2} = -\frac{4}{8} - \frac{1}{4} < 0$, so max.	E1	1	Using c's x_M and c's $\frac{d^2y}{dx^2}$ to show $\frac{d^2y}{dx^2}$ is negative and stating conclusion ie max.
(b)(iii)	$\int \left(\frac{2}{x^2} - \frac{x}{4} \right) dx = -2x^{-1} - \frac{x^2}{8} (+c)$	M1		Attempt to integrate $\frac{dy}{dx}$ with at least one of the two terms integrated correctly.
	$(y =) -2x^{-1} - \frac{x^2}{8} (+c)$	A1		$-2x^{-1} - \frac{x^2}{8}$ OE ; condone unsimplified
	When $x = 2, y = 2.5 \Rightarrow 2.5 = -1 - 0.5 + c$	M1		Subst. $x = c$'s (b) , $y = 2.5$ into $y = F(x) + c$ ' in attempt to find constant of integration, where $F(x)$ follows attempted integration of expression for $\frac{dy}{dx}$
	$y = -2x^{-1} - \frac{x^2}{8} + 4$	A1	4	ACF but with signs and coeffs simplified
	Total		10	

Q5	Solution	Mark	Total	Comment
(a)	$132 = 160p + q$ $20 = 20p + q$	M1		Seen or used
	$112 = 140p$ $p = \frac{112}{140} \left(= \frac{4}{5} \right)$ $q = 4$	M1		Seen or used
(b)	$160 = \frac{4}{5}u_1 + 4$ $u_1 = 195$	m1		Valid method to solve the correct two simultaneous eqns in p and q to at least the stage $112 = 140p$ OE or $28 = 7q$ OE PI by correct values for both p and q from two correct simultaneous equations
		A1		ACF
		A1	5	$q = 4$
		B1F	1	Ft on $u_1 = \frac{160 - c's q}{c's p}$, provided u_1 is exact and p and q are both positive.
	Total		6	

Q6	Solution	Mark	Total	Comment
(a)	$\sin^{-1} 0.6 = 0.64(35\dots) (= \beta)$	B1	3	PI by one correct value for x to at least 2dp or 2sf $x + 0.7 = \beta$ and $x + 0.7 = \pi - \beta$ where β is the c's value for $\sin^{-1} 0.6$
	$x + 0.7 = \beta, \quad x + 0.7 = \pi - \beta (=2.4(98\dots))$	M1		
	$x = -0.056, 1.8$ (to 2 sf)	A1		Must be correct 2sf values ie $-0.056, 1.8$ Ignore any values outside given interval. SC NMS Condone $>2sf$ and mark as B1 B1 max. $\{-0.056(498\dots); 1.7(9809\dots)\}$
(b)(i)	$5 \cos^2 \theta - \cos \theta = 1 - \cos^2 \theta$	M1	4	Replacing $\sin^2 \theta$ by $1 - \cos^2 \theta$ $(2 \cos \theta \pm 1)(3 \cos \theta \pm 1)$ PI by the two 'correct' roots with correct/incorrect signs
	$6 \cos^2 \theta - \cos \theta - 1 = 0$ $(2 \cos \theta - 1)(3 \cos \theta + 1) (= 0)$	A1 m1		
(b)(ii)	(Possible values of $\cos \theta =$) $\frac{1}{2}, -\frac{1}{3}$	A1		The two correct values of $\cos \theta$.
	When $\cos \theta = -\frac{1}{3}, \sin^2 \theta = \frac{8}{9}$ $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{(\pm) \sqrt{\frac{8}{9}}}{-\frac{1}{3}}$	B1 M1		$\tan \theta = \frac{\sin \theta}{\cos \theta}$ used ; could be used with either of c's values of $\cos \theta$ from (b)(i) and a corresponding value of $\sin \theta$
	So a (+'ve) value for $\tan \theta$ is $-\sqrt{\frac{8}{9}} \div \left(-\frac{1}{3}\right) = \sqrt{8} = 2\sqrt{2}$	A1	3	CSO A.G. Be convinced.
	Total		10	
(a)	Eg NMS $x = -0.06, 1.80$ scores B0B1			
(b)(ii) Alt	$\sec \theta = -3, \sec^2 \theta = 9$ (B1); $\tan^2 \theta = \sec^2 \theta - 1 = 9 - 1$ (M1); (+'ve) value of $\tan \theta$ is $\sqrt{8} = 2\sqrt{2}$ (A1CSO)			

Q7	Solution	Mark	Total	Comment
(a)(i)	Translation $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	E2,1,0	2	E2: 'translat...' and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ OE. If not E2 award E1 for 'translat... in y-dir' OE. More than one transformation scores 0/2
(a)(ii)	Stretch (I) in x-direction (II) scale factor 9 (III)	M1 A1	2	Need (I) and either (II) or (III) Need (I) and (II) and (III) More than one transformation scores 0/2
(b)(i)	$\int_0^9 (1 + \sqrt{x}) dx = 9 + 18 = 27$	B1	1	27
(b)(ii)	$h = 2.25$ $f(x) = 4^{\frac{x}{9}}$ $I \approx \frac{h}{2} \{f(0)+f(9)+2[f(2.25)+f(4.5)+f(6.75)]\}$ $\frac{h}{2}$ with $\{\dots\} = 1 + 4 + 2\left(4^{\frac{1}{4}} + 4^{\frac{1}{2}} + 4^{\frac{3}{4}}\right)$ $= 5 + 2(\sqrt{2} + 2 + 2\sqrt{2}) = 9 + 6\sqrt{2}$ $(I \approx \frac{2.25}{2} [9 + 8.48\dots] = 1.125 \times 17.485\dots)$ $(= 19.67\dots) = 19.7$ (to 1 dp)	B1 M1 A1 A1	4	$h = 2.25$ OE stated or used. (PI by x-values 0, 2.25, 4.5, 6.75, 9 provided no contradiction) $h/2 \{f(0)+f(9)+2[f(2.25)+f(4.5)+f(6.75)]\}$ OE summing of areas of the 'trapezia'.. OE Accept 2sf or better evidence for surds. Can be implied by later <u>correct</u> work provided >1 term or a single term which rounds to 19.7 CAO Must be 19.7 SC 5strips used: Max B0M1A0, 19.6 A1
(b)(iii)	Area of shaded region \approx $\int_0^9 (1 + \sqrt{x}) dx - \int_0^9 4^{\frac{x}{9}} dx$ $= 27 - 19.7 = 7.3$ Since trapezia cover larger area than area under lower curve, 19.7 is overestimate so subtracting this from the true area, 27, under upper curve will lead to an underestimate of the true area of shaded region.	M1 A1F E1	3	Ft on [c's (b)(i) - c's (b)(ii)] provided this gives a value >0. Need both the final answer ' underestimate ' plus mention of the fact that the trapezium rule gives overestimate as trapezia cover larger area-cand could show this on a diagram. (E1 is dep on M1 but not on the A1F)
Total			12	
(a)(i)	Example: 'translating 1 in positive y' OE (E2)			
(b)(ii)	For guidance, separate trap. 2.71(5..) + 3.84(0..)+5.43(1..)+7.68(1..). NB 3/4 possible if values to 2sf			
(b)(ii)	MR of f(x), but NOT from an attempted integration, max B1M1A0A0			

Q8	Solution	Mark	Total	Comment
	<p>Gradient of the line $3y - 2x = 1$ is $\frac{2}{3}$</p> $\frac{dy}{dx} = \frac{1}{2}x^{-0.5}$ <p>At A, $\frac{1}{2}x^{-0.5} = \frac{2}{3}$</p> $A\left(\frac{9}{16}, \frac{3}{4}\right)$ <p>Eqn of tang at A: $y - \frac{3}{4} = \frac{2}{3}\left(x - \frac{9}{16}\right)$</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>5</p>	<p>(Gradient) $\frac{2}{3}$ seen or used. Condone 0.66, 0.67 or better for $\frac{2}{3}$.</p> <p>Correct differentiation of $x^{\frac{1}{2}}$</p> <p>c's $\frac{dy}{dx}$ expression = c's numerical gradient of given line.</p> <p>Correct exact coordinates of A</p> <p>ACF eg $y = \frac{2}{3}x + \frac{3}{8}$ or eg $3y - 2x = \frac{9}{8}$ must be exact</p>
	Total		5	
Examples	<p>Cand. writes $0.5x^{-0.5} = k$, and stops, where $k = -\frac{2}{3}$ or 2 or -2. Mark these types as (B0, B1, M1A0A0)</p>			

Q9	Solution	Mark	Total	Comment
(a)	$3x \log 2 = \log 5$ $x = 0.773(976\dots) = 0.774$ (to 3sf)	M1 A1	2	OE eg $3x = \log_2 5$ or eg $x \log 8 = \log 5$ Condone > 3sf. If use of logarithms not explicitly seen then score 0/2
(b)	$\log_a \frac{k}{2} = \frac{2}{3}$ $\frac{k}{2} = a^{\frac{2}{3}}$ $a^{\frac{2}{3}} = \frac{k}{2} \Rightarrow a = \left(\frac{k}{2}\right)^{\frac{3}{2}}$	M1 A1 m1 A1	4	Either $\log k - \log 2 = \log \frac{k}{2}$ or $\frac{2}{3} = \log a^{\frac{2}{3}}$ seen at any stage OE eqn with logs eliminated with no incorrect work $a^{\frac{m}{n}} = C \Rightarrow a = C^{\frac{n}{m}}$ $a = \left(\frac{k}{2}\right)^{\frac{3}{2}}$ OE exact form with no obvious incorrect working
(c)(i)	$(1 + 2x)^3 = 1 + 3(2x) + 3(2x)^2 + (2x)^3$ $= 1 + 6x + 12x^2 + 8x^3$	B3,2,1	3	B3: expansion correct and simplified B2: 3 of the 4 terms correct and simplified B2; 4 terms correct but not all simplified B1 2 of the 4 terms correct and simplified (ignore the ordering of the terms)
(c)(ii)	$[(1 + 2n)^3 - 8n] = 1 - 2n + 12n^2 + 8n^3$ $\log(1 + 2n) + \log 4(1 + n^2) = \log 4(1 + n^2)(1 + 2n)$ Given equation becomes $1 - 2n + 12n^2 + 8n^3 = 8n^3 + 4n^2 + 8n + 4$ $8n^2 - 10n - 3 (=0)$ $(4n + 1)(2n - 3) (=0)$ $n = -\frac{1}{4}, n = \frac{3}{2}$	B1F M1 A1 A1 A1	5	Ft at most two incorrect coefficients in (c)(i) Log law 1 applied correctly to RHS of given eqn., ignore base. Those who rearrange the terms first before applying log law 2 correctly must also attempt to deal with the resulting fraction in a correct manner. Correct three term quadratic PI by correct two roots from a correct quadratic equation Need both as the final two values of n with no extras
Total			14	
(b)	Example: $\log k - \log 2 = \frac{\log k}{\log 2} = \frac{2}{3}, \frac{\log k}{\log 2} = \log a^{\frac{2}{3}}$ (M1), $\frac{k}{2} = a^{\frac{2}{3}}$ (A0), $a = \left(\frac{k}{2}\right)^{\frac{3}{2}}$ (m1) (A0)			