



General Certificate of Education
Advanced Level Examination
June 2013

Mathematics

MPC4

Unit Pure Core 4

Monday 10 June 2013 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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1 (a) (i) Express $\frac{5-8x}{(2+x)(1-3x)}$ in the form $\frac{A}{2+x} + \frac{B}{1-3x}$, where A and B are integers. (3 marks)

(ii) Hence show that $\int_{-1}^0 \frac{5-8x}{(2+x)(1-3x)} dx = p \ln 2$, where p is rational. (4 marks)

(b) (i) Given that $\frac{9-18x-6x^2}{2-5x-3x^2}$ can be written as $C + \frac{5-8x}{2-5x-3x^2}$, find the value of C . (1 mark)

(ii) Hence find the exact value of the area of the region bounded by the curve $y = \frac{9-18x-6x^2}{2-5x-3x^2}$, the x -axis and the lines $x = -1$ and $x = 0$.

You may assume that $y > 0$ when $-1 \leq x \leq 0$. (2 marks)

2 The acute angles α and β are given by $\tan \alpha = \frac{2}{\sqrt{5}}$ and $\tan \beta = \frac{1}{2}$.

(a) (i) Show that $\sin \alpha = \frac{2}{3}$, and find the exact value of $\cos \alpha$. (2 marks)

(ii) Hence find the exact value of $\sin 2\alpha$. (2 marks)

(b) Show that the exact value of $\cos(\alpha - \beta)$ can be expressed as $\frac{2}{15}(k + \sqrt{5})$, where k is an integer. (4 marks)

3 (a) Find the binomial expansion of $(1+6x)^{-\frac{1}{3}}$ up to and including the term in x^2 . (2 marks)

(b) (i) Find the binomial expansion of $(27+6x)^{-\frac{1}{3}}$ up to and including the term in x^2 , simplifying the coefficients. (3 marks)

(ii) Given that $\sqrt[3]{\frac{2}{7}} = \frac{2}{\sqrt[3]{28}}$, use your binomial expansion from part (b)(i) to obtain an approximation to $\sqrt[3]{\frac{2}{7}}$, giving your answer to six decimal places. (2 marks)



- 4** A curve is defined by the parametric equations $x = 8e^{-2t} - 4$, $y = 2e^{2t} + 4$.
- (a)** Find $\frac{dy}{dx}$ in terms of t . (3 marks)
- (b)** The point P , where $t = \ln 2$, lies on the curve.
- (i)** Find the gradient of the curve at P . (1 mark)
- (ii)** Find the coordinates of P . (2 marks)
- (iii)** The normal at P crosses the x -axis at the point Q . Find the coordinates of Q . (3 marks)
- (c)** Find the Cartesian equation of the curve in the form $xy + 4y - 4x = k$, where k is an integer. (3 marks)
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- 5** The polynomial $f(x)$ is defined by $f(x) = 4x^3 - 11x - 3$.
- (a)** Use the Factor Theorem to show that $(2x + 3)$ is a factor of $f(x)$. (2 marks)
- (b)** Write $f(x)$ in the form $(2x + 3)(ax^2 + bx + c)$, where a , b and c are integers. (2 marks)
- (c) (i)** Show that the equation $2 \cos 2\theta \sin \theta + 9 \sin \theta + 3 = 0$ can be written as $4x^3 - 11x - 3 = 0$, where $x = \sin \theta$. (3 marks)
- (ii)** Hence find all solutions of the equation $2 \cos 2\theta \sin \theta + 9 \sin \theta + 3 = 0$ in the interval $0^\circ < \theta < 360^\circ$, giving your solutions to the nearest degree. (4 marks)



- 6 The points A , B and C have coordinates $(3, -2, 4)$, $(1, -5, 6)$ and $(-4, 5, -1)$ respectively.

The line l passes through A and has equation $\mathbf{r} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 7 \\ -7 \\ 5 \end{bmatrix}$.

- (a) Show that the point C lies on the line l . (2 marks)
- (b) Find a vector equation of the line that passes through points A and B . (3 marks)
- (c) The point D lies on the line through A and B such that the angle CDA is a right angle. Find the coordinates of D . (5 marks)
- (d) The point E lies on the line through A and B such that the area of triangle ACE is three times the area of triangle ACD .

Find the coordinates of the two possible positions of E . (4 marks)

- 7 The height of the tide in a certain harbour is h metres at time t hours. Successive high tides occur every 12 hours.

The **rate of change** of the height of the tide can be modelled by a function of the form $a \cos(kt)$, where a and k are constants. The largest value of this rate of change is 1.3 metres per hour.

Write down a differential equation in the variables h and t . State the values of the constants a and k . (3 marks)

- 8 (a) Find $\int t \cos\left(\frac{\pi}{4}t\right) dt$. (4 marks)

- (b) The platform of a theme park ride oscillates vertically. For the first 75 seconds of the ride,

$$\frac{dx}{dt} = \frac{t \cos\left(\frac{\pi}{4}t\right)}{32x}$$

where x metres is the height of the platform above the ground after time t seconds.

At $t = 0$, the height of the platform above the ground is 4 metres.

Find the height of the platform after 45 seconds, giving your answer to the nearest centimetre. (6 marks)

