

1. A company manufactures bolts with a mean diameter of 5 mm. The company wishes to check that the diameter of the bolts has not decreased. A random sample of 10 bolts is taken and the diameters, x mm, of the bolts are measured. The results are summarised below.

$$\sum x = 49.1 \quad \sum x^2 = 241.2$$

Using a 1% level of significance, test whether or not the mean diameter of the bolts is less than 5 mm.

(You may assume that the diameter of the bolts follows a normal distribution.)

(8)



2. An emission-control device is tested to see if it reduces CO_2 emissions from cars. The emissions from 6 randomly selected cars are measured with and without the device. The results are as follows.

Car	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
Emissions without device	151.4	164.3	168.5	148.2	139.4	151.2
Emissions with device	148.9	162.7	166.9	150.1	140.0	146.7

- (a) State an assumption that needs to be made in order to carry out a *t*-test in this case. (1)

- (b) State why a paired *t*-test is suitable for use with these data. (1)

- (c) Using a 5% level of significance, test whether or not there is evidence that the device reduces CO_2 emissions from cars. (8)

- (d) Explain, in context, what a type II error would be in this case. (2)



3. Define, in terms of H_0 and/or H_1 ,

(a) the size of a hypothesis test, (1)

(b) the power of a hypothesis test. (1)

The probability of getting a head when a coin is tossed is denoted by p .

This coin is tossed 12 times in order to test the hypotheses $H_0: p = 0.5$ against $H_1: p \neq 0.5$, using a 5% level of significance.

(c) Find the largest critical region for this test, such that the probability in each tail is less than 2.5%. (4)

(d) Given that $p = 0.4$

(i) find the probability of a type II error when using this test,

(ii) find the power of this test. (4)

(e) Suggest two ways in which the power of the test can be increased. (2)



4. A farmer set up a trial to assess whether adding water to dry feed increases the milk yield of his cows. He randomly selected 22 cows. Thirteen of the cows were given dry feed and the other 9 cows were given the feed with water added. The milk yields, in litres per day, were recorded with the following results.

	Sample size	Mean	s^2
Dry feed	13	25.54	2.45
Feed with water added	9	27.94	1.02

You may assume that the milk yield from cows given the dry feed and the milk yield from cows given the feed with water added are from independent normal distributions.

- (a) Test, at the 10% level of significance, whether or not the variances of the populations from which the samples are drawn are the same. State your hypotheses clearly. **(5)**
- (b) Calculate a 95% confidence interval for the difference between the two mean milk yields. **(7)**
- (c) Explain the importance of the test in part (a) to the calculation in part (b). **(2)**



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5. A machine fills jars with jam. The weight of jam in each jar is normally distributed. To check the machine is working properly the contents of a random sample of 15 jars are weighed in grams. Unbiased estimates of the mean and variance are obtained as

$$\hat{\mu} = 560 \quad s^2 = 25.2$$

Calculate a 95% confidence interval for,

- (a) the mean weight of jam, (4)

- (b) the variance of the weight of jam. (5)

A weight of more than 565 g is regarded as too high and suggests the machine is not working properly.

- (c) Use appropriate confidence limits from parts (a) and (b) to find the highest estimate of the proportion of jars that weigh too much. (5)



6. A continuous uniform distribution on the interval $[0, k]$ has mean $\frac{k}{2}$ and variance $\frac{k^2}{12}$.

A random sample of three independent variables X_1, X_2 and X_3 is taken from this distribution.

(a) Show that $\frac{2}{3}X_1 + \frac{1}{2}X_2 + \frac{5}{6}X_3$ is an unbiased estimator for k . **(3)**

An unbiased estimator for k is given by $\hat{k} = aX_1 + bX_2$ where a and b are constants.

(b) Show that $\text{Var}(\hat{k}) = (a^2 - 2a + 2) \frac{k^2}{6}$ **(6)**

(c) Hence determine the value of a and the value of b for which \hat{k} has minimum variance, and calculate this minimum variance. **(6)**



