4721 Mark Scheme January 2009

4721 Core Mathematics 1

1	$3\sqrt{5} + \frac{20\sqrt{5}}{5}$ $= 7\sqrt{5}$	B1	$3\sqrt{5}$ soi
	$=7\sqrt{5}$	M1	Attempt to rationalise $\frac{20}{\sqrt{5}}$
		A1 3	cao
2 (i)	x^2	B1 1	cao
(ii)	$\frac{3y^4 \times 1000y^3}{2y^5}$	B1	$1000y^3$ soi
	$= 1500 y^2$	В1	1500
	1000	B1 3 4	y ²
3	Let $y = x^{\frac{1}{3}}$	*M1	Attempt a substitution to obtain a quadratic or
	$3y^2 + y - 2 = 0$		factorise with $\sqrt[3]{x}$ in each bracket
	(3y-2)(y+1) = 0	DM1	Correct method to find roots
	$y = \frac{2}{3}, y = -1$	A1	Both values correct
	$x = \left(\frac{2}{3}\right)^3, x = (-1)^3$	DM1	Attempt cube of at least one value
	$x = \frac{8}{27}, x = -1$	A1 ft 5	Both answers correctly followed through
		5	SR If M1* not awarded, B1 $x = -1$ from T & I
4 (i)		B1	Excellent curve in one quadrant or roughly correct curves in correct 2 quadrants
		B1 2	Completely correct
	1		1
(ii)	$y = \frac{1}{\left(x+3\right)^2}$	M1	$(x\pm 3)^2$
		A1 2	$y = \frac{1}{(x+3)^2}$
(iii)	(1, 4)	B1 B1 2 6	Correct x coordinate Correct y coordinate

	T -	1		
5 (i)	$\frac{dy}{dx} = -50x^{-6}$	M1		kx^{-6}
	dx	A1	2	Fully correct answer
(**)	1	B1		$\sqrt[4]{x} = x^{\frac{1}{4}}$ soi
(ii)	$y = x^{\frac{1}{4}}$ $\frac{dy}{dx} = \frac{1}{4}x^{-\frac{3}{4}}$			$\sqrt[4]{x} = x^4$ soi
	$\int dy _1 _{r}^{-\frac{3}{4}}$	B1		$\frac{1}{4}x^c$
	$\frac{1}{dx} = \frac{1}{4}x$	B1	3	4
				$\frac{1}{4}x^{c}$ $kx^{-\frac{3}{4}}$
(iii)	$y = (x^2 + 3x)(1 - 5x)$	M1		Attempt to multiply out fully
	$= 3x - 14x^2 - 5x^3$	A1		Correct expression (may have 4 terms)
				(yy
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3 - 28x - 15x^2$	M1		Two terms correctly differentiated from their
	u.			expanded expression
		A1	4	Completely correct (3 terms)
			9	
6(i)	$5(x^2+4x)-8$	B1		p = 5
(2)	$=5[(x+2)^2-4]-8$	B1		
	$= 5(x+2)^2 - 20 - 8$	M1		$(x+2)^2$ seen or $q = 2$ $-8-5q^2$ or $-\frac{8}{5}-q^2$ r = -28
	$= 5(x+2)^2 - 28$	IVII		5
	-3(x+2) - 26	A1	4	r = -28
(ii)	x = -2	D1.6	1	
	202 4 5 0	B1 ft	1	
(iii)	$20^2 - 4 \times 5 \times -8$	M1		Uses $b^2 - 4ac$
	= 560	A1	2	560
(iv)	2 real roots	B1	1	
			8	2 real roots
7(i)	30 + 4k - 10 = 0	M1		Attempt to substitute $x = 10$ into equation of line
	$\therefore k = -5$	A1	2	
(ii)				
	$\sqrt{(10-2)^2+(-5-1)^2}$	M1		Correct method to find line length using Pythagoras'
	$=\sqrt{64+36}$			theorem
	=10	A1	2	cao, dependent on correct value of k in (i)
(iii)	_10			
	Centre (6, -2)	B1		
	Radius 5	B1	2	
(iv)				
	Midpoint of AB = $(6, -2)$	B1		One correct statement of verification
	Length of $AB = 2 x$ radius	B1	2	Complete verification
	Both A and B lie on circumference		8	-
	Centre lies on line $3x + 4y - 10 = 0$		ᆸ	

8 (i)	$x = \frac{8 \pm \sqrt{(-8)^2 - (4 \times -1 \times 5)}}{-2}$	M1		Correct method to solve quadratic
	$=\frac{8\pm\sqrt{84}}{-2}$	A1		$x = \frac{8 \pm \sqrt{84}}{-2}$
	$= -4 - \sqrt{21} \text{ or } = -4 + \sqrt{21}$	A1	3	Both roots correct and simplified
(ii)	$x \le -4 - \sqrt{21} \ , \ x \ge -4 + \sqrt{21}$	M1 A1	2	Identifying $x \le$ their lower root, $x \ge$ their higher root $x \le -4 - \sqrt{21}$, $x \ge -4 + \sqrt{21}$
		AI	2	(not wrapped, no 'and')
(iii)				
(111)		B1		Roughly correct negative cubic with max and min
		B1		(-4, 0)
		B1		(0, 20)
		B1		Cubic with 3 distinct real roots
	ı	B1	5	Completely correct graph
			10	
9	dy 22 + 2	M1		Attempt to differentiate
	$\frac{dy}{dx} = 3x^2 + 2px$	A1		Correct expression cao
	When $x = 4$, $\frac{dy}{dx} = 0$	M1		Setting their $\frac{dy}{dx} = 0$
	$\therefore 3 \times 4^2 + 8p = 0$	M1		Substitution of $x = 4$ into their $\frac{dy}{dx} = 0$ to evaluate p
	8p = -48			dx

A1

M1

A1

7

7

When x = 4, 6x - 12 > 0

Minimum point

Looks at sign of $\frac{d^2y}{dx^2}$, derived correctly from their

 $\frac{dy}{dx}$, or other correct method

Minimum point CWO

10(i)	$\frac{dy}{dx} = 2x + 1$ $= 5$	M1 A1 2	Attempt to differentiate <i>y</i> cao
(ii)	Gradient of normal = $-\frac{1}{5}$ When $x = 2$, $y = 6$	B1 ft B1	ft from a non-zero numerical value in (i) May be embedded in equation of line
	$y - 6 = -\frac{1}{5}(x - 2)$ $x + 5y - 32 = 0$	M1 A1 4	Equation of line, any non-zero gradient, their y coordinate Correct equation in correct form
(iii)	$\begin{vmatrix} x^2 + x = kx - 4 \\ x^2 + (1 - k)x + 4 = 0 \end{vmatrix}$	*M1	Equating $y_1 = y_2$
	One solution => $b^2 - 4ac = 0$ $(1-k)^2 - 4 \times 1 \times 4 = 0$	DM1 DM1	Statement that discriminant = 0 Attempt (involving k) to use a, b, c from their equation
	$(1-k)^2 = 16$ $1-k = \pm 4$ k = -3 or 5	A1 DM1 A1 6	Correct equation (may be unsimplified) Correct method to find <i>k</i> , dep on 1 st 3Ms Both values correct
		12	