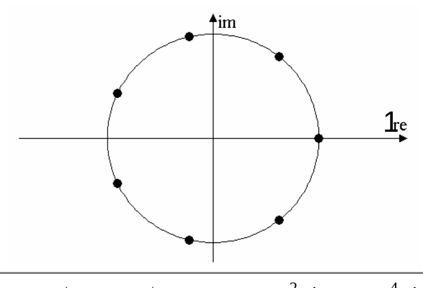
Mark Scheme 4727
June 2007

1 **(i)** $zz^* = r\,\mathrm{e}^{\mathrm{i}\theta}.r\,\mathrm{e}^{-\mathrm{i}\theta} = r^2 = \lvert z\rvert^2$	B1 **1**	For verifying result **AG**
(ii) Circle Centre $0\ (+0\mathrm{i})$ *OR* (0, 0) *OR* *O*, radius 3	B1 B1 **2** $\boxed{3}$	For stating circle For stating correct centre and radius
2 *EITHER*: $(\mathbf{r} =)\,[3+t, 1+4t, -2+2t]$ $8(3+t)-7(1+4t)+10(-2+2t)=7$ $\Rightarrow (0t)+(-3)=7 \Rightarrow$ contradiction l is parallel to Π, no intersection	M1 M1 A1 A1 B1 **5**	For parametric form of l seen or implied For substituting into plane equation For obtaining a contradiction For conclusion from correct working
OR: $[1, 4, 2]\,.\,[8, -7, 10] = 0$ $\Rightarrow l$ is parallel to Π $(3, 1, -2)$ into Π $\Rightarrow\ 24-7-20 \neq 7$ l is parallel to Π, no intersection	M1 A1 M1 A1 B1	For finding scalar product of direction vectors For correct conclusion For substituting point into plane equation For obtaining a contradiction For conclusion from correct working
OR:Solve $\dfrac{x-3}{1}=\dfrac{y-1}{4}=\dfrac{z+2}{2}$ and $8x-7y+10z=7$ eg $y-2z=3$, $2y-2=4z+8$ eg $4z+4=4z+8$ l is parallel to Π, no intersection	 M1 A1 M1 A1 B1 $\boxed{5}$	 For eliminating one variable For eliminating another variable For obtaining a contradiction For conclusion from correct working
3 Aux. equation $m^2-6m+8\ (=0)$ $m=2, 4$ CF $(y=)\ A\mathrm{e}^{2x}+B\mathrm{e}^{4x}$ PI $(y=)\ C\mathrm{e}^{3x}$ $9C-18C+8C=1 \Rightarrow C=-1$ GS $y=A\mathrm{e}^{2x}+B\mathrm{e}^{4x}-\mathrm{e}^{3x}$	M1 A1 A1√ M1 A1 B1√ **6** $\boxed{6}$	For auxiliary equation seen For correct roots For correct CF. f.t. from their m For stating and substituting PI of correct form For correct value of C For GS. f.t. from their CF + PI with 2 arbitrary constants in CF and none in PI

4 **(i)** $q(st) = qp = s$	B1		For obtaining s
$(qs)t = tt = s$	B1	2	For obtaining s
(ii) METHOD 1			
Closed: see table	B1		For stating closure with reason
Identity = r	B1		For stating identity r
Inverses: $p^{-1} = s,\ q^{-1} = t,\ (r^{-1} = r),$	M1		For checking for inverses
$s^{-1} = p,\ t^{-1} = q$	A1	4	For stating inverses *OR* For giving sufficient explanation to justify each element has an inverse eg r occurs once in each row and/or column
METHOD 2			
Identity = r	B1		For stating identity r
	M1		For attempting to establish a generator $\neq r$
eg $p^2 = t,\ p^3 = q,\ p^4 = s$	A1		For showing powers of p (*OR* q, s or t) are different elements of the set
$\Rightarrow p^5 = r$, so p is a generator	A1		For concluding p^5 (*OR* q^5, s^5 or t^5) = r
(iii) e, d, d^2, d^3, d^4	B2	2	For stating all elements **AEF** eg d^{-1}, d^{-2}, dd
		$\boxed{8}$	

5 **(i)** $(\cos 6\theta =)\ \mathrm{Re}(c + \mathrm{i}\,s)^6$	M1		For expanding (real part of) $(c+\mathrm{i}\,s)^6$ at least 4 terms and 1 evaluated binomial coefficient needed
$(\cos 6\theta =)\ c^6 - 15c^4s^2 + 15c^2s^4 - s^6$	A1		For correct expansion
$(\cos 6\theta =)$ $c^6 - 15c^4(1-c^2) + 15c^2(1-c^2)^2 - (1-c^2)^3$	M1		For using $s^2 = 1 - c^2$
$(\cos 6\theta =)\ 32c^6 - 48c^4 + 18c^2 - 1$	A1	4	For correct result **AG**
(ii) $64x^6 - 96x^4 + 36x^2 - 3 = 0 \Rightarrow \cos 6\theta = \frac{1}{2}$	M1		For obtaining a numerical value of $\cos 6\theta$
$\Rightarrow (\theta =)\ \frac{1}{18}\pi, \frac{5}{18}\pi, \frac{7}{18}\pi$ etc.	A1		For any correct solution of $\cos 6\theta = \frac{1}{2}$
$\cos 6\theta = \frac{1}{2}$ has multiple roots	M1		For stating or implying at least 2 values of θ
largest x requires smallest θ $\Rightarrow$ largest positive root is $\cos\frac{1}{18}\pi$	A1	4	For identifying $\cos\frac{1}{18}\pi$ **AEF** as the largest positive root from a list of 3 positive roots *OR* from general solution *OR* from consideration of the cosine function
		$\boxed{8}$	

6 **(i)** $\mathbf{n} = l_1 \times l_2$	B1	For stating or implying in **(i)** or **(ii)** that **n** is perpendicular to l_1 and l_2
$\mathbf{n} = [2, -1, 1] \times [4, 3, 2]$	M1*	For finding vector product of direction vectors
$\mathbf{n} = k[-1, 0, 2]$	A1	For correct vector (any k)
$[3, 4, -1] \,.\, k[-1, 0, 2] = -5k$	M1 (*dep)	For substituting a point of l_1 into $\mathbf{r}.\mathbf{n}$
$\mathbf{r}.[-1, 0, 2] = -5$	A1 **5**	For obtaining correct p. **AEF** in this form
(ii) $[5, 1, 1] \,.\, k[-1, 0, 2] = -3k$	M1	For using same **n** and substituting a point of l_2
$\mathbf{r}.[-1, 0, 2] = -3$	A1√ **2**	For obtaining correct p. **AEF** in this form f.t. on incorrect **n**
(iii) $d = \dfrac{\lvert -5+3 \rvert}{\sqrt{5}}$ *OR* $d = \dfrac{\lvert [2, -3, 2] . [-1, 0, 2] \rvert}{\sqrt{5}}$ *OR* d from $(5, 1, 1)$ to $\Pi_1 = \dfrac{\lvert 5(-1)+1(0)+1(2)+5 \rvert}{\sqrt{5}}$ *OR* d from $(3, 4, -1)$ to $\Pi_2 = \dfrac{\lvert 3(-1)+4(0)-1(2)+3 \rvert}{\sqrt{5}}$ *OR* $[3-t, 4, -1+2t] . [-1, 0, 2] = -3 \Rightarrow t = \frac{2}{5}$ *OR* $[5-t, 1, 1+2t] . [-1, 0, 2] = -5 \Rightarrow t = -\frac{2}{5}$	M1	For using a distance formula from their equations Allow omission of | | *OR* For finding intersection of $\mathbf{n}_1$ and Π_2 or $\mathbf{n}_2$ and Π_1
$d = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5} = 0.894427...$	A1√ **2**	For correct distance **AEF** f.t. on incorrect **n**
(iv) d is the shortest *OR* perpendicular distance between l_1 and l_2	B1 **1** $\boxed{10}$	For correct statement
7 **(i)** $(z - e^{i\phi})(z - e^{-i\phi}) \equiv z^2 - (2)z\dfrac{(e^{i\phi} + e^{-i\phi})}{(2)} + 1$ $\equiv z^2 - (2\cos\phi)z + 1$	B1 **1**	For correct justification **AG**
(ii) $z = e^{\frac{2}{7}k\pi i}$	B1	For general form *OR* any one non-real root
for $k = 0, 1, 2, 3, 4, 5, 6$ *OR* $0, \pm 1, \pm 2, \pm 3$	B1	For other roots specified (k=0 may be seen in any form, eg 1, e^0, $e^{2\pi i}$) For answers in form $\cos\theta + i\sin\theta$ allow maximum B1 B0
	B1 B1 **4**	For any 7 points equally spaced round unit circle (circumference need not be shown) For 1 point on $+^{ve}$ real axis, and other points in correct quadrants
(iii) $\left(z^7 - 1 =\right) (z-1)(z - e^{\frac{2}{7}\pi i})(z - e^{\frac{4}{7}\pi i})$ $(z - e^{\frac{6}{7}\pi i})(z - e^{\frac{-2}{7}\pi i})(z - e^{\frac{-4}{7}\pi i})(z - e^{\frac{-6}{7}\pi i})$	M1	For using linear factors from **(ii),** seen or implied
$= (z - e^{\frac{2}{7}\pi i})(z - e^{\frac{-2}{7}\pi i}) \times (z - e^{\frac{4}{7}\pi i})(z - e^{\frac{-4}{7}\pi i})$ $(z - e^{\frac{6}{7}\pi i})(z - e^{\frac{-6}{7}\pi i}) \times$	M1	For identifying at least one pair of complex conjugate factors
$\times (z-1)$	B1	For linear factor seen
$= (z^2 - (2\cos\frac{2}{7}\pi)z + 1) \times$	A1	For any one quadratic factor seen
$(z^2 - (2\cos\frac{4}{7}\pi)z + 1) \times (z^2 - (2\cos\frac{6}{7}\pi)z + 1) \times$ $\times (z-1)$	A1 **5** $\boxed{10}$	For the other 2 quadratic factors and expression written as product of 4 factors

8 **(i)** Integrating factor $e^{\int \tan x\,(\mathrm{d}x)}$	B1	For correct IF
$= e^{-\ln\cos x}$	M1	For integrating to ln form
$= (\cos x)^{-1}$ *OR* $\sec x$	A1	For correct simplified IF **AEF**
$\Rightarrow \dfrac{\mathrm{d}}{\mathrm{d}x}\left(y(\cos x)^{-1}\right) = \cos^2 x$	B1√	For $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(y.\text{their IF}\right) = \cos^3 x\,.\,\text{their IF}$
$y(\cos x)^{-1} = \int \frac{1}{2}(1+\cos 2x)\,(\mathrm{d}x)$	M1 M1	For integrating LHS For attempting to use $\cos 2x$ formula *OR* parts for $\int \cos^2 x\,\mathrm{d}x$
$y(\cos x)^{-1} = \frac{1}{2}x + \frac{1}{4}\sin 2x\,(+c)$	A1	For correct integration both sides **AEF**
$y = \left(\frac{1}{2}x + \frac{1}{4}\sin 2x + c\right)\cos x$	A1 **8**	For correct general solution **AEF**
(ii) $2 = \left(\frac{1}{2}\pi + c\right).-1 \Rightarrow c = -2 - \frac{1}{2}\pi$	M1	For substituting $(\pi, 2)$ into their GS and solve for c
$y = \left(\frac{1}{2}x + \frac{1}{4}\sin 2x - 2 - \frac{1}{2}\pi\right)\cos x$	A1 **2**	For correct solution **AEF**
	$\boxed{10}$	
9 **(i)** $3^n \times 3^m = 3^{n+m}$, $n+m \in \mathbb{Z}$	B1	For showing closure
$\left(3^p \times 3^q\right)\times 3^r = \left(3^{p+q}\right)\times 3^r = 3^{p+q+r}$	M1	For considering 3 distinct elements, seen bracketed 2+1 or 1+2
$= 3^p \times \left(3^{q+r}\right) = 3^p \times \left(3^q \times 3^r\right) \Rightarrow$ associativity	A1	For correct justification of associativity
Identity is 3^0	B1	For stating identity. Allow 1
Inverse is 3^{-n}	B1	For stating inverse
$3^n \times 3^m = 3^{n+m} = 3^{m+n} = 3^m \times 3^n \Rightarrow$ commutativity	B1 **6**	For showing commutativity
(ii) (a) $3^{2n} \times 3^{2m} = 3^{2n+2m}\left(= 3^{2(n+m)}\right)$	B1*	For showing closure
Identity, inverse OK	B1 (*dep) **2**	For stating other two properties satisfied and hence a subgroup
(b) For 3^{-n},	M1	For considering inverse
$-n \notin$ subset	A1 **2**	For justification of not being a subgroup 3^{-n} must be seen here or in **(i)**
(c) *EITHER*: eg $3^{1^2} \times 3^{2^2} = 3^5$	M1	For attempting to find a specific counter-example of closure
$\neq 3^{r^2} \Rightarrow$ not a subgroup	A1 **2**	For a correct counter-example and statement that it is not a subgroup
OR: $3^{n^2} \times 3^{m^2} = 3^{n^2+m^2}$	M1	For considering closure in general
$\neq 3^{r^2}$ eg $1^2 + 2^2 = 5 \Rightarrow$ not a subgroup	A1	For explaining why $n^2 + m^2 \neq r^2$ in general and statement that it is not a subgroup
	$\boxed{12}$	