

Mark Scheme 4727

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<p>1 (a) Identity = $1+0i$ Inverse = $\frac{1}{1+2i}$ $= \frac{1}{1+2i} \times \frac{1-2i}{1-2i} = \frac{1}{5} - \frac{2}{5}i$</p>	<p>B1 B1 B1 3</p>	<p>For correct identity. Allow 1 For $\frac{1}{1+2i}$ seen or implied For correct inverse AEFcartesian</p>
<p>(b) Identity = $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Inverse = $\begin{pmatrix} -3 & 0 \\ 0 & 0 \end{pmatrix}$</p>	<p>B1 B1 2 5</p>	<p>For correct identity For correct inverse</p>
<p>2 (a) $(z_1 z_2 =) 6e^{\frac{5}{12}\pi i}$ $\left(\frac{z_1}{z_2} = \frac{2}{3} e^{-\frac{1}{12}\pi i} \right) = \frac{2}{3} e^{\frac{23}{12}\pi i}$</p>	<p>B1 B1 M1 A1 4</p>	<p>For modulus = 6 For argument = $\frac{5}{12}\pi$ For subtracting arguments For correct answer</p>
<p>(b) $(w^{-5} =) 2^{-5} \operatorname{cis}\left(-\frac{5}{8}\pi\right)$ $= \frac{1}{32} \left(\cos \frac{11}{8}\pi + i \sin \frac{11}{8}\pi \right)$</p>	<p>M1 A1 A1 3 7</p>	<p>For use of de Moivre For $-\frac{5}{8}\pi$ seen or implied For correct answer (allow 2^{-5} and $\operatorname{cis} \frac{11}{8}\pi$)</p>

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<p>3 EITHER $\mathbf{c} - \mathbf{a} = \pm[11, 3, -2]$ $(\mathbf{c} - \mathbf{a}) \times [8, 3, -6]$ $\mathbf{n} = \pm[-12, 50, 9]$ $d = \frac{ \mathbf{n} }{ [8, 3, -6] }$ $= \frac{\sqrt{2725}}{\sqrt{109}}$ $(d =) 5$</p>	<p>B1 M1* A1 √ M1 (dep*) A1 A1</p>	<p>For vector joining lines For attempt at vector product of $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ For obtaining \mathbf{n}. f.t. from incorrect $\mathbf{c} - \mathbf{a}$ For dividing \mathbf{n} by magnitude of $[8, 3, -6]$ For either magnitude correct For correct distance CAO</p>
<p>OR $\mathbf{c} - \mathbf{a} = \pm[11, 3, -2]$ $(\mathbf{c} - \mathbf{a}) \cdot [8, 3, -6]$ $\cos \theta = \pm \frac{109}{\sqrt{134}\sqrt{109}} = \pm \sqrt{\frac{109}{134}}$ $d = \sqrt{134} \sin \theta$ $(d =) 5$</p>	<p>B1 M1* A1 √ M1 (dep*) A1 A1</p>	<p>For vector joining lines For attempt at scalar product of $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ For correct $\cos \theta$ AEF. f.t. from incorrect $\mathbf{c} - \mathbf{a}$ For using trigonometry for perpendicular distance For correct expression for d in terms of θ For correct distance CAO</p>
<p>OR $\mathbf{c} - \mathbf{a} = \pm[11, 3, -2]$ $(\mathbf{c} - \mathbf{a}) \cdot [8, 3, -6]$ $x = \frac{109}{\sqrt{109}} = \sqrt{109}$ $d = \sqrt{134 - 109}$ $(d =) 5$</p>	<p>B1 M1* A1 √ M1 (dep*) A1 A1</p>	<p>For vector joining lines For attempt at scalar product of $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ For finding projection of $\mathbf{c} - \mathbf{a}$ onto $[8, 3, -6]$ f.t. from incorrect $\mathbf{c} - \mathbf{a}$ For using Pythagoras for perpendicular distance For correct expression for d For correct distance CAO</p>
<p>OR $\mathbf{CP} = \pm[-11 + 8t, -3 + 3t, 2 - 6t]$ $\mathbf{CP} \cdot [8, 3, -6] = 0$ $t = \pm 1$ OR $P = (9, 5, -1)$ $d = \sqrt{3^2 + 0^2 + 4^2}$ $(d =) 5$</p>	<p>B1 M1* A1 √ M1 (dep*) A1 A1 6</p>	<p>For finding a vector from $C(12, 5, 3)$ to a point on the line For using scalar product for perpendicularity For correct point. f.t. from incorrect CP For finding magnitude of CP For correct expression for d For correct distance CAO SR Obtain $\mathbf{CP} = [11, 3, -2] - [8, 3, -6] = \pm[3, 0, 4]$ B1 Verify $[3, 0, 4] \cdot [8, 3, -6] = 0$ M1* $d = \sqrt{3^2 + 0^2 + 4^2} = 5$ M1(dep*) A1 A1 (maximum 5 / 6)</p>

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<p>4 Integrating factor $e^{\int -\frac{x^2}{1+x^3} dx}$</p> $= e^{-\frac{1}{3}\ln(1+x^3)} = (1+x^3)^{-\frac{1}{3}}$ $\Rightarrow \frac{d}{dx} \left(y(1+x^3)^{-\frac{1}{3}} \right) = \frac{x^2}{(1+x^3)^{\frac{1}{3}}}$ $\Rightarrow y(1+x^3)^{-\frac{1}{3}} = \frac{1}{2}(1+x^3)^{\frac{2}{3}} (+c)$ $\Rightarrow 1 = \frac{1}{2} + c \Rightarrow c = \frac{1}{2}$ $\Rightarrow y = \frac{1}{2}(1+x^3) + \frac{1}{2}(1+x^3)^{\frac{1}{3}}$	<p>M1 A1 M1 M1 A1 M1 A1 √ A1</p> <p style="text-align: center;">8</p>	<p>For correct process for finding integrating factor</p> <p>For correct IF, simplified (here or later)</p> <p>For multiplying through by their IF</p> <p>For integrating RHS to obtain $A(1+x^3)^k$ OR $\ln A(1+x^3)^k$</p> <p>For correct integration (+c not required here)</p> <p>For substituting (0, 1) into GS (including + c)</p> <p>For correct c. f.t. from their GS</p> <p>For correct solution. AEF in form $y = f(x)$</p>
<p>5 (i) EITHER $\mathbf{a} = [2, 3, 5]$, $\mathbf{b} = \pm[2, 2, 0]$</p> $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \pm k[-10, 10, -2]$ <p>Use (2, 1, 5) OR (0, -1, 5)</p> $\Rightarrow 5x - 5y + z = 10$	<p>B1 M1 A1 √ M1 A1</p>	<p>For stating 2 vectors in the plane</p> <p>For finding perpendicular to plane</p> <p>For correct \mathbf{n}. f.t. from incorrect \mathbf{b}</p> <p>For substituting a point into equation $ax + by + cz = d$ where $[a, b, c] =$ their \mathbf{n}</p> <p>For correct cartesian equation AEF</p>
<p>OR $\mathbf{a} = [2, 3, 5]$, $\mathbf{b} = \pm[2, 2, 0]$</p> <p>e.g. $\mathbf{r} = [2, 1, 5] + \lambda[2, 2, 0] + \mu[2, 3, 5]$</p> $[x, y, z] = [2 + 2\lambda + 2\mu, 1 + 2\lambda + 3\mu, 5 + 5\mu]$ $\Rightarrow 5x - 5y + z = 10$	<p>B1 M1 A1 √ M1 A1 5</p>	<p>For stating 2 vectors in the plane</p> <p>For stating parametric equation of plane</p> <p>For writing 3 equations in x, y, z f.t. from incorrect \mathbf{b}</p> <p>For eliminating λ and μ</p> <p>For correct cartesian equation AEF</p>
<p>(ii) $[2t, 3t - 4, 5t - 9]$</p>	<p>B1 1</p>	<p>For stating a point A on l_1 with parameter t AEF</p>
<p>(iii) $\pm[2t + 5, 3t - 7, 5t - 13]$</p> $\pm[2t + 5, 3t - 7, 5t - 13] \cdot [2, 3, 5] = 0$ $\Rightarrow t = 2$ $\frac{x+5}{9} = \frac{y-3}{-1} = \frac{z-4}{-3} \text{ OR}$ $\frac{x-4}{9} = \frac{y-2}{-1} = \frac{z-1}{-3}$	<p>M1 M1 A1 A1 4</p> <p style="text-align: center;">10</p>	<p>For finding direction of l_2 from A and (-5, 3, 4)</p> <p>For using scalar product for perpendicularity with any vector involving t</p> <p>For correct value of t</p> <p>For a correct equation AEFcartesian</p> <p>SR For $2p + 3q + 5r = 0$ and no further progress award B1</p>

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<p>6 (i) $(m^2 + 4 = 0 \Rightarrow) m = \pm 2i$</p> <p>CF = $A \cos 2x + B \sin 2x$</p> <p>PI = $p \sin x (+ q \cos x)$</p> <p>$-p \sin x (-q \cos x) + 4p \sin x (+4q \cos x) = \sin x$</p> <p>$\Rightarrow p = \frac{1}{3}, q = 0$</p> <p>$\Rightarrow y = A \cos 2x + B \sin 2x + \frac{1}{3} \sin x$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1 $\sqrt{6}$</p>	<p>For correct solutions of auxiliary equation (may be implied by correct CF)</p> <p>For correct CF (AETrig but not $Ae^{2ix} + Be^{-2ix}$ only)</p> <p>State a trial PI with at least $p \sin x$</p> <p>For substituting PI into DE</p> <p>For correct p and q (which may be implied)</p> <p>For using GS = CF + PI, with 2 arbitrary constants in CF and none in PI</p>
<p>(ii) $(0, 0) \Rightarrow A = 0$</p> <p>$\frac{dy}{dx} = 2B \cos 2x + \frac{1}{3} \cos x \Rightarrow \frac{4}{3} = 2B + \frac{1}{3}$</p> <p>$A = 0, B = \frac{1}{2}$</p> <p>$\Rightarrow y = \frac{1}{2} \sin 2x + \frac{1}{3} \sin x$</p>	<p>B1 $\sqrt{}$</p> <p>M1</p> <p>A1</p> <p>A1 4</p> <p>10</p>	<p>For correct equation in A and/or B f.t. from their GS</p> <p>For differentiating their GS and substituting values for x and $\frac{dy}{dx}$</p> <p>For correct A and B Allow $A = -\frac{1}{4}i, B = \frac{1}{4}i$ from CF $Ae^{2ix} + Be^{-2ix}$</p> <p>For stating correct solution CAO</p>
<p>7 (i) $C + iS = 1 + e^{i\theta} + e^{2i\theta} + e^{3i\theta} + e^{4i\theta} + e^{5i\theta}$</p> <p>$= \frac{e^{6i\theta} - 1}{e^{i\theta} - 1}$</p> <p>$= \frac{e^{3i\theta} - e^{-3i\theta}}{e^{\frac{1}{2}i\theta} - e^{-\frac{1}{2}i\theta}} \cdot \frac{e^{3i\theta}}{e^{\frac{1}{2}i\theta}} = \frac{e^{3i\theta} - e^{-3i\theta}}{e^{\frac{1}{2}i\theta} - e^{-\frac{1}{2}i\theta}} e^{\frac{5}{2}i\theta}$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1 4</p>	<p>For using de Moivre, showing at least 3 terms</p> <p>For recognising GP</p> <p>For correct GP sum</p> <p>For obtaining correct expression AG</p>
<p>(ii) $C + iS = \frac{2i \sin 3\theta}{2i \sin \frac{1}{2}\theta} \cdot e^{\frac{5}{2}i\theta}$</p> <p>Re $\Rightarrow C = \sin 3\theta \cos \frac{5}{2}\theta \operatorname{cosec} \frac{1}{2}\theta$</p> <p>Im $\Rightarrow S = \sin 3\theta \sin \frac{5}{2}\theta \operatorname{cosec} \frac{1}{2}\theta$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>B1 4</p>	<p>For expressing numerator and denominator in terms of sines</p> <p>For $k \sin 3\theta$ and $k \sin \frac{1}{2}\theta$</p> <p>For correct expression AG</p> <p>For correct expression</p>
<p>(iii) $C = S \Rightarrow \sin 3\theta = 0, \tan \frac{5}{2}\theta = 1$</p> <p>$\theta = \frac{1}{3}\pi, \frac{2}{3}\pi$</p> <p>$\theta = \frac{1}{10}\pi, \frac{1}{2}\pi, \frac{9}{10}\pi$</p>	<p>M1</p> <p>A1</p> <p>A2 4</p> <p>12</p>	<p>For either equation deduced AEF</p> <p>Ignore values outside $0 < \theta < \pi$</p> <p>For both values correct and no extras</p> <p>For all values correct and no extras. Allow A1 for any 1 value OR all correct with extras</p>

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<p>8 (i) $r^4 \cdot a \neq a \cdot r^4$</p>	<p>B1 1</p>	<p>For stating the non-commutative product in the given table, or justifying another correct one</p>																									
<p>(ii) Possible subgroups order 2, 5</p>	<p>B1 B1 2</p>	<p>For either order stated For both orders stated, and no more (Ignore 1)</p>																									
<p>(iii) (a) $\{e, a\}$ (b) $\{e, r, r^2, r^3, r^4\}$</p>	<p>B1 B1 2</p>	<p>For correct subgroup For correct subgroup</p>																									
<p>(iv) order of $r^3 = 5$ $(ar)^2 = ar \cdot ar = r^4 a \cdot ar = e$ \Rightarrow order of $ar = 2$ $(ar^2)^2 = ar^2 ar \cdot r = ar^2 r^4 a \cdot r = ara \cdot r = e$ \Rightarrow order of $ar^2 = 2$</p>	<p>B1 M1 A1 A1 4</p>	<p>For correct order For attempt to find $(ar)^m = e$ OR $(ar^2)^m = e$ For correct order For correct order</p>																									
<p>(v)</p> <table border="1" style="margin-left: 20px;"> <tr> <td></td> <td>ar</td> <td>ar^2</td> <td>ar^3</td> <td>ar^4</td> </tr> <tr> <td>ar</td> <td>e</td> <td>r</td> <td>r^2</td> <td>r^3</td> </tr> <tr> <td>ar^2</td> <td>r^4</td> <td>e</td> <td>r</td> <td>r^2</td> </tr> <tr> <td>ar^3</td> <td>r^3</td> <td>r^4</td> <td>e</td> <td>r</td> </tr> <tr> <td>ar^4</td> <td>r^2</td> <td>r^3</td> <td>r^4</td> <td>e</td> </tr> </table>		ar	ar^2	ar^3	ar^4	ar	e	r	r^2	r^3	ar^2	r^4	e	r	r^2	ar^3	r^3	r^4	e	r	ar^4	r^2	r^3	r^4	e	<p>B1 B1 B1 B1 B1 5</p> <p style="text-align: center;">14</p>	<p>If the border elements $ar ar^2 ar^3 ar^4$ are not written, it will be assumed that the products arise from that order For all 16 elements of the form e or r^m For all 4 elements in leading diagonal = e For no repeated elements in any completed row or column For any two rows or columns correct For all elements correct</p>
	ar	ar^2	ar^3	ar^4																							
ar	e	r	r^2	r^3																							
ar^2	r^4	e	r	r^2																							
ar^3	r^3	r^4	e	r																							
ar^4	r^2	r^3	r^4	e																							