

General Certificate of Education
January 2007
Advanced Subsidiary Examination



MATHEMATICS
Unit Further Pure 1

MFP1

Friday 26 January 2007 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Solve the following equations, giving each root in the form $a + bi$:

(i) $x^2 + 16 = 0$; *(2 marks)*

(ii) $x^2 - 2x + 17 = 0$. *(2 marks)*

(b) (i) Expand $(1 + x)^3$. *(2 marks)*

(ii) Express $(1 + i)^3$ in the form $a + bi$. *(2 marks)*

(iii) Hence, or otherwise, verify that $x = 1 + i$ satisfies the equation

$$x^3 + 2x - 4i = 0 \quad \text{span style="float: right;">*(2 marks)*$$

2 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

(a) Calculate:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) \mathbf{BA} . *(3 marks)*

(b) Describe fully the geometrical transformation represented by each of the following matrices:

(i) \mathbf{A} ; *(2 marks)*

(ii) \mathbf{B} ; *(2 marks)*

(iii) \mathbf{BA} . *(2 marks)*

3 The quadratic equation

$$2x^2 + 4x + 3 = 0$$

has roots α and β .

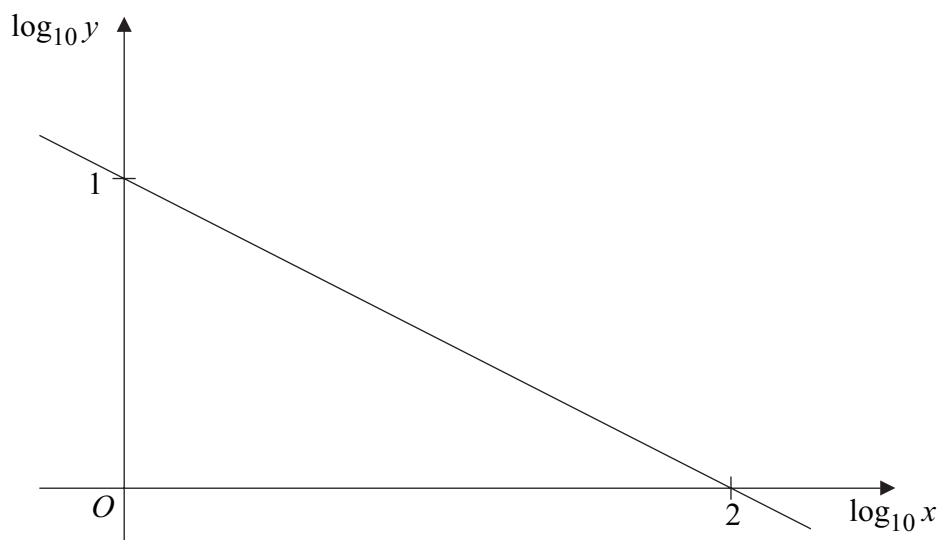
- (a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. (2 marks)
- (b) Show that $\alpha^2 + \beta^2 = 1$. (3 marks)
- (c) Find the value of $\alpha^4 + \beta^4$. (3 marks)

4 The variables x and y are related by an equation of the form

$$y = ax^b$$

where a and b are constants.

- (a) Using logarithms to base 10, reduce the relation $y = ax^b$ to a linear law connecting $\log_{10}x$ and $\log_{10}y$. (2 marks)
- (b) The diagram shows the linear graph that results from plotting $\log_{10}y$ against $\log_{10}x$.



Find the values of a and b . (4 marks)

Turn over ►

5 A curve has equation

$$y = \frac{x}{x^2 - 1}$$

(a) Write down the equations of the three asymptotes to the curve. (3 marks)

(b) Sketch the curve.

(You are given that the curve has no stationary points.) (4 marks)

(c) Solve the inequality

$$\frac{x}{x^2 - 1} > 0 \quad (3 \text{ marks})$$

6 (a) (i) Expand $(2r - 1)^2$. (1 mark)

(ii) Hence show that

$$\sum_{r=1}^n (2r - 1)^2 = \frac{1}{3}n(4n^2 - 1) \quad (5 \text{ marks})$$

(b) Hence find the sum of the squares of the odd numbers between 100 and 200. (4 marks)

7 The function f is defined for all real numbers by

$$f(x) = \sin\left(x + \frac{\pi}{6}\right)$$

(a) Find the general solution of the equation $f(x) = 0$. (3 marks)

(b) The quadratic function g is defined for all real numbers by

$$g(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}x - \frac{1}{4}x^2$$

It can be shown that $g(x)$ gives a good approximation to $f(x)$ for small values of x .

(i) Show that $g(0.05)$ and $f(0.05)$ are identical when rounded to four decimal places. (2 marks)

(ii) A chord joins the points on the curve $y = g(x)$ for which $x = 0$ and $x = h$. Find an expression in terms of h for the gradient of this chord. (2 marks)

(iii) Using your answer to part (b)(ii), find the value of $g'(0)$. (1 mark)

8 A curve C has equation

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

- (a) Find the y -coordinates of the points on C for which $x = 10$, giving each answer in the form $k\sqrt{3}$, where k is an integer. (3 marks)
- (b) Sketch the curve C , indicating the coordinates of any points where the curve intersects the coordinate axes. (3 marks)
- (c) Write down the equation of the tangent to C at the point where C intersects the positive x -axis. (1 mark)
- (d) (i) Show that, if the line $y = x - 4$ intersects C , the x -coordinates of the points of intersection must satisfy the equation

$$16x^2 - 200x + 625 = 0 \quad (3 \text{ marks})$$

- (ii) Solve this equation and hence state the relationship between the line $y = x - 4$ and the curve C . (2 marks)

END OF QUESTIONS

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