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**General Certificate of Education**

**Mathematics 6360**

**MFP3      Further Pure 3**

**Mark Scheme**

*2010 examination - January series*

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## Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MFP3**

Q	Solution	Marks	Total	Comments
<b>1(a)</b>	$y_1 = 2 + 0.1 \times [3 \ln(2 \times 3 + 2)] = 2 + 0.3 \ln 8$ $= 2.6238(3\dots)$ $y(3.1) = 2.6238$ (to 4dp)	M1A1 A1	3	Condone greater accuracy
<b>(b)</b>	$k_1 = 0.1 \times 3 \ln 8 = 0.6238(32\dots)$ $k_2 = 0.1 \times f(3.1, 2.6238(32\dots))$ $\dots = 0.1 \times 3.1 \times \ln 8.8238(32\dots)$ $[= 0.6750(1\dots)]$ $y(3.1) = 2 + \frac{1}{2} [0.6238(3\dots) + 0.6750(1\dots)]$ $= 2.6494(2\dots) = 2.6494$ to 4dp	B1F M1 A1F  m1 A1	5	PI ft from (a), 4dp or better  PI; ft on $0.1 \times 3.1 \times \ln[6.2 + \text{answer(a)}]$  CAO Must be 2.6494
<b>Total</b>			<b>8</b>	
<b>2(a)</b>	$\frac{dy}{dx} = \frac{1}{4+3x} \times 3$	M1		Chain rule
	$\frac{d^2y}{dx^2} = -3(4+3x)^{-2} \times 3 = -9(4+3x)^{-2}$	M1A1	3	M1 for quotient (PI) or chain rule used
<b>(b)</b>	$\ln(4+3x) = \ln 4 + y'(0)x + y''(0)\frac{1}{2}x^2 + \dots$  First three terms: $\ln 4 + \frac{3}{4}x - \frac{9}{32}x^2$	M1  A1F	2	Clear attempt to use Maclaurin's theorem with numerical values for $y'(0)$ and $y''(0)$  ft on c's answers to (a) provided $y'(0)$ and $y''(0)$ are $\neq 0$ . Accept 1.38(6..) for $\ln 4$
<b>(c)</b>	$\ln(4-3x) = \ln 4 - \frac{3}{4}x - \frac{9}{32}x^2$	B1F	1	ft $x \rightarrow -x$ in c's answer to (b)
<b>(d)</b>	$\ln\left(\frac{4+3x}{4-3x}\right) = \ln(4+3x) - \ln(4-3x)$  $\approx \ln 4 + \frac{3}{4}x - \frac{9}{32}x^2 - \ln 4 + \frac{3}{4}x + \frac{9}{32}x^2$  $\approx \frac{3}{2}x$	M1  A1	2	CSO AG
<b>Total</b>			<b>8</b>	

**MFP3 (cont)**

Q	Solution	Marks	Total	Comments
3(a)	$u = \frac{dy}{dx} \Rightarrow \frac{du}{dx} = \frac{d^2y}{dx^2}$ $x \frac{du}{dx} + 2u = 3x \Rightarrow \frac{du}{dx} + \frac{2}{x}u = 3$	M1  A1	2	CSO AG Substitution into LHS of DE and completion
3(b)	<p>IF is <math>\exp\left(\int \frac{2}{x} dx\right)</math></p> $= e^{2\ln x}; = x^2$ $\frac{d}{dx}(ux^2) = 3x^2$ $ux^2 = x^3 + A \Rightarrow u = x + Ax^{-2}$	M1  A1;A1  M1  A1	5	<p><math>\exp\left(\int \frac{k}{x} dx\right)</math>, for <math>k = \pm 2, \pm 1</math> and integration attempted</p> <p>LHS as differential of <math>u \times</math> IF</p> <p>Must have an arbitrary constant</p>
(c)	$\frac{dy}{dx} = x + Ax^{-2}$ $\frac{dy}{dx} = x + Ax^{-2} \Rightarrow y = \frac{1}{2}x^2 - \frac{A}{x} + B$	M1  A1F	2	<p>and with integration attempted</p> <p>ft only if IF is M1A0A0</p>
<b>Total</b>			<b>9</b>	
4(a)	$\sin 3x = 3x - \frac{1}{3!}(3x)^3 + \dots = 3x - 4.5x^3 + \dots$	B1	1	
(b)	$\cos 2x = 1 - \frac{1}{2!}(2x)^2 + \dots$ $\lim_{x \rightarrow 0} \left[ \frac{3x \cos 2x - \sin 3x}{5x^3} \right] =$ $\lim_{x \rightarrow 0} \frac{3x - 6x^3 - 3x + 4.5x^3 + \dots}{5x^3}$ $= \lim_{x \rightarrow 0} \frac{-1.5 + (o(x^2)) \dots}{5}$ $= -\frac{3}{10}$	B1  M1  m1  A1	4	<p>Using expansions</p> <p>Division by <math>x^3</math> stage to reach relevant form of quotient before taking limit.</p> <p>CSO OE</p>
<b>Total</b>			<b>5</b>	

**MFP3 (cont)**

Q	Solution	Marks	Total	Comments
<p><b>5(a)</b></p>	$y_{PI} = pxe^{-2x} \Rightarrow \frac{dy}{dx} = pe^{-2x} - 2pxe^{-2x}$	M1	4	Product Rule used
	$\Rightarrow \frac{d^2y}{dx^2} = -2pe^{-2x} - 2pe^{-2x} + 4pxe^{-2x}$	A1		
	$-4pe^{-2x} + 4pxe^{-2x} + 3pe^{-2x} - 6pxe^{-2x} + 2pxe^{-2x} = 2e^{-2x}$	M1		
	$-pe^{-2x} = 2e^{-2x} \Rightarrow p = -2$	A1F		Sub. into DE ft one slip in differentiation
<p><b>5(b)</b></p>	<p>Aux. eqn. <math>m^2 + 3m + 2 = 0</math>  <math>\Rightarrow m = -1, -2</math></p>	B1	8	ft on real values of $m$ only  Their CF + their PI must have 2 arb consts Must be using GS; ft on wrong non-zero values for $p$ and $m$  Must be using GS; ft on wrong non-zero values for $p$ and $m$  Must be using GS; ft on wrong non-zero values for $p$ and $m$ and slips in finding $y'(x)$
	<p>CF is <math>Ae^{-x} + Be^{-2x}</math></p>	M1		
	<p>GS <math>y = Ae^{-x} + Be^{-2x} - 2xe^{-2x}</math>.</p>	B1F		
	<p>When <math>x = 0, y = 2 \Rightarrow A + B = 2</math></p>	B1F		
	$\frac{dy}{dx} = -Ae^{-x} - 2Be^{-2x} - 2e^{-2x} + 4xe^{-2x}$	B1F		
	<p>When <math>x = 0, \frac{dy}{dx} = 0 \Rightarrow -A - 2B - 2 = 0</math></p>	B1F		
	<p>Solving simultaneously, 2 eqns each in two arbitrary constants  <math>A = 6, B = -4; y = 6e^{-x} - 4e^{-2x} - 2xe^{-2x}</math>.</p>	m1 A1		
	<b>Total</b>		<b>12</b>	

**MFP3 (cont)**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>6(a)</b>	The interval of integration is infinite	E1	1	OE
<b>(b)(i)</b>	$x = \frac{1}{y} \Rightarrow 'dx = -y^{-2} dy'$			
	$\int \frac{\ln x^2}{x^3} dx \Rightarrow \int (y^3 \ln y^{-2})(-y^{-2})dy$	M1		
	$= \int -y \ln y^{-2} dy = \int 2y \ln y dy$	A1	2	CSO AG
<b>(ii)</b>	$\int 2y \ln y dy = y^2 \ln y - \int y^2 \left(\frac{1}{y}\right) dy$	M1		...= $ky^2 \ln y \pm \int f(y) dy$ with $f(y)$ not involving the 'original' $\ln y$
	..... = $y^2 \ln y - \frac{1}{2}y^2 + c$	A1		
	$\int_0^1 2y \ln y dy = \lim_{a \rightarrow 0} \int_a^1 2y \ln y dy$	A1		Condone absence of '+ c'
	$= \left(0 - \frac{1}{2}\right) - \lim_{a \rightarrow 0} \left[ a^2 \ln a - \frac{a^2}{2} \right]$	M1		
	$= -\frac{1}{2}$ since $\lim_{a \rightarrow 0} a^2 \ln a = 0$	A1	5	CSO Must see clear indication that cand has correctly considered $\lim_{a \rightarrow 0} a^k \ln a = 0$
<b>(iii)</b>	So $\int_1^\infty \frac{\ln x^2}{x^3} dx = \frac{1}{2}$	B1F	1	ft on minus c's value as answer to (b)(ii)
<b>Total</b>			<b>9</b>	
<b>7</b>	Aux. eqn. $m^2 + 4 = 0 \Rightarrow m = \pm 2i$ CF is $A \cos 2x + B \sin 2x$	B1 M1 A1F		OE. If $m$ is real give M0 ft on incorrect complex value for $m$
	PI: Try $ax^2 + b + c \sin x$	M1 M1		Award even if extra terms, provided the relevant coefficients are shown to be zero.
	$2a - c \sin x + 4ax^2 + 4b + 4c \sin x = 8x^2 + 9 \sin x$			
	$a = 2, b = -1,$	A1		Dep on relevant M mark
	$c = 3$	A1		Dep on relevant M mark
	$(y =) A \cos 2x + B \sin 2x + 2x^2 - 1 + 3 \sin x$	B1F	8	Their CF + their PI. Must be exactly two arbitrary constants
<b>Total</b>			<b>8</b>	

Q	Solution	Marks	Total	Comments
8(a)	$4 \sin \theta (1 - \sin \theta) = 1$ $4 \sin^2 \theta - 4 \sin \theta + 1 = 0$ $(2 \sin \theta - 1)^2 = 0 \Rightarrow \sin \theta = 0.5$  $\theta = \frac{\pi}{6}, \theta = \frac{5\pi}{6}, r = 2$ $[P(2, \frac{\pi}{6}) \quad Q(2, \frac{5\pi}{6})]$	M1 A1 m1  A2,1	5	Elimination of $r$ or $\theta$ { $r = 4[1 - (1/r)]$ } { $r^2 - 4r + 4 = 0$ } Valid method to solve quadratic eqn. PI { $(r - 2)^2 = 0 \Rightarrow r = 2$ }  A1 for any two of the three.  SC: Verification of $P(2, \frac{\pi}{6})$ scores max of B1 & a further B1 if $Q(2, \frac{5\pi}{6})$ stated
8(b)	Area triangle OPQ = $\frac{1}{2} \times 2 \times r_Q \times \sin POQ$  Angle $POQ = \frac{5\pi}{6} - \frac{\pi}{6} (= \frac{2\pi}{3})$  Area triangle OPQ = $2 \sin \frac{2\pi}{3} = \sqrt{3}$ Unshaded area bounded by line OP and arc OP = $\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} [4(1 - \sin \theta)]^2 d\theta$  $= 8 \int (1 - 2 \sin \theta + \sin^2 \theta) d\theta$ $= 8 \int \left( 1 - 2 \sin \theta + \frac{1 - \cos 2\theta}{2} \right) d\theta$ $= 8 \left[ \theta + 2 \cos \theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right] (+ c)$ $8 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 - \sin \theta)^2 d\theta =$ $8 \times \left[ \frac{3\theta}{2} + 2 \cos \theta - \frac{\sin 2\theta}{4} \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= 8 \times \left\{ \frac{3\pi}{4} - \left( \frac{3\pi}{12} + 2 \cos \frac{\pi}{6} - \frac{1}{4} \sin \frac{2\pi}{6} \right) \right\}$ $= 8 \times \left( \frac{\pi}{2} - \sqrt{3} + \frac{\sqrt{3}}{8} \right) \quad \{ = 4\pi - 7\sqrt{3} \}$  Shaded area = Area of triangle OPQ - $2 \times \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} [4(1 - \sin \theta)]^2 d\theta$  Shaded area = $\sqrt{3} - 16 \left( \frac{\pi}{2} - \sqrt{3} + \frac{\sqrt{3}}{8} \right) = 15\sqrt{3} - 8\pi$	M1  m1  A1  M1  B1  M1  A1F    m1  A1F  M1  A1		11
<b>Total</b>			<b>16</b>	
<b>TOTAL</b>			<b>75</b>	