Version 1.0

General Certificate of Education (A-level)
January 2011

# Mathematics MFP1

(Specification 6360)

Further Pure 1

# Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.



Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334).
Registered address: AQA, Devas Street, Manchester M15 6EX.

**Key to mark scheme abbreviations**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| $-x$ EE | deduct $x$ marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MFP1**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)** | $\alpha+\beta=6$, $\alpha\beta=18$ | B1B1 | 2 | |
| **(b)** | Sum of new roots $=6^2-2(18)=0$<br>Product $=18^2=324$<br>Equation $x^2+324=0$ | M1A1F<br>B1F<br>A1F | <br><br>4 | ft wrong value(s) in (a)<br>ditto<br>'= 0' needed here;<br>ft wrong value(s) for sum/product |
| **(c)** | $\alpha^2$ and $\beta^2$ are $\pm 18\mathrm{i}$ | B1 | 1 | |
| | **Total** | | **7** | |
| **2(a)** | $\int 2x^{-3}\,\mathrm{d}x=-x^{-2}\ (+c)$ | M1A1 | | M1 for correct index |
| | $\int_p^q 2x^{-3}\,\mathrm{d}x=p^{-2}-q^{-2}$ | A1F | 3 | OE; ft wrong coefficient of $x^{-2}$ |
| **(b)(i)** | As $p\to 0$, $p^{-2}\to\infty$, so no value | B1 | | |
| **(ii)** | As $q\to\infty$, $q^{-2}\to 0$, so value is ¼ | M1A1F | 3 | ft wrong coefficient of $x^{-2}$<br>or reversal of limits |
| | **Total** | | **6** | |
| **3(a)(i)** | $\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$ | B1 | 1 | |
| **(ii)** | $\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$ | B1 | 1 | |
| **(b)(i)** | $\mathbf{AB}=\begin{bmatrix}-20 & 14\\ 14 & -10\end{bmatrix}$ | M1A1 | 2 | M1A0 if 3 entries correct |
| **(ii)** | $\mathbf{A}+\mathbf{B}=\begin{bmatrix}0 & 5\\ -5 & 0\end{bmatrix}$ | B1 | | |
| | $(\mathbf{A}+\mathbf{B})^2=\begin{bmatrix}-25 & 0\\ 0 & -25\end{bmatrix}$ | B1 | | |
| | $\ldots=-25\mathbf{I}$ | B1F | 3 | ft if c's $(\mathbf{A}+\mathbf{B})^2$ is of the form $k\mathbf{I}$ |
| **(c)(i)** | Rot'n 90° clockwise, enlargem't SF 5 | B2, 1 | 2 | OE |
| **(ii)** | Rotation 180°, enlargement SF 25 | B2, 1F | 2 | Accept 'enlargement SF −25';<br>ft wrong value of $k$ |
| **(iii)** | Enlargement SF 625 | B2, 1F | 2 | B1 for pure enlargement; ft ditto |
| | **Total** | | **13** | |
| **4** | $\sin\left(-\frac{\pi}{6}\right)=-\frac{1}{2}$ | B1 | | OE; dec/deg penalised at 6th mark |
| | $\sin\left(-\frac{5\pi}{6}\right)=-\frac{1}{2}$ | B1F | | OE; ft wrong first value |
| | Use of $2n\pi$ | M1 | | (or $n\pi$) at any stage |
| | Going from $4x-\frac{2\pi}{3}$ to $x$ | m1 | | including division of all terms by 4 |
| | GS $x=\frac{\pi}{8}+\frac{1}{2}n\pi$ or $x=-\frac{\pi}{24}+\frac{1}{2}n\pi$ | A1A1 | 6 | OE |
| | **Total** | | **6** | |

**MFP1(cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **5(a)(i)** | $z_1^{\ 2} = \frac{1}{4} - \mathrm{i} + \mathrm{i}^2 = -\frac{3}{4} - \mathrm{i}$ | M1A1 | 2 | M1 for use of $\mathrm{i}^2 = -1$ |
| **(ii)** | LHS $= -\frac{3}{4} - \mathrm{i} + \frac{1}{2} + \mathrm{i} + \frac{1}{4} = 0$ | M1A1 | 2 | AG; M1 for $z$* correct |
| **(b)** | LHS $= -\frac{3}{4} + \mathrm{i} + \frac{1}{2} - \mathrm{i} + \frac{1}{4} = 0$ | M1A1 | 2 | AG; M1 for $z_2^{\ 2}$ correct |
| **(c)** | $z$ real $\Rightarrow z^* = z$ | M1 | | Clearly stated |
| | Discr't zero or correct factorisation | A1 | 2 | AG |
| | **Total** | | **8** | |
| **6(a)** | Sketch of ellipse | M1 | | centred at origin |
| | Correct relationship to circle | A1 | | |
| | Coords $\left(\pm 2\sqrt{2}, 0\right), \left(0, \pm\sqrt{2}\right)$ | B2,1 | 4 | Accept $\sqrt{8}$ for $2\sqrt{2}$ ;<br>B1 for any 2 of $x = \pm 2\sqrt{2}$ , $y = \pm\sqrt{2}$<br>allow B1 if all correct except for use of decimals (at least one DP) |
| **(b)(i)** | Replacing $x$ by $\frac{x}{2}$ | M1 | | or by $2x$ |
| | $E$ is $\left(\frac{x}{2}\right)^2 + y^2 = 2$ | A1 | 2 | OE |
| **(ii)** | Tangent is $\frac{x}{2} + y = 2$ | M1A1 | 2 | M1 for complete valid method |
| | **Total** | | **8** | |
| **7(a)** | Denom never zero, so no vert asymp | E1 | | |
| | Horizontal asymptote is $y = 0$ | B1 | 2 | |
| **(b)** | $x - 4 = k(x^2 + 9)$ | M1 | | |
| | Hence result clearly shown | A1 | 2 | AG |
| **(c)** | Real roots if $b^2 - 4ac \geq 0$ | E1 | | PI (at any stage) |
| | Discriminant $= 1 - 4k(9k + 4)$ | M1 | | |
| | ... $= -(36k^2 + 16k - 1)$ | m1 | | m1 for expansion |
| | ... $= -(18k - 1)(2k + 1)$ | m1 | | m1 for correct factorisation |
| | Result (AG) clearly justified | A1 | 5 | eg by sketch or sign diagram |
| **(d)** | $k = -\frac{1}{2} \Rightarrow -\frac{1}{2}x^2 - x - \frac{1}{2} = 0$ | M1A1 | | or equivalent using $k = \frac{1}{18}$ |
| | ... $\Rightarrow (x+1)^2 = 0 \Rightarrow x = -1$ | A1 | | |
| | $k = \frac{1}{18} \Rightarrow \frac{1}{18}x^2 - x + \frac{9}{2} = 0$ | A1 | | |
| | ... $\Rightarrow (x-9)^2 = 0 \Rightarrow x = 9$ | A1 | | |
| | SPs are $\left(-1, -\frac{1}{2}\right), \left(9, \frac{1}{18}\right)$ | A1 | 6 | correctly paired |
| | **Total** | | **15** | |

**MFP1(cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **8(a)** | $x_2 = 50 - \dfrac{50^3 + 2(50^2) + 50 - 100\ 000}{3(50^2) + 4(50) + 1}$ | B1<br>B1 | | For numerator (PI by value 30050)<br>For denominator (PI by value 7701) |
| | $x_2 \approx 46.1$ | B1 | 3 | Allow AWRT 46.1 |
| **8(b)(i)** | $\Sigma r(3r+1) = 3\Sigma r^2 + \Sigma r$ | M1 | | |
| | $... = 3\left(\frac{1}{6}n\right)(n+1)(2n+1) + \frac{1}{2}n(n+1)$ | m1 | | correct formulae substituted |
| | $... = \frac{1}{2}n(n+1)(2n+1+1)$ | m1m1 | | m1 for each factor ($n$ and $n+1$) |
| | $... = n(n+1)^2$ convincingly shown | A1 | 5 | AG |
| **(ii)** | Correct expansion of $n(n+1)^2$ | B1 | 1 | and conclusion drawn (AG) |
| **(c)** | Attempt at value of $S_{46}$<br>Attempt at value of $S_{45}$<br>$S_{45} < 100000 < S_{46}$, so $N = 46$ | M1<br>m1<br>A1 | 3 | |
| | **Alternative method**<br>Root of equation in (a) is 45.8<br>So lowest integer value is 46 | (B3) | | Allow AWRT 45.7 or 45.8 |
| | **Total** | | **12** | |
| | **TOTAL** | | **75** | |