

1. A small bead is threaded on a smooth, straight horizontal wire which passes through the point $A(-3, 1)$ and the point $B(2, 5)$ in the x - y plane. The bead moves under the action of a horizontal force \mathbf{F} of magnitude 8.5 N whose line of action is parallel to the line with equation $15x - 8y + 4 = 0$. The unit on both the x and y axes has length one metre. Find the work done by \mathbf{F} as it moves the bead from A to B .

(8)



2. A particle P moves in a plane so that its position vector, \mathbf{r} metres at time t seconds, satisfies the differential equation

$$\frac{d\mathbf{r}}{dt} + \mathbf{r} = t\mathbf{i} + e^{-t}\mathbf{j}$$

When $t = 0$ the particle is at the point with position vector $(\mathbf{i} + \mathbf{j})$ m.

Find \mathbf{r} in terms of t .

(9)



4. A spacecraft is travelling in a straight line in deep space where all external forces can be assumed to be negligible. The spacecraft decelerates by ejecting fuel at a constant speed k relative to the spacecraft, in the direction of motion of the spacecraft. At time t , the spacecraft has speed v and mass m .

(a) Show, from first principles, that while the spacecraft is ejecting fuel,

$$\frac{dv}{dm} - \frac{k}{m} = 0 \tag{5}$$

At time $t = 0$, the spacecraft has speed U and mass M .

(b) Find the mass of the spacecraft when it comes to rest. (6)

Given that $m = Me^{-\alpha t^2}$, where α is a positive constant, and that the spacecraft comes to rest at time $t = T$,

(c) find, in terms of U and T only, the distance travelled by the spacecraft in decelerating from speed U to rest. (6)



5. A uniform rod AB , of mass m and length $2a$, is free to rotate in a vertical plane about a fixed smooth horizontal axis L . The axis L is perpendicular to the rod and passes through the point P of the rod, where $AP = \frac{2}{3}a$.

(a) Find the moment of inertia of the rod about L . (3)

The rod is held at rest with B vertically above P and is slightly displaced.

(b) Find the angular speed of the rod when PB makes an angle θ with the upward vertical. (4)

(c) Find the magnitude of the angular acceleration of the rod when PB makes an angle θ with the upward vertical. (3)

(d) Find, in terms of g and a only, the angular speed of the rod when the force acting on the rod at P is perpendicular to the rod. (5)



