Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2014

Mathematics MPC4

Unit Pure Core 4

Thursday 12 June 2014 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A curve is defined by the parametric equations $x = \frac{t^2}{2} + 1$, $y = \frac{4}{t} - 1$.

(a) Find the gradient at the point on the curve where $t = 2$.

[3 marks]

(b) Find a Cartesian equation of the curve.

[2 marks]

QUESTION PART REFERENCE

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2 (a) Given that $\dfrac{4x^3 - 2x^2 + 16x - 3}{2x^2 - x + 2}$ can be expressed as $Ax + \dfrac{B(4x-1)}{2x^2 - x + 2}$, find the values of the constants A and B.

[3 marks]

(b) The gradient of a curve is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4x^3 - 2x^2 + 16x - 3}{2x^2 - x + 2}$$

The point $(-1, 2)$ lies on the curve. Find the equation of the curve.

[4 marks]

QUESTION PART REFERENCE

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3 (a) Find the binomial expansion of $(1-4x)^{\frac{1}{4}}$ up to and including the term in x^2.
[2 marks]

(b) Find the binomial expansion of $(2+3x)^{-3}$ up to and including the term in x^2.
[3 marks]

(c) Hence find the binomial expansion of $\dfrac{(1-4x)^{\frac{1}{4}}}{(2+3x)^{3}}$ up to and including the term in x^2.
[2 marks]

QUESTION PART REFERENCE

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4 A painting was valued on 1 April 2001 at $£5000$.

The value of this painting is modelled by

$$V = Ap^t$$

where $£V$ is the value t years after 1 April 2001, and A and p are constants.

(a) Write down the value of A.

[1 mark]

(b) According to the model, the value of this painting on 1 April 2011 was $£25\,000$.

Using this model:

(i) show that $p^{10} = 5$;

[1 mark]

(ii) use logarithms to find the year in which the painting will be valued at $£75\,000$.

[4 marks]

(c) A painting by another artist was valued at $£2500$ on 1 April 1991. The value of this painting is modelled by

$$W = 2500q^t$$

where $£W$ is the value t years after 1 April 1991, and q is a constant.

(i) Show that, according to the two models, the value of the two paintings will be the same T years after 1 April 1991,

$$\text{where } T = \frac{\ln\left(\frac{5}{2}\right)}{\ln\left(\frac{p}{q}\right)}$$

[4 marks]

(ii) Given that $p = 1.029q$, find the year in which the two paintings will have the same value.

[1 mark]

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5 (a) (i) Express $3\sin x + 4\cos x$ in the form $R\sin(x + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$, giving your value of α to the nearest $0.1°$.

[3 marks]

(ii) Hence solve the equation $3\sin 2\theta + 4\cos 2\theta = 5$ in the interval $0° < \theta < 360°$, giving your solutions to the nearest $0.1°$.

[3 marks]

(b) (i) Show that the equation $\tan 2\theta \tan\theta = 2$ can be written as $2\tan^2\theta = 1$.

[2 marks]

(ii) Hence solve the equation $\tan 2\theta \tan\theta = 2$ in the interval $0° \leqslant \theta \leqslant 180°$, giving your solutions to the nearest $0.1°$.

[2 marks]

(c) (i) Use the Factor Theorem to show that $2x - 1$ is a factor of $8x^3 - 4x + 1$.

[1 mark]

(ii) Show that $4\cos 2\theta \cos\theta + 1$ can be written as $8x^3 - 4x + 1$ where $x = \cos\theta$.

[1 mark]

(iii) Given that $\theta = 72°$ is a solution of $4\cos 2\theta\cos\theta + 1 = 0$, use the results from parts **(c)(i)** and **(c)(ii)** to show that the exact value of $\cos 72°$ is $\dfrac{(\sqrt{5} - 1)}{p}$ where p is an integer.

[3 marks]

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6 The line l_1 has equation $\mathbf{r} = \begin{bmatrix} 4 \\ -5 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$.

The line l_2 has equation $\mathbf{r} = \begin{bmatrix} 7 \\ -8 \\ 6 \end{bmatrix} + \mu \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$.

The point P lies on l_1 where $\lambda = -1$. The point Q lies on l_2 where $\mu = 2$.

(a) Show that the vector $\overrightarrow{PQ}$ is parallel to $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.

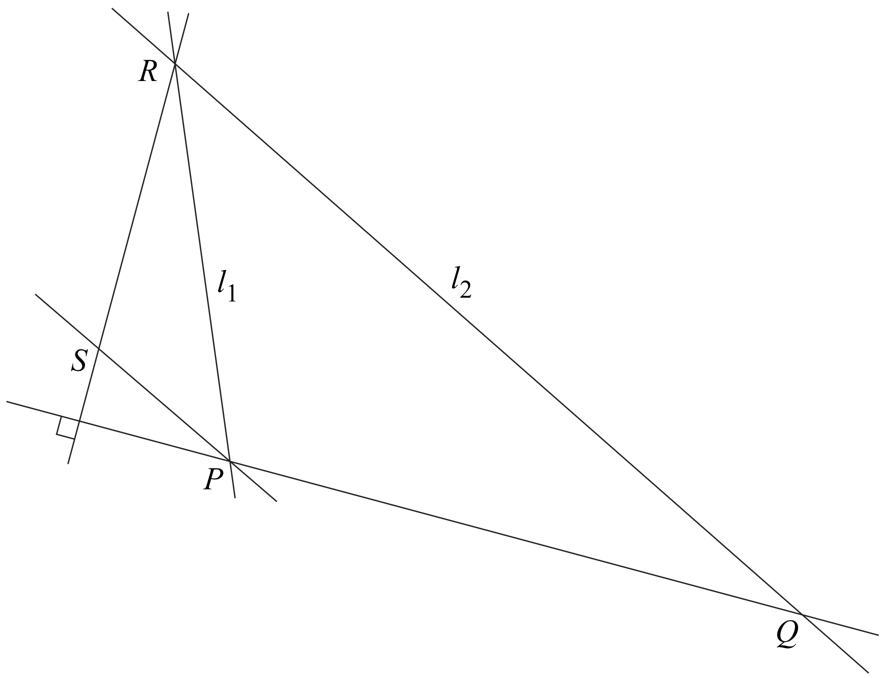
[3 marks]

(b) The lines l_1 and l_2 intersect at the point $R(3, b, c)$.

(i) Show that $b = -2$ and find the value of c.

[3 marks]

(ii) The point S lies on a line through P that is parallel to l_2. The line RS is perpendicular to the line PQ.



Find the coordinates of S.

[4 marks]

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7 A curve has equation $\cos 2y + y\mathrm{e}^{3x} = 2\pi$.

The point $A\left(\ln 2, \dfrac{\pi}{4}\right)$ lies on this curve.

(a) (i) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

[6 marks]

(ii) Hence find the exact value of the gradient of the curve at A.

[1 mark]

(b) The normal at A crosses the y-axis at the point B. Find the exact value of the y-coordinate of B.

[2 marks]

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8 (a) Express $\dfrac{16x}{(1-3x)(1+x)^2}$ in the form $\dfrac{A}{1-3x}+\dfrac{B}{1+x}+\dfrac{C}{(1+x)^2}$.

[4 marks]

(b) Solve the differential equation

$$\frac{dy}{dx}=\frac{16x\,e^{2y}}{(1-3x)(1+x)^2}$$

where $y=0$ when $x=0$.

Give your answer in the form $f(y)=g(x)$.

[7 marks]

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END OF QUESTIONS