General Certificate of Education
Advanced Level Examination
January 2013

Mathematics MM2B

Unit Mechanics 2B

Monday 28 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 Tim is playing cricket. He hits a ball at a point A. The speed of the ball immediately after being hit is $11\,\mathrm{m\,s^{-1}}$.

The ball strikes a tree at a point B. The height of B is 5 metres above the height of A.

The ball is to be modelled as a particle of mass 0.16 kg being acted upon only by gravity.

(a) Calculate the initial kinetic energy of the ball. *(2 marks)*

(b) Calculate the potential energy gained by the ball as it moves from the point A to the point B. *(2 marks)*

(c) (i) Find the kinetic energy of the ball immediately before it strikes the tree. *(2 marks)*

(ii) Hence find the speed of the ball immediately before it strikes the tree. *(2 marks)*

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2 A particle moves in a horizontal plane. The vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in the horizontal plane. At time t seconds, the velocity of the particle, $\mathbf{v}\,\mathrm{m\,s^{-1}}$, is given by

$$\mathbf{v} = 12\cos\left(\frac{\pi}{3}t\right)\mathbf{i} - 9t^2\mathbf{j}$$

(a) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(b) The particle, which has mass 4 kg, moves under the action of a single force, $\mathbf{F}$ newtons.

(i) Find an expression for the force $\mathbf{F}$ in terms of t. *(2 marks)*

(ii) Find the magnitude of $\mathbf{F}$ when $t = 3$. *(2 marks)*

(c) When $t = 3$, the particle is at the point with position vector $(4\mathbf{i} - 2\mathbf{j})$ m.

Find the position vector, $\mathbf{r}$ metres, of the particle at time t. *(5 marks)*

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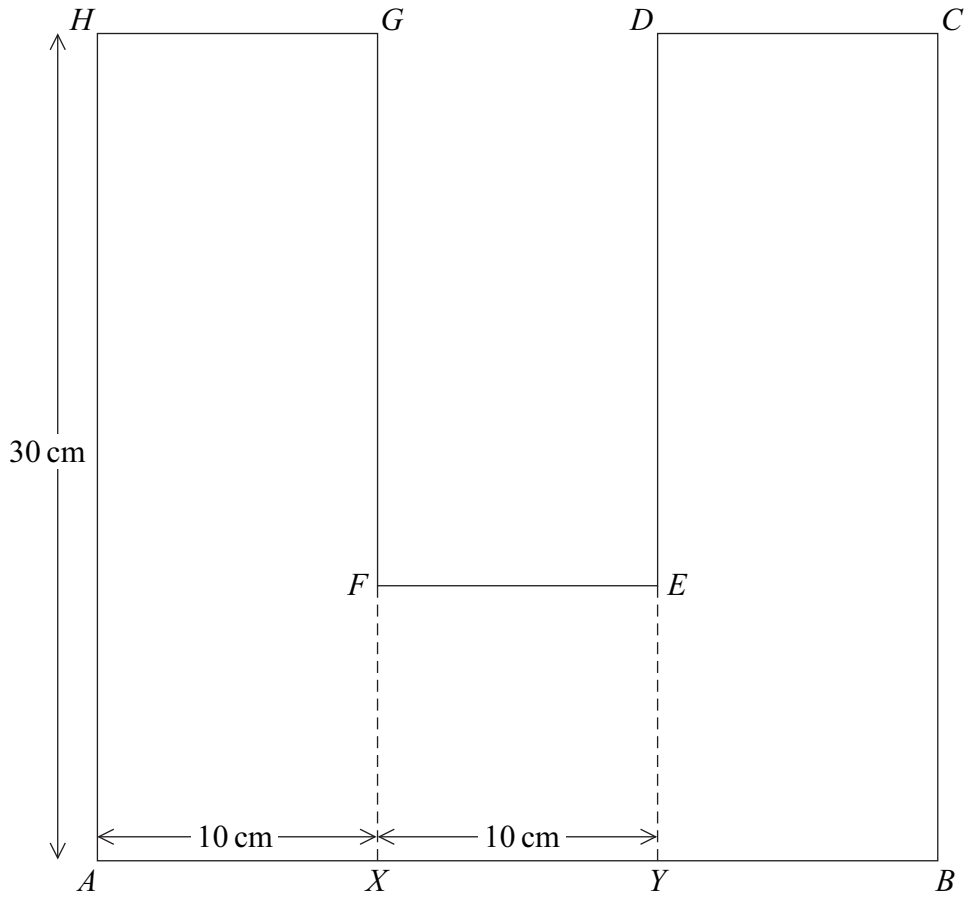
3 A van, of mass 1500 kg, travels at a constant speed of $22\,\mathrm{m\,s^{-1}}$ up a slope inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{25}$.

The van experiences a resistance force of 8 000 N.

Find the power output of the van's engine, giving your answer in kilowatts. *(5 marks)*

4 The diagram shows a uniform lamina which is in the shape of two identical rectangles *AXGH* and *YBCD* and a square *XYEF*, arranged as shown.

The length of *AX* is 10 cm, the length of *XY* is 10 cm and the length of *AH* is 30 cm.



(a) Explain why the centre of mass of the lamina is 15 cm from *AH*. *(1 mark)*

(b) Find the distance of the centre of mass of the lamina from *AB*. *(3 marks)*

(c) The lamina is freely suspended from the point *H*.

Find, to the nearest degree, the angle between *HG* and the horizontal when the lamina is in equilibrium. *(4 marks)*

5 A particle, of mass 12 kg, is moving along a straight horizontal line. At time t seconds, the particle has speed $v\,\mathrm{m\,s^{-1}}$. As the particle moves, it experiences a resistance force of magnitude $4v^{\frac{1}{3}}$. No other horizontal force acts on the particle.

The initial speed of the particle is $8\,\mathrm{m\,s^{-1}}$.

(a) Show that

$$v = \left(4 - \frac{2}{9}t\right)^{\frac{3}{2}}$$ *(6 marks)*

(b) Find the value of t when the particle comes to rest. *(1 mark)*

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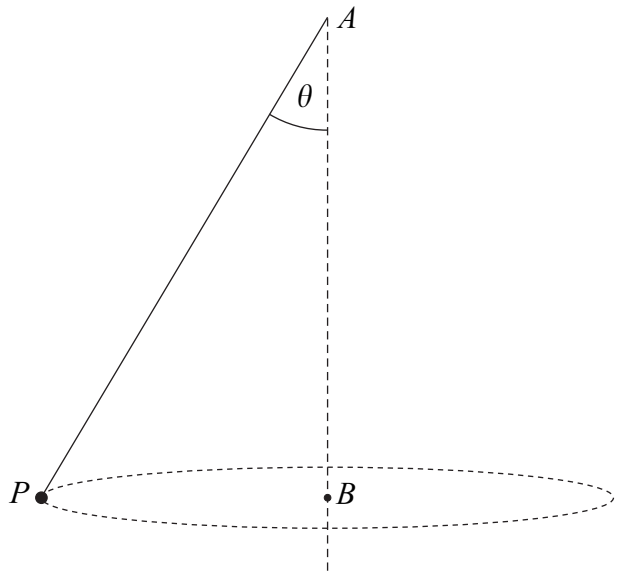
6 A light inextensible string has one end attached to a particle, P, of mass 2 kg. The other end of the string is attached to the fixed point A. The point A is vertically above the point B. The particle moves at a constant speed in a horizontal circle of radius 0.8 m and centre B. The tension in the string is 34 N.

The string is inclined at an angle θ to the vertical, as shown in the diagram.



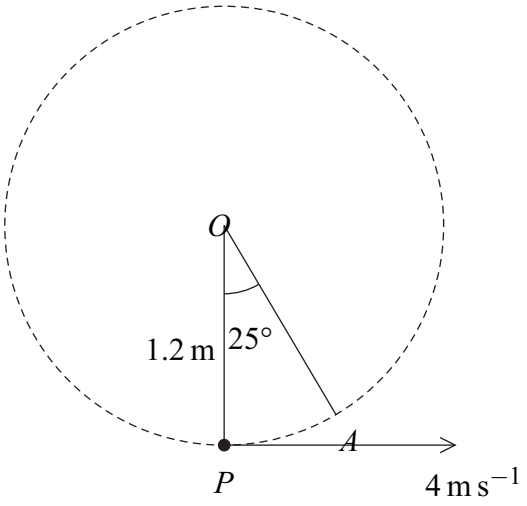
(a) Find the angle θ. *(3 marks)*

(b) Find the speed of the particle. *(3 marks)*

(c) Find the time taken for the particle to make one complete revolution. *(2 marks)*

7 A small ball, of mass 3 kg, is suspended from a fixed point O by a light inextensible string of length 1.2 m. Initially, the string is taut and the ball is at the point P, vertically below O. The ball is then set into motion with an initial horizontal velocity of $4\,\mathrm{m\,s^{-1}}$.

The ball moves in a vertical circle, centre O. The point A, on the circle, is such that angle AOP is 25°, as shown in the diagram.



(a) Find the speed of the ball at the point A. *(4 marks)*

(b) Find the tension in the string when the ball is at the point A. *(3 marks)*

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8 (a) An elastic string has natural length l and modulus of elasticity λ. The string is stretched from length l to length $l + e$.

Show, by integration, that the work done in stretching the string is $\dfrac{\lambda e^2}{2l}$. *(3 marks)*

(b) A particle, of mass 5 kg, is attached to one end of a light elastic string. The other end of the string is attached to a fixed point O.

The string has natural length 1.6 m and modulus of elasticity 392 N.

(i) Find the extension of the string when the particle hangs in equilibrium. *(2 marks)*

(ii) The particle is pulled down to a point A, which is 2.2 m below the point O.

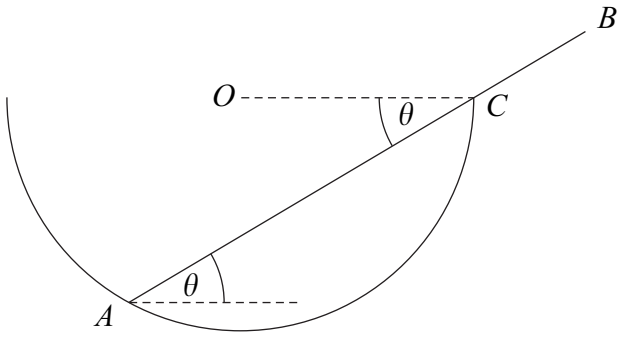
Calculate the elastic potential energy in the string. *(3 marks)*

(iii) The particle is released when it is at rest at the point A.

Calculate the distance of the particle from the point A when its speed first reaches $0.8\,\mathrm{m\,s^{-1}}$. *(5 marks)*

Turn over ▶

9 A smooth hollow hemisphere, of radius a and centre O, is fixed so that its rim is in a horizontal plane. A smooth uniform rod AB, of mass m, is in equilibrium, with one end A resting on the inside of the hemisphere and the point C on the rod being in contact with the rim of the hemisphere. The rod, of length l, is inclined at an angle θ to the horizontal, as shown in the diagram.



(a) Explain why the reaction between the rod and the hemisphere at point A acts through O. *(1 mark)*

(b) Draw a diagram to show the forces acting on the rod. *(2 marks)*

(c) Show that $l = \dfrac{4a\cos 2\theta}{\cos\theta}$. *(5 marks)*