

1. A string AB of length 5cm is cut, in a random place C , into two pieces. The random variable X is the length of AC .
- (a) Write down the name of the probability distribution of X and sketch the graph of its probability density function. (3)
- (b) Find the values of $E(X)$ and $\text{Var}(X)$. (3)
- (c) Find $P(X > 3)$. (1)
- (d) Write down the probability that AC is 3 cm long. (1)



3. An engineering company manufactures an electronic component. At the end of the manufacturing process, each component is checked to see if it is faulty. Faulty components are detected at a rate of 1.5 per hour.

(a) Suggest a suitable model for the number of faulty components detected per hour. (1)

(b) Describe, in the context of this question, two assumptions you have made in part (a) for this model to be suitable. (2)

(c) Find the probability of 2 faulty components being detected in a 1 hour period. (2)

(d) Find the probability of at least one faulty component being detected in a 3 hour period. (3)



- 5. (a) Write down the conditions under which the Poisson distribution may be used as an approximation to the Binomial distribution. (2)

A call centre routes incoming telephone calls to agents who have specialist knowledge to deal with the call. The probability of the caller being connected to the wrong agent is 0.01

- (b) Find the probability that 2 consecutive calls will be connected to the wrong agent. (2)

- (c) Find the probability that more than 1 call in 5 consecutive calls are connected to the wrong agent. (3)

The call centre receives 1000 calls each day.

- (d) Find the mean and variance of the number of wrongly connected calls. (3)

- (e) Use a Poisson approximation to find, to 3 decimal places, the probability that more than 6 calls each day are connected to the wrong agent. (2)



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7. (a) (i) Write down two conditions for $X \sim \text{Bin}(n, p)$ to be approximated by a normal distribution $Y \sim N(\mu, \sigma^2)$. **(2)**

(ii) Write down the mean and variance of this normal approximation in terms of n and p . **(2)**

A factory manufactures 2000 DVDs every day. It is known that 3% of DVDs are faulty.

(b) Using a normal approximation, estimate the probability that at least 40 faulty DVDs are produced in one day. **(5)**

The quality control system in the factory identifies and destroys every faulty DVD at the end of the manufacturing process. It costs £0.70 to manufacture a DVD and the factory sells non-faulty DVDs for £11.

(c) Find the expected profit made by the factory per day. **(3)**



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Question 7 continued

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8. The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{6}x & 0 < x \leq 3 \\ 2 - \frac{1}{2}x & 3 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Sketch the probability density function of X . (3)
- (b) Find the mode of X . (1)
- (c) Specify fully the cumulative distribution function of X . (7)
- (d) Using your answer to part (c), find the median of X . (3)



