

Version 1.0



**General Certificate of Education (A-level)  
January 2011**

**Mathematics**

**MFP3**

**(Specification 6360)**

**Further Pure 3**

***Mark Scheme***

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**Key to mark scheme abbreviations**

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct $x$ marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

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**MFP3**

Q	Solution	Marks	Total	Comments
<b>1</b>	$k_1 = 0.1 \times (3 + \sqrt{4}) \quad (=0.5)$ $k_2 = 0.1 f(3.1, 4.5)$ $k_2 = 0.1 \times (3.1 + \sqrt{4.5}) = 0.522132\dots$ $y(3.1) = y(3) + \frac{1}{2}[k_1 + k_2]$ $= 4 + 0.5 \times 1.022132\dots$ $y(3.1) = 4.511$	M1 M1 A1  m1  A1	      5	PI accept 3dp or better  Dep on previous two Ms and numerical values for $k$ 's Must be 4.511
<b>Total</b>			<b>5</b>	
<b>2(a)</b>	$p \cos x - q \sin x + 5p \sin x + 5q \cos x = 13 \cos x$ $p + 5q = 13; \quad 5p - q = 0$ $p = \frac{1}{2}; \quad q = \frac{5}{2}$	M1 m1  A1	   3	Differentiation and subst. into DE Equating coeffs.  OE Need both
<b>(b)</b>	Aux. eqn. $m + 5 = 0$ $(y_{CF} =) Ae^{-5x}$ $(y_{GS} =) Ae^{-5x} + \frac{1}{2} \sin x + \frac{5}{2} \cos x$	M1 A1  M1 A1  B1F	     3	PI. Or solving $y'(x) + 5y = 0$ as far as $y =$ OE  c's CF + c's PI with exactly one arbitrary constant OE
<b>Total</b>			<b>6</b>	
<b>3(a)</b>	$r + r \cos \theta = 2$ $r + x = 2$ $r = 2 - x$ $x^2 + y^2 = (2 - x)^2$ $y^2 = 4 - 4x$	M1 B1 A1 M1 A1	    5	$r \cos \theta = x$ stated or used  $r^2 = x^2 + y^2$ used Must be in the form $y^2 = f(x)$ but accept ACF for $f(x)$ .
<b>(b)</b>	Equation of line: $r \cos \theta = \frac{3}{4} \Rightarrow x = \frac{3}{4}$  $y^2 = 4 - 4\left(\frac{3}{4}\right) = 1 \Rightarrow y = \pm 1; \quad \left[\text{Pts} \left(\frac{3}{4}, \pm 1\right)\right]$ Distance between pts $(0.75, 1)$ and $(0.75, -1)$ is 2  <u>Altn:</u> At pts of intersection, $r = \frac{5}{4}$ and $\cos \theta = \frac{3}{5}$ OE (M1A1) Distance $PQ = 2r \sin \theta$ (M1) $= 2 \times \frac{5}{4} \times \frac{4}{5} = 2$ (A1)	M1 A1  M1  A1  (M1A1) (M1) (A1)	       4	Use of $r \cos \theta = x$ $4x = 3$ OE    (M1 elimination of either $r$ or $\theta$ ) (For A condone slight prem approx.) Or use of cosine rule or Pythag. Must be from exact values.
<b>Total</b>			<b>9</b>	

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**MFP3(cont)**

Q	Solution	Marks	Total	Comments
4	IF is $\int \frac{-2}{x} dx$ $= e^{-2\ln(x)} (+c) = e^{\ln(x)^{-2}} (+c)$ $= (k)x^{-2}$  $x^{-2} \frac{dy}{dx} - 2x^{-3}y = 2xe^{2x}$ $\frac{d}{dx}(x^{-2}y) = 2xe^{2x}$  $x^{-2}y = \int 2xe^{2x} dx$ $= \int x d(e^{2x}) = xe^{2x} - \int e^{2x} dx$  $x^{-2}y = xe^{2x} - \frac{1}{2}e^{2x} (+c)$ When $x = 2, y = e^4$ so $\frac{1}{4}e^4 = 2e^4 - \frac{1}{2}e^4 + c$ $c = -\frac{5}{4}e^4$  $y = x^3e^{2x} - \frac{1}{2}x^2e^{2x} - \frac{5}{4}x^2e^4$	M1  A1  A1F    M1    M1 A1   A1  m1   A1	9	Award even if negative sign missing  OE Condone missing $c$  Ft earlier sign error    LHS as $d/dx(y \times IF)$ PI   Integration by parts in correct dirn   ACF  Boundary condition used to find $c$ after integration.   Must be in the form $y = f(x)$
	<b>Total</b>		<b>9</b>	

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**MFP3(cont)**

Q	Solution	Marks	Total	Comments
<b>5(a)</b>	$\frac{12x+8-12x-3}{(4x+1)(3x+2)} = \frac{5}{(4x+1)(3x+2)}$	B1	1	Accept $C = 5$
<b>(b)</b>	$\int \frac{10}{(4x+1)(3x+2)} dx = 2 \int \left( \frac{4}{4x+1} - \frac{3}{3x+2} \right) dx$ $= 2 [\ln(4x+1) - \ln(3x+2)] (+c)$ $I = \lim_{a \rightarrow \infty} \int_1^a \left( \frac{10}{(4x+1)(3x+2)} \right) dx$ $= 2 \lim_{a \rightarrow \infty} [\ln(4a+1) - \ln(3a+2)] - (\ln 5 - \ln 5)$ $= 2 \lim_{a \rightarrow \infty} \left[ \ln \left( \frac{4a+1}{3a+2} \right) \right] = 2 \lim_{a \rightarrow \infty} \left[ \ln \left( \frac{4 + \frac{1}{a}}{3 + \frac{2}{a}} \right) \right]$ $= 2 \ln \frac{4}{3} = \ln \frac{16}{9}$	M1  A1  M1   m1,m1  A1	6	OE  $\infty$ replaced by $a$ and $\lim_{a \rightarrow \infty}$ (OE)  Limiting process shown. Dependent on the previous M1M1  CSO
	<b>Total</b>		<b>7</b>	

MFP3(cont)

Q	Solution	Marks	Total	Comments
6	$\text{Area} = \frac{1}{2} \int (2 \sin 2\theta \sqrt{\cos \theta})^2 d\theta$ $= \frac{1}{2} \int_0^{\frac{\pi}{2}} (4 \cos \theta \sin^2 2\theta) d\theta$ $= \frac{1}{2} \int_0^{\frac{\pi}{2}} (16 \sin^2 \theta \cos^3 \theta) d\theta$ $= \int_0^{\frac{\pi}{2}} (8 \sin^2 \theta (1 - \sin^2 \theta)) d\sin \theta$ $= \left[ \frac{8 \sin^3 \theta}{3} - \frac{8 \sin^5 \theta}{5} \right]_0^{\frac{\pi}{2}}$ $= \left( \frac{8}{3} - \frac{8}{5} \right) - 0 = \frac{16}{15}$ <p><b>Alternatives for the last four marks</b></p> $\text{Area} = \int_0^{\frac{\pi}{2}} (\cos \theta - \cos 4\theta \cos \theta) d\theta$ $\int (\cos 4\theta \cos \theta) d\theta$ $= -\frac{1}{15} (\cos 4\theta \sin \theta - 4 \sin 4\theta \cos \theta)$ $\text{Area} = (1-0) + \frac{1}{15} [(1-0) - (0)] = \frac{16}{15}$	<p>M1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>m1</p> <p>A1F</p> <p>A1</p> <p>(M1)</p> <p>(m1)</p> <p>(A1F)</p> <p>(A1)</p>	<p>7</p>	<p>Use of <math>\frac{1}{2} \int r^2 d\theta</math></p> <p><math>r^2 = 4 \cos \theta \sin^2 2\theta</math> or better</p> <p>Correct limits</p> <p><math>\sin^2 2\theta = k \sin^2 \theta \cos^2 \theta</math> (<math>k &gt; 0</math>)</p> <p>Substitution or another valid method to integrate <math>\sin^2 \theta \cos^3 \theta</math></p> <p>Correct integration of <math>p \sin^2 \theta \cos^3 \theta</math></p> <p>CSO AG</p> <p><math>2 \cos \theta \sin^2 2\theta = \lambda \cos \theta + \mu \cos 4\theta \cos \theta</math> (<math>\lambda, \mu \neq 0</math>)</p> <p>Integration by parts twice or use of <math>\cos 4\theta \cos \theta = \frac{1}{2} (\cos 5\theta + \cos 3\theta)</math></p> <p>Correct integration of <math>p \cos 4\theta \cos \theta</math> [eg <math>p \left[ \frac{1}{10} \sin 5\theta + \frac{1}{6} \sin 3\theta \right]</math>]</p> <p>CSO AG <math>\left\{ 1 - \frac{1}{10} + \frac{1}{6} = \frac{16}{15} \right\}</math></p>
	<b>Total</b>		<b>7</b>	

**MFP3(cont)**

Q	Solution	Marks	Total	Comments
7(a)(i)	$\cos x + \sin x = 1 + x - \frac{1}{2}x^2 - \frac{1}{6}x^3$	B1	1	Accept coeffs unsimplified, even 3! for 6.
(ii)	$\ln(1+3x) = 3x - \frac{1}{2}(3x)^2 + \frac{1}{3}(3x)^3 = 3x - \frac{9}{2}x^2 + 9x^3$	B1	1	Accept coeffs unsimplified
(b)(i)	$y = e^{\tan x}, \quad \frac{dy}{dx} = \sec^2 x e^{\tan x}$	M1 A1		Chain rule ACF eg $y \sec^2 x$
	$\frac{d^2 y}{dx^2} = 2 \sec^2 x \tan x e^{\tan x} + \sec^4 x e^{\tan x}$ $= \sec^2 x e^{\tan x} (2 \tan x + \sec^2 x)$ $= \frac{dy}{dx} (2 \tan x + 1 + \tan^2 x)$	m1 A1		Product rule OE ACF
	$\frac{d^2 y}{dx^2} = (1 + \tan x)^2 \frac{dy}{dx}$	A1	5	AG Completion; CSO any valid method.
(ii)	$\frac{d^3 y}{dx^3} = 2(1 + \tan x) \sec^2 x \frac{dy}{dx} + (1 + \tan x)^2 \frac{d^2 y}{dx^2}$	M1		
	When $x = 0, \frac{d^3 y}{dx^3} = 2(1)(1)(1) + (1)(1) = 3$	A1	2	CSO
(iii)	$y(0) = 1; y'(0) = 1; y''(0) = 1; y'''(0) = 3;$ $y(x) \approx y(0) + x y'(0) + \frac{1}{2} x^2 y''(0) + \frac{1}{3!} x^3 y'''(0)$	M1		
	$e^{\tan x} \approx 1 + x + \frac{1}{2} x^2 + \frac{1}{2} x^3$	A1	2	CSO AG
(c)	$\lim_{x \rightarrow 0} \left[ \frac{e^{\tan x} - (\cos x + \sin x)}{x \ln(1 + 3x)} \right]$			
	$= \lim_{x \rightarrow 0} \frac{1 + x + \frac{x^2}{2} + \frac{x^3}{2} - 1 - x + \frac{x^2}{2} + \frac{x^3}{6}}{x \left( 3x - \frac{9}{2} x^2 + \dots \right)}$	M1		Using series expns.
	$= \lim_{x \rightarrow 0} \left[ \frac{x^2 + \frac{2}{3} x^3 + \dots}{3x^2 - \frac{9}{2} x^3 \dots} \right] = \lim_{x \rightarrow 0} \left[ \frac{1 + \frac{2}{3} x + \dots}{3 - \frac{9}{2} x \dots} \right]$ $= \frac{1}{3}$	m1 A1	3	Dividing numerator and denominator by $x^2$ to get constant terms. OE following a slip.
<b>Total</b>			<b>14</b>	



**MFP3(cont)**

Q	Solution	Marks	Total	Comments
8(a)	$\frac{dx}{dt} \frac{dy}{dx} = \frac{dy}{dt}$	M1		Chain rule
	$e^t \frac{dy}{dx} = \frac{dy}{dt} \Rightarrow x \frac{dy}{dx} = \frac{dy}{dt}$	A1	2	CSO AG
(b)	$\frac{d}{dt} \left( x \frac{dy}{dx} \right) = \frac{d^2 y}{dt^2}; \frac{dx}{dt} \frac{d}{dx} \left( x \frac{dy}{dx} \right) = \frac{d^2 y}{dt^2}$	M1		OE $\frac{d}{dx} \left( x \frac{dy}{dx} \right) = \frac{dx}{dx} \frac{d^2 y}{dt^2}$
	$\frac{dx}{dt} \left( \frac{dy}{dx} + x \frac{d^2 y}{dx^2} \right) = \frac{d^2 y}{dt^2}$	m1		Product rule (dep on previous M)
	$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = \frac{d^2 y}{dt^2}$	A1		OE
	$x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 4y = 2 \ln x$ becomes			
	$\frac{d^2 y}{dt^2} - x \frac{dy}{dx} - 3x \frac{dy}{dx} + 4y = 2 \ln x$			
	$\Rightarrow \frac{d^2 y}{dt^2} - 4 \frac{dy}{dt} + 4y = 2 \ln e^t$ (using (a))	m1		
	$\Rightarrow \frac{d^2 y}{dt^2} - 4 \frac{dy}{dt} + 4y = 2t$	A1	5	CSO AG
(c)	Auxl eqn $m^2 - 4m + 4 = 0$	M1		PI
	$(m - 2)^2 = 0, m = 2$	A1		PI
	CF: $(y_c =) (At + B)e^{2t}$	M1		Ft wrong value of $m$ provided equal roots and 2 arb. constants in CF. Condone $x$ for $t$ here
	PI Try $(y_p =) at + b$	M1		If extras, coeffs. must be shown to be 0.
	$-4a + 4at + 4b = 2t \Rightarrow a = b = \frac{1}{2}$	A1		Correct PI. Condone $x$ for $t$ here
	GS $\{y\} = (At + B)e^{2t} + 0.5(t + 1)$	B1F	6	Ft on c's CF + PI, provided PI is non-zero and CF has two arbitrary constants and RHS is fn of $t$ only
(d)	$\Rightarrow y = (A \ln x + B)x^2 + 0.5(\ln x + 1)$	M1		
	$y = 1.5$ when $x = 1 \Rightarrow B = 1$	A1F		Ft one earlier slip
	$y'(x) = (A \ln x + B) 2x + Ax + 0.5 x^{-1}$	m1		Product rule
	$y'(1) = 0.5 \Rightarrow A = -2$	A1F		Ft one earlier slip
	$y = (1 - 2 \ln x)x^2 + \frac{1}{2}(\ln x + 1)$	A1	5	ACF
	<b>Total</b>		<b>18</b>	
	<b>TOTAL</b>		<b>75</b>	