General Certificate of Education (A-level) June 2013

Statistics SS03

(Specification 6380)

Statistics 3

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and accuracy
Е	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
−x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
С	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	H_0 : Population median time = 32	B1		
	H_1 : Population median time < 32			must mention population
	1 tail test 10% level			
	signs			
	+ + + +	M1		for signs or signed differences
	test stat = $8 - 4+$	A1		for test stat 8 or 4
	D: (12.05) 1.1	3.41		C CD: 1.1
	Bin (12, 0.5) model	M1		for use of Bin model
	$P(\le 4+) = 0.194 > 0.10$	M1		for comparison ts,
	$1(\leq 7) = 0.197 > 0.10$	1711		0.193-0.194, and 10%
				0.175 0.17 i, and 1070
	Accept H ₀ . No significant evidence to			
	suggest median time to complete			
	crossword has decreased.	A1	6	ts/cv correct
				Alt method
				Use of cr {0, 1, 2, 3} or {9, 10, 11, 12}
				with prob 0.073 used.
	Total		6	

Q	Solution	Marks	Total	Comments
2(a)	H_0 : Population average difference = 0	B1		May refer to mean/median
	H₁: Population average difference ≠ 0 2 tail test 5 % level			
	2 tan test 3 % level			
	A B C D E			
	diff +3.30.8 -0.9 +0.3	M1		For differences
	rank 9 5 6 2	m1		Ranks: smallest rank 1
	F G H I J	1		T-4-1 - f ()
	diff +0.7 +0.4 +1.7 -0.1 +1.1	m1 A1		Total of ranks (any) One correct
		Ai		One correct
	$T_{+} = 9 + 2 + 4 + 3 + 8 + 7 = 33$			
	$T_{-}=5 + 6 + 1 = 12$			
	test stat $T = 12$			
	critical value = 6	B1		For cv
	test stat > 6	m1		ft (must be positive ts) 'correct' T with cv comparison (smaller T / smaller cv larger T / larger cv)
	Accept H_0 There is no significant evidence of a	A1		ts/cv correct
	difference in average percentage of total expenditure spent on 'Highways' between 2002 and 2012.	E1	9	In context – only if conclusion correct
(b)(i)	<u>Differences</u> are symmetrically distributed.	B1		
(ii)	A paired sign test	B1	2	
(c)(i)	0	B1		
(ii)	1+2+3+4+5+6+7+8+9+10=55	M1A1	3	
	Total		14	

Q	Solution	Marks	Total	Comments
3(a)(i)	H ₀ : Samples are taken from identical populations	B1		Or equivalent referring to population medians
	H ₁ : Samples are not taken from identical populations (average emissions lower when device fitted) 1 tail 5%			
	Ranks	M1		Ranks as one group either way
	New device fitted Device not fitted 139.1 5 8 145.4 12 1 134.6 3 10 144.0 11 2 128.9 1 12 138.7 4 9 139.8 8 5 139.7 7 6 129.5 2 11 139.6 6 7	A1		Ranks correct
		m1		Ranks totalled or reversed
	$U_{\text{fitted}} = 29 - \frac{6 \times 7}{2} = 8$ $U_{\text{not}} = 49 - \frac{6 \times 7}{2} = 28$	m1 A1		Attempt to find U Either U correct
	U = 8			
	cv = 7 for $n = 6$, $m = 6$ 1 tail 5%	B1		cv correct cao
	U > 7 (or cv=29 and 28 < 29)	M1		correct comparison ft – upper ts /29 lower ts /7
	Accept H ₀	A1		ts/cv correct
	No significant evidence of a reduction in average CO ₂ emissions for cars fitted with new device.	E1	10	In context – only if conclusion correct
(ii)	The 12 new cars can be regarded as a random sample. OR They were randomly selected.	B1	1	Disallow 'random' no context
(b)	A Type II error would be to conclude that H_0 is true, that is there is no reduction in average CO_2 emissions for cars fitted with	В1		Type II correctly explained
	new device, when in fact H ₀ is not true and there is a reduction in average CO ₂ emissions for cars fitted with new device	E1	2	In context
	Total		13	

Q	Solution				Marks	Total	Comments
4 (a)							
		Minor	Serious	total	B1		One total correct 13/12
	New	9	3	12	M1		One frequency correct
	Current	4	9	13			
	Total	13	12	25	A1	3	All Correct
(b)	H ₀ : Mouthweategory of the category of the ca	mouth inf vash type	ection is associate	ed with	B1		H ₀ Indep/No Assoc H ₁ Not Indep/ Assoc
	Expected New Current	Minor 6.24 6.76	5.76 6.24		M1 A1		Method for expected freqs
	$ts = \sum \frac{(O - E - 0.5)^2}{E}$ $= \frac{2.26^2}{6.24} + \frac{2.26^2}{5.76} + \frac{2.26^2}{6.76} + \frac{2.26^2}{6.24}$ $= 3.28$ $cv df = 1 5\% cv = 3.84$			M1		ts 'correct' with/without Yates	
				M1		Yates used correctly	
					A1		ts correct (3.1 – 3.4)
				B1		cv cao	
	ts < 3.84 A	ccept H ₀					No Yates used can gain M1 A1 M1 B1 ts = 4.89
	No significa mouthwash mouth infect	is associa		itegory of	E1	8	Conclusion correct in contest and ts/cv correct
				Total		11	

Q	Solution	Marks	Total	Comments
5(a)	H ₀ : Samples from identical populations	B1		
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	M1 A1		For ranks as one group 10 or more correct
	Totals of ranks $T_{\rm I} = 75 \ 27 \ T_{\rm II} = 33 \ 52 \ T_{\rm III} = 28 \ 57$ $n_{\rm I} = 6 \ n_{\rm II} = 5 \ n_{\rm III} = 5$	m1		Totals – can be 27 52 57
	$\sum_{i=1}^{m} \frac{T_i^2}{n_i} = \frac{75^2}{6} + \frac{33^2}{5} + \frac{28^2}{5} = 1312.1$	m1 m1		Numerators correct Denominators correct
	$H = \frac{12}{16 \times 17} \times 1312.1 - (3 \times 17)$	m1		H formula correctly used $ft \sum_{i=1}^{m} \frac{T_i^2}{n_i}$
	= 6.89	A1		(6.7 – 7.1) ts/cv correct
	Critical value from $\chi_2^2 = 5.991$ H > 5.991	B1		For cv cao
	Reject H ₀ . Significant evidence to suggest that samples are not from identical populations. Significant difference in average score for at least 2 of the three methods involved.	E1	10	Conclusion correct in context
(b)	Use Approach I since <u>average of ranks</u> is highest (lowest ft) (so <u>lowest average scores</u>)	B1 E1	2	Approach I Reason (vice versa for reversed ranks). Allow reference to average scores.
	Total		12	

Q	Solution	Marks	Total	Comments
6(a)(i)	H ₀ : Colour preference is independent of personality H ₁ : Colour preference is not independent of personality 1 tail 5%	В1		H ₀ Indep / No Assoc H ₁ Not Indep / Assoc
	Exp R Y G B Intro 47 9.4 18.8 18.8 Extro 153 30.6 61.2 61.2	M1 M1 A1		Any one E_i correct At least 5 correct All correct SC1 integers
	$ts = \sum \frac{(O-E)^2}{E}$ $= \frac{11^2}{47} + \frac{1.4^2}{9.4} + \frac{5.2^2}{18.8} + \frac{7.2^2}{18.8} + \frac{11^2}{153} + \frac{1.4^2}{153} + 1.$	M1 M1		Numerators OK ft Denominators OK ft and added
	$\frac{1.4^{2}}{30.6} + \frac{5.2^{2}}{61.2} + \frac{7.2^{2}}{61.2}$ $= 9.12$ $df = 3 5\% cv = 7.815$	A1 B1		(9.0-9.3) For cv
	ts > 7.815 Reject H ₀ Sig evidence to suggest colour preference is not independent of personality	E1	9	or = 0.0277 In context
(ii)	Introverts far more likely than expected to prefer blue or green (introverts far less likely than expected to choose red)	B1 E1	2	Alt Extroverts are more likely than expected to prefer red

Q	Solution	Marks	Total	Comments
6(b)(i)	d 0 0 1.5 0 0 1 1 1 2.5	M1		Differences
	$\sum d^2 = 11.5$	M1		Formula correct
	SRCC $r_s = 1 - \frac{6 \times \sum d^2}{9 \times 80} = 0.904$	A1		Formula correct
	or SRCC $r_s = 0.904$ (from calc)	(B3)	3	SC1 0.9 SC2 0.90 if no method shown
(ii)	H ₀ : Rank orders of personality score and	B1		Hypothesis
	happiness score are independent.			
	H ₁ : Rank orders of personality score and			
	happiness score are not independent. 2 tail 1%			
	cv = 0.8167	B1		cv cao
	test stat $r_s = 0.904$			
	$r_s > cv$	M1		comparison ft seen or implied
	Reject H ₀ Significant evidence at 1% level to suggest an association (positive)	A1		ts/cv correct
	between rank orders of personality score	F.1	-	
	and happiness score. Students with a	E1	5	in context – vice versa OK
	higher extrovert personality score tend to			
	have a higher happiness score. Total		19	
	TOTAL		75	