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General Certificate of Education
June 2010

Statistics SS03

Statistics 3

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

SS03

Q	Solution	Marks	Total	Comments
1(a)	H_0 $\eta_{\text{difference}}$ = 0 difference (on–off target) H_1 $\eta_{\text{difference}}$ > 0 1 tail 5%	B1		Not mean Can be in words Must be consistent with signs
	Signs + + + + + + + - + - 8^+ / 2^- signs – test values	M1 A1		signs test stat correct and identified or used
	Binomial (10, 0.5) model P (≥ 8+) = P(≤ 2 -) = 0.0547 > 0.05 for one tail test Accept H_o . There is not sufficient evidence, at the 5% level, to suggest that the median difference is greater than 0. On average, teams do not have more shots on target than shots off target.	M1 M1 E1	6	Binomial model used, B (10,0.5), and probability attempted Comparison of Binomial probability with 0.05 or use of cr with probs Correct conclusion in context
(b)(i)				
	Team A 3.5 B 1 C 3.5 D 9.5 E 7 **Number of goals** 1 2 3 4 5 **Team** F 2 G 6 H 8 I 5 J 9.5 **Number of goals** 6 8 8 8 10 r_s = 0.562	M1 M1 A1 B3	6	attempt at ranks (any) 12 or more correct all correct **alternative** d = 2.5, -1, 0.5, 5.5, 2, -4, -2, 0 , -3, -0.5 $\sum d^2 = 71$ B1 $r_s = 1 - \frac{6\times 71}{10\times 99} = 0.570$ M1, A1 or 0.57 **SC** ft incorrect ranks B1, M1 **SC** No working 0.562 6/6 0.56 4/6 0.6 1/6
(ii)	H_o Rank orders of number of shots and number of goals scored are independent. H_1 Rank orders of number of shots and number of goals scored are not independent – there is a positive association 1 tail 1% cv = 0.7333 test stat r_s = 0.562 (or 0.570) r_s < cv Accept H_o No significant evidence at 1% level to suggest a positive association between rank orders of number of shots and number of goals scored.	B1 B1 M1 A1 E1	5	or equivalent 1 tail 0.6485, 0.7667, 0.7000, 0.7818 allow comparison if $0 \leqslant r < 1$ no ft can ft

SS03 (cont)

Q	Solution	Marks	Total	Comments
1(b)(iii)	The correlation coefficient does not indicate a significant positive association. Journalist wrong. (B1 E1**)** **or** There is evidence of a positive correlation but it was not found to be significant at 1%. Journalist could have a valid point. (B1 E1)	no ft B1 E1	2	Mention journalist wrong with valid reason B1 reason $\rightarrow$ SRCC 0.5/0.6 E1 journalist wrong Mention possibility of positive correlation so journalist might have a valid point Comment + reason B1 reason $\rightarrow$ test Acc H_O E1 Journalist correct
(iv)	Type II error is to accept H_O when actually H_O is not true. This would mean that the conclusion to the test in part (b)(ii) that there is no significant positive association between number of shots and number of goals is incorrect and there is actually a positive association between the two.	B1 E1	2	Do not need 'positive'
	Total		**21**	

SS03 (cont)

Q	Solution	Marks	Total	Comments
2(a)	H_o Development of Type 2 diabetes is independent of alcohol consumption H_1 Development of Type 2 diabetes is not independent of alcohol consumption 1 tail 1%	B1		Disallow 'nonsense' Allow H_o independent H_o no association H_1 not independent H_1 association
		M1		E method SC ft wrong totals
		A1		3 correct M1A1
		A1		All E correct
	ts = $\sum \frac{(O-E)^2}{E}$	m1		ts sum with correct denominators ft wrong E_i
	$= \frac{14.2^2}{23.80} + \frac{14.2^2}{396.20} + \frac{25.68^2}{37.18} + \frac{25.68^2}{627.32} + \frac{11.48^2}{23.52} + \frac{11.48^2}{391.48}$ = 33.48	A1		for ts in range 30.0 – 36.0
	cv df = 2 1% cv = 9.210	B1		for cv
	ts > 9.210	m1		for comparison ts/cv – allow for any
	Reject H_o	A1		df = 2 upper cv or p value .00000005 < 0.01
	Sig evidence to suggest that development of Type 2 diabetes is not independent of alcohol consumption.	E1	10	Explanation in context ft
(b)	**Study** Conclusions cannot be generalised to whole population.	E2		or only 85 women had Type 2 E1 only 85 so can't be generalised E2
	Sources Association sources indicate those who drink the least (less than 5g) and those that drink the most (more than 30g) are more likely than expected to develop Type 2 diabetes. Drinking more will not help unless it is to within the category 'between 5 and 30'.	E2		Only drinking 5-30 will reduce chance E2 Drinking less 5 or more 30 increases chance E2 SC ft 'reject H_o' for E1 only
			4	
(c)(i)	**No change** as df is still 2 since test on 3×2 contingency table with no pooling. **cv =9.210**	B1		**No change** ft any cv in range
(ii)	Test statistic will be **10**× larger so **ts = 334.8**	M1 A1		10× M1 A1 correct ft 300-360
(iii)	Conclusion would be **the same** because the ts is further into the critical region. 33.48 > 9.210 and also 334.8> 9.210	B1	4	the same → requires cv same, ts bigger
	Total		**18**	

	Yes	No
Less 5	23.80	396.20
5 -30	37.68	627.32
More 30	23.52	391.48

SS03 (cont)

Q	Solution	Marks	Total	Comments
3	H_0 Samples from identical populations H_1 Samples not from identical populations 5% sig level	B1		Or H_0 $\eta_A = \eta_B = \eta_C$ H_1 at least two of η_A, η_B, η_C do differ Or equivalent inwards Not mean
	Ranks **A** 1; **B** 5½; **C** 5½ 2; 7; 9 3; 11; 12 4; 14; 13 8; 16; 15 10; 17; 18	M1 A1		Ranks (can be reversed - bracketed) **SC** ranks groups independently M1 A1 for at least 13 correct
	$T_A = 28 (86)$ $T_B = 70½ (43½)$ $T_C = 72½ (41½$ $n_A = 6$ $n_B = 6$ $n_C = 6$	m1		Dep ranks Totals of ranks **SC** m1 if ranks groups independently
	$\sum_{i=1}^{m} \frac{T_i^2}{n_i} = \frac{28^2}{6} + \frac{70\frac{1}{2}^2}{6} + \frac{72\frac{1}{2}^2}{6} = 1835.1$	m1		dep ranks
	$H = \frac{12}{18 \times 19} \times 1835.1 - (3 \times 19) = 7.39$	m1 A1		dep ranks test stat H = 7.00-7.80 $\frac{12}{N(N+1)} \sum_{i=1}^{m} \frac{T_i^2}{n_i} - 3(N+1)$
	Critical value from $\chi^2_2 = 5.991$	B1		4.605 …. 10.597 df = 2 upper tail
	$H > 5.99$	m1		
	Sig evidence to reject H_0 and conclude that samples are not from identical populations. At least 2 differ.	A1		
	Group A had the highest median score and, as there is significant evidence that at least two of the median scores (from groups A, B or C) do differ, it would seem likely that group A children achieved higher scores for improvement in reading on average. **or** It appears that children who are praised as much as possible when reading and are not criticised improve significantly more than the children in the other groups.	B1 E1	12	**SC** E1 for ft 'Accept' H_0 **SC** E1 if only 'difference exists' in context identification of A/C or 'at least 2 differ'(ft explanation if ranks reversed) Explanation in context of reason A/C selected.
	Total		**12**	

SS03 (cont)

Q	Solution	Marks	Total	Comments
4(a)	minimum T = 1+2+3+4+5+6+7+8 = 36 maximum T = 9+10+11+12+13+14+15+16 = 100	M1 A1 M1 A1	4	SC3 U = 36–36 = 0 U = 100–36 = 64
(b)(i)	H_o Samples are from two populations with identical distributions H_1 Samples are from two populations that do not have identical distributions	B1		Or ref to pop. averages
	$U = 31 - \frac{6\times 7}{2} = 10$ (lower tail) $U = 140 - \frac{12\times 13}{2} = 62$	M1 A1		
	$n = 6,\ m = 12$ lower tail cv = 15 test stat U = 10 U < 15	B1 M1		For consistent upper/lower cv cv 11, 14, 16, 18, 22, 13 for M1
	Reject H_o There is sufficient evidence to suggest a difference in heights between the two populations of children.	A1	6	
(ii)	There is a significant difference in the heights of children who are the youngest in their family and those who are either an only child or not the youngest. Those who are **the youngest in their family** appear to be **shorter** when compared to children of the same age who are either an only child or not the youngest in their family.	E1	1	No ft on incorrect conclusion
	Total		**11**	

SS03 (cont)

Q	Solution	Marks	Total	Comments
5(a)	H_o pop mean/median, $\mu / \eta = 56$ H_1 pop mean/median, $\mu / \eta < 56$ 1 tail 1%	B1 B1		Or words/pop average Consistent sign with diffs
	diff -9, 0, -18, -13, -36, -16 **rank** 2, ., 9, 6, 12, 7	M1		For differences (can +/– be reversed)
	+1, -10, -24, -31, +10, -17, -12 1, 3.5, 10, 11, 3.5, 8, 5	m1		For ranks |smallest| = rank 1 (allow rank1 for 0)
	$T_+ = 1 + 3.5 = 4.5$ $T_- = 2 + 9 + \ldots + 5 = 73.5$	m1		For totals of ranks (any)
	Test stat T = 4.5 $n = 12$ cv = 10 T < 10	A1 B1 M1		Either total correct For cv 7, 10, 13, 14
	Significant evidence at 1% level to reject H_o. Conclude that new tablet is faster, on average, than existing tablet.	E1	9	Correct conclusion in context
(b)(i)	Wilcoxon signed-test is preferred because the magnitudes of the differences are taken into account whereas, with the sign test, only the signs of the differences are used.	E1 E1		Reduces expt. error E1
(ii)	Data not symmetrically distributed therefore Wilcoxon signed-rank cannot be carried out. **or** Data given only as signs/preferences so only sign test possible.	B1		
(iii)	*z* test	B1	4	Or ts seen OE
	Total		**13**	
	TOTAL		**75**	