GCE

Mathematics

Unit **4727**: Further Pure Mathematics 3

Advanced GCE

Mark Scheme for June 2014

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Question			Answer	Marks	Guidance	
1	(i)		$\begin{pmatrix}2\\1\\-1\end{pmatrix}\times\begin{pmatrix}3\\5\\2\end{pmatrix}=\begin{pmatrix}7\\-7\\7\end{pmatrix}=7\begin{pmatrix}1\\-1\\1\end{pmatrix}$	M1 A1		M1 requires evidence of method for cross product or at least 2 correct values calculated
			(eg) $z=0\Rightarrow 2x+y=4, 3x+5y=13\Rightarrow x=1, y=2$	M1		or any valid point e.g.(0, 3, -1), (3, 0, 2)
			$\mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$	A1	oe vector form	Must have full equation including '**r** ='
			Alternative: Find one point	M1		
			Find a second point and vector between points	M1		
			multiple of $\begin{pmatrix}1\\-1\\1\end{pmatrix}$	A1		
			$\mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$	A1		
			Alternative: Solve simultaneously	M1	to at least expressions for x,y,z parametrically, or two relationship between 2 variables.	
			Point **and** direction found	M1 A1		
			$\mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$	A1		
				[4]		

Question			Answer	Marks	Guidance	
1	(ii)		$\dfrac{\lvert 2\times 2+5--2-4\rvert}{\sqrt{2^2+1^2+1^2}}=\dfrac{7}{\sqrt{6}}$	M1 A1	Condone lack of absolute signs for M1 oe surd form. isw	2.86 with no workings scores M1
			Alternative: find parameter for perpendicular meets plane and use to find distance	M1 **[2]**	For complete method with calculation errors	look for $\lambda = -7/6$
2			$u=y^2 \Rightarrow \dfrac{du}{dx}=2y\dfrac{dy}{dx}$	M1	Correctly finds	Or $\dfrac{dy}{dx}=\dfrac{1}{2}u^{-\frac{1}{2}}\dfrac{du}{dx}$
			so DE $\Rightarrow 2y\dfrac{dy}{dx}-4y^2=2e^x$	M1	or for complete unsimplified substitution	
			$\Rightarrow \dfrac{du}{dx}-4u=2e^x$	A1		Can be implied by next A1
			$I=\exp\int -4\,dx=e^{-4x}$	A1ft		Must have form $\dfrac{du}{dx}+f(x)u=g(x)$ for this mark and any further marks Can be implied by subsequent work
			$e^{-4x}\dfrac{du}{dx}-4e^{-4x}u=2e^{-3x}$	M1*	Multiples through by IF of form e^{kx}, simplifying RHS	
			$ue^{-4x}=-\frac{2}{3}e^{-3x}(+A)$	*M1dep*	Integrates	
			$u=-\frac{2}{3}e^x+Ae^{4x}$	M1dep *	Rearranges to make u or y^2 the subject	No more than 1 numerical error at this step
			$y=\sqrt{-\frac{2}{3}e^x+Ae^{4x}}$	A1	Cao	ignore use of '±'
			Alternative from 4th mark to 6th mark			
			CF: (u=…) Ae^{4x}	A1		
			PI: $u=ke^x$, $\dfrac{du}{dx}=ke^x$	M1*	PI chosen & differentiated correctly	
			$ke^x-4ke^x=2e^x$, $k=-\dfrac{2}{3}$	M1 dep* **[8]**	Substitutes and solves	

Question			Answer	Marks	Guidance	
3	(i)		$z^6 = 1 \Rightarrow z = e^{2k\pi i/6}$	M1		
			$k = 0,1,2,3,4,5$	A1	Oe exactly 6 roots	accept roots 1, -1 given as integers.
			Diagram	B1	6 roots in right quadrant,	
				B1	correct angles and moduli	as evidenced by labels, circles, or accurate diagram, or by co-ordinates
				[4]		
3	(ii)		$(1+i)^6 = \left(\sqrt{2}\,e^{\frac{1}{4}\pi i}\right)^6$	M1	Attempts modulus-argument form, getting at least 1 correct	
			$8e^{\frac{6}{4}\pi i}$	M1	for $(\text{mod})^6$ and arg x 6	
			$= -8i$	A1	**ag**	complete argument including start line
			Alternative:			
			$(1+i)^6 = 1+6i+15i^2+20i^3+15i^4+6i^5+i^6$	M1		
			$= 1+6i-15-20i+15+6i\ -1$	M1	no more than 1 term wrong	**Sc 2** for only lines 2 & 3correct
			$= -8i$	A1	**ag**	
			Alternative: $(1+i)^2 = 2i$	M1		
			$(1+i)^6 = (2i)^3$	M1		
			$= -8i$	A1	**ag**	
				[3]		

Question			Answer	Marks	Guidance	
3	(iii)		$z^6 = -8\text{i} \Rightarrow z = (1+\text{i})\text{e}^{2k\pi\text{i}/6}$	M1		
			$= \sqrt{2}e^{i\frac{\pi}{4}}\text{e}^{2k\pi\text{i}/6}$	M1		
			$\sqrt{2}\,\text{e}^{\text{i}\pi(1/4+k/3)},\ k = 0,1,2,3,4,5$	A1	or equivalent k	
			Alternative: $z^6 = 8\text{e}^{\text{i}\pi(\frac{3}{2}+2k)}$	M1		
			$\sqrt{2}\,\text{e}^{\text{i}\pi(1/4+k/3)},\ k = 0,1,2,3,4,5$	M1 A1 **[3]**		or equivalent: e.g. $\sqrt{2}\,\text{e}^{\text{i}\pi(-1/12+k/3)}$ accept unsimplified modulus

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>4</td><td>(i)</td><td></td><td><table><tr><td>element</td><td>(1)</td><td>3</td><td>7</td><td>9</td><td>11</td><td>13</td><td>17</td><td>19</td></tr><tr><td>inverse</td><td>(1)</td><td>7</td><td>3</td><td>9</td><td>11</td><td>17</td><td>13</td><td>19</td></tr></table></td><td>B1
B1
B1
[3]</td><td>2 or more
4 or more
all 7 correct</td><td>Ignore 1</td></tr>
<tr><td>4</td><td>(ii)</td><td></td><td>(1 has order 1)
9,11,19 have order 2

$3^2 = 9 \Rightarrow 3^4 = 1$ so order 4
similarly 7,13,17 order 4

no element of order 8 so not cyclic</td><td>M1

B1

A1

[3]</td><td>Correctly identifies order of all elements

justifies order for at least 1 element of order 4

www</td><td>Allow one error

must show workings towards a^4 for demonstration that these elements are order 4`

condone “no generator” in place of ”no element or order 8”</td></tr>
<tr><td>4</td><td>(iii)</td><td></td><td>

{1,13, 9, 17} and {1, 3, 9, 7}

$1 \leftrightarrow 1,\ 9 \leftrightarrow 9,\ 3 \leftrightarrow 13,\ 7 \leftrightarrow 17$</td><td>M1
B1
A1
M1
A1
[5]</td><td>For two sets which both contain “1” and all (4) elements’ inverses
One subgroup of order 4

for correspondence of “their” elements of same order
or $3 \leftrightarrow 17,\ 7 \leftrightarrow 13$</td><td></td></tr>
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Question			Answer	Marks	Guidance	
5			AE: $\lambda^2+5\lambda+6=0$			
			$\lambda=-2,-3$	B1		
			CF: $Ae^{-2x}+Be^{-3x}$	B1ft		
			PI: $y=ae^{-x}$	B1ft		
			$ae^{-x}-5ae^{-x}+6ae^{-x}=e^{-x}$	M1	Differentiate and substitute	
			$2a=1$			
			$a=\frac{1}{2}$	A1		
			GS: $(y=)\frac{1}{2}e^{-x}+Ae^{-2x}+Be^{-3x}$	A1ft		ft must be of form "ke^{-x} plus a standard CF form" with 2 arbitrary constants
			$x=0,y=0\Rightarrow\frac{1}{2}+A+B=0$	M1	Use condition on GS	Must have 2 arbitrary constants
			$y'=-\frac{1}{2}e^{-x}-2Ae^{-2x}-3Be^{-3x}$	M1*	Differentiate their GS of form $y=ke^{-x}+Ae^{mx}+Be^{nx}$ where k, m, n are non-zero constants and m, n not 1	
			$x=0,y'=0\Rightarrow-\frac{1}{2}-2A-3B=0$			
			$A=-1,\ B=\frac{1}{2}$	M1dep*	Use condition and attempt to find A, B	
			$y=\frac{1}{2}e^{-x}-e^{-2x}+\frac{1}{2}e^{-3x}$	A1	www	Must have 'y ='
				[10]		

Question			Answer	Marks	Guidance	
6	(i)		$l \parallel \begin{pmatrix}2\\3\\5\end{pmatrix}$ $\Pi \perp \begin{pmatrix}4\\-1\\-1\end{pmatrix}$ so $\begin{pmatrix}2\\3\\5\end{pmatrix}\bullet\begin{pmatrix}4\\-1\\-1\end{pmatrix}=0 \Rightarrow l \parallel \Pi$	M1	dot product of correct vectors = 0	
			$(1, -2, 7)$ on l but $4\times 1 - -2 - 7 = -1 \neq 8$ so not in Π	M1	substitute point on line into Π and calculate d	
			hence l not in Π	A1	Full argument includes key components	Argument can be about a general point on line
				[3]		
6	(ii)		$(\mathbf{r}=)\begin{pmatrix}1\\-2\\7\end{pmatrix}+\lambda\begin{pmatrix}4\\-1\\-1\end{pmatrix}$	B1		
			closest point where meets Π $4(1+4\lambda)-(-2-\lambda)-(7-\lambda)=8$	M1	parametric form of (x, y, z) substituted into plane	
			$\Rightarrow \lambda = \frac{1}{2}$	A1ft		
			$\Rightarrow \mathbf{r} = \begin{pmatrix}3\\-\frac{5}{2}\\\frac{13}{2}\end{pmatrix}$	A1		
				[4]		
6	(iii)		$\mathbf{r} = \begin{pmatrix}3\\-\frac{5}{2}\\\frac{13}{2}\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\5\end{pmatrix}$	B1ft	oe	must have "$\mathbf{r} =$"
				[1]		

Question			Answer	Marks	Guidance	
7	(i)		$2\mathrm{i}\sin\theta = \mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}$	B1	any equivalent form	If use z, must define it
			$2\mathrm{i}\sin n\theta = \mathrm{e}^{\mathrm{i}n\theta} - \mathrm{e}^{-\mathrm{i}n\theta}$			
			$(2\mathrm{i}\sin\theta)^5 = \left(\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}\right)^5$			
			$= \mathrm{e}^{\mathrm{i}5\theta} - 5\mathrm{e}^{\mathrm{i}3\theta} + 10\mathrm{e}^{\mathrm{i}\theta} - 10\mathrm{e}^{-\mathrm{i}\theta} + 5\mathrm{e}^{-\mathrm{i}3\theta} - \mathrm{e}^{-\mathrm{i}5\theta}$	M1*	binomial expansion	can be unsimplified
			$32i\sin^5\theta = (e^{5i\theta} - e^{-5i\theta}) - 5(e^{3i\theta} - e^{-3i\theta}) + 10(e^{i\theta} - e^{-i\theta})$	M1dep*	grouping terms	Award **B1 then sc M1A1** for candidates who omit this stage from otherwise complete argument
			$= 2\mathrm{i}\sin 5\theta - 5(2\mathrm{i}\sin 3\theta) + 10(2\mathrm{i}\sin\theta)$			
			$\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$	A1	AG	must convince on the $\frac{1}{16}$ and on the elimination of i
				[4]		
7	(ii)		$16\sin^5\theta - 10\sin\theta = \sin 5\theta - 5\sin 3\theta$	M1*	Attempts to eliminate $\sin 5\theta$ and $\sin 3\theta$	
			$16\sin^5\theta - 6\sin\theta = 0$	A1		Or $16\sin^5\theta = 6\sin\theta$
			$\sin\theta = 0, \pm\sqrt[4]{\frac{3}{8}}$	M1dep*	must have 3 values for $\sin\theta$	
			$\theta = 0, \pm 0.899$	A1		
				[4]		

Question		Answer	Marks	Guidance	
8	(i)	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is identity	B1		
		$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}^{-1} = \frac{1}{a^2+b^2}\begin{pmatrix} a & b \\ -b & a \end{pmatrix} \in G$	M1 A1	for M1, must at least get matrix in form $\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$, or state existence of inverse due to non-singularity	
		$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} c & -d \\ d & c \end{pmatrix} = \begin{pmatrix} ac-bd & -bc-ad \\ bc+ad & ac-bd \end{pmatrix}$	M1		
		and $(ac-bd)^2 + (bc+ad)^2 = a^2c^2 + b^2d^2 + b^2c^2 + a^2d^2$ $= (a^2+b^2)(c^2+d^2) \neq 0$	M1 A1 **[6]**	Must not attempt to prove commutativity in (i)	
8	(ii)	$\begin{pmatrix} c & -d \\ d & c \end{pmatrix}\begin{pmatrix} a & -b \\ b & a \end{pmatrix} = \begin{pmatrix} ac-bd & -bc-ad \\ bc+ad & ac-bd \end{pmatrix}$ $= \begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} c & -d \\ d & c \end{pmatrix}$ so commutative	M1 A1 **[2]**		must also consider matrices reversed, but may be seen in (i)
8	(iii)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	M1	g^2 must be correct	
		$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	M1	allow 1 error in getting g^4	
		order 4	A1 **[3]**		

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