General Certificate of Education
January 2008
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Friday 25 January 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Question 7 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 It is given that $z_1 = 2 + \mathrm{i}$ and that $z_1{}^*$ is the complex conjugate of z_1.

Find the real numbers x and y such that

$$x + 3\mathrm{i}y = z_1 + 4\mathrm{i}z_1{}^*$$

(4 marks)

2 A curve satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2^x$$

Starting at the point $(1, 4)$ on the curve, use a step-by-step method with a step length of 0.01 to estimate the value of y at $x = 1.02$. Give your answer to six significant figures. *(5 marks)*

3 Find the general solution of the equation

$$\tan 4\left(x - \frac{\pi}{8}\right) = 1$$

giving your answer in terms of π. *(5 marks)*

4 (a) Find

$$\sum_{r=1}^{n} (r^3 - 6r)$$

expressing your answer in the form

$$kn(n+1)(n+p)(n+q)$$

where k is a fraction and p and q are integers. *(5 marks)*

(b) It is given that

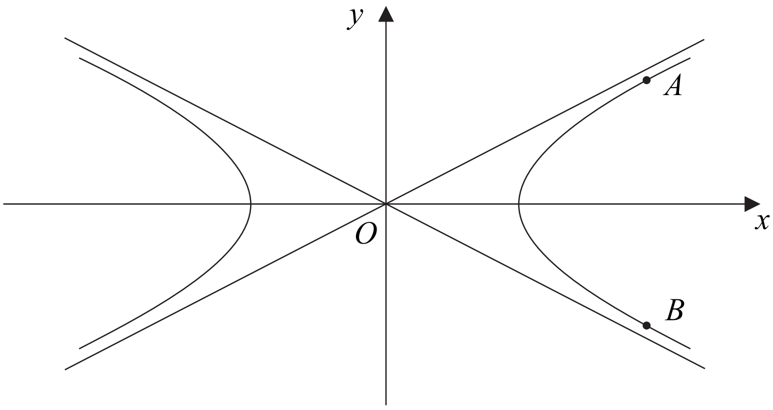
$$S = \sum_{r=1}^{1000} (r^3 - 6r)$$

Without calculating the value of S, show that S is a multiple of 2008. *(2 marks)*

5 The diagram shows the hyperbola

$$\frac{x^2}{4} - y^2 = 1$$

and its asymptotes.



(a) Write down the equations of the two asymptotes. *(2 marks)*

(b) The points on the hyperbola for which $x = 4$ are denoted by A and B.

Find, in surd form, the y-coordinates of A and B. *(2 marks)*

(c) The hyperbola and its asymptotes are translated by two units in the positive y direction.

Write down:

(i) the y-coordinates of the image points of A and B under this translation; *(1 mark)*

(ii) the equations of the hyperbola and the asymptotes after the translation. *(3 marks)*

Turn over for the next question

6 The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} \sqrt{3} & 3 \\ 3 & -\sqrt{3} \end{bmatrix}$$

(a) (i) Show that

$$\mathbf{M}^2 = p\mathbf{I}$$

where p is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(3 marks)*

(ii) Show that the matrix $\mathbf{M}$ can be written in the form

$$q\begin{bmatrix} \cos 60° & \sin 60° \\ \sin 60° & -\cos 60° \end{bmatrix}$$

where q is a real number. Give the value of q in surd form. *(3 marks)*

(b) The matrix $\mathbf{M}$ represents a combination of an enlargement and a reflection.

Find:

(i) the scale factor of the enlargement; *(1 mark)*

(ii) the equation of the mirror line of the reflection. *(1 mark)*

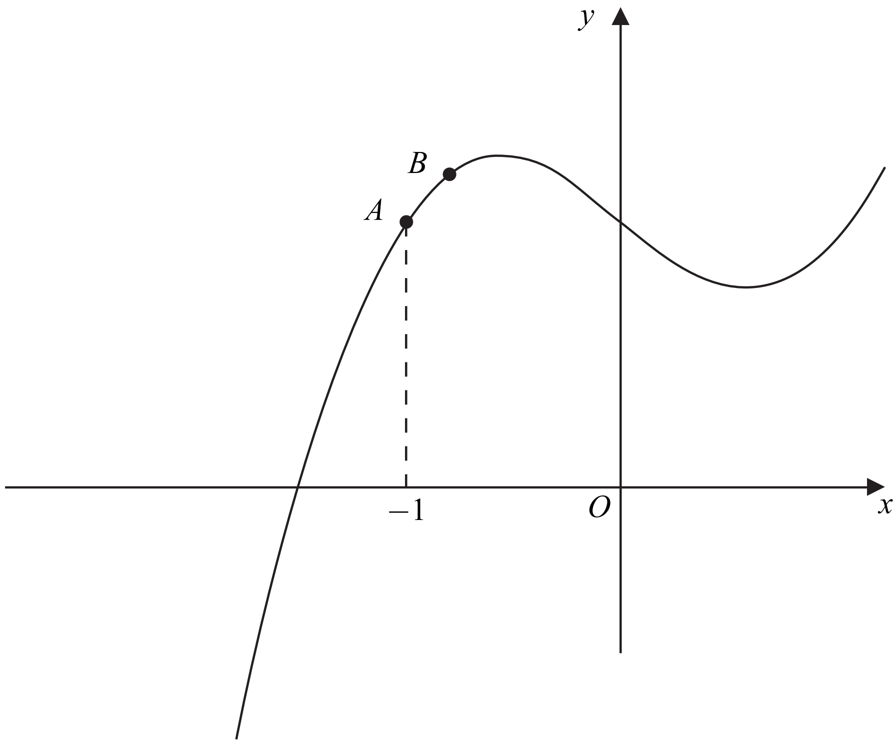
(c) Describe fully the geometrical transformation represented by $\mathbf{M}^4$. *(2 marks)*

7 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows the curve

$$y = x^3 - x + 1$$

The points A and B on the curve have x-coordinates -1 and $-1 + h$ respectively.



(a) (i) Show that the y-coordinate of the point B is

$$1 + 2h - 3h^2 + h^3$$

(3 marks)

(ii) Find the gradient of the chord AB in the form

$$p + qh + rh^2$$

where p, q and r are integers. *(3 marks)*

(iii) Explain how your answer to part (a)(ii) can be used to find the gradient of the tangent to the curve at A. State the value of this gradient. *(2 marks)*

(b) The equation $x^3 - x + 1 = 0$ has one real root, α.

(i) Taking $x_1 = -1$ as a first approximation to α, use the Newton-Raphson method to find a second approximation, x_2, to α. *(2 marks)*

(ii) On **Figure 1**, draw a straight line to illustrate the Newton-Raphson method as used in part (b)(i). Show the points $(x_2, 0)$ and $(\alpha, 0)$ on your diagram.

(2 marks)

Turn over ▶

8 (a) (i) It is given that α and β are the roots of the equation

$$x^2 - 2x + 4 = 0$$

Without solving this equation, show that α^3 and β^3 are the roots of the equation

$$x^2 + 16x + 64 = 0$$ *(6 marks)*

(ii) State, giving a reason, whether the roots of the equation

$$x^2 + 16x + 64 = 0$$

are real and equal, real and distinct, or non-real. *(2 marks)*

(b) Solve the equation

$$x^2 - 2x + 4 = 0$$ *(2 marks)*

(c) Use your answers to parts (a) and (b) to show that

$$(1 + \mathrm{i}\sqrt{3})^3 = (1 - \mathrm{i}\sqrt{3})^3$$ *(2 marks)*

9 A curve C has equation

$$y = \frac{2}{x(x-4)}$$

(a) Write down the equations of the three asymptotes of C. *(3 marks)*

(b) The curve C has one stationary point. By considering an appropriate quadratic equation, find the coordinates of this stationary point.

(No credit will be given for solutions based on differentiation.) *(6 marks)*

(c) Sketch the curve C. *(3 marks)*

END OF QUESTIONS

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Surname		Other Names				
Centre Number						Candidate Number
Candidate Signature						

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Insert

Insert for use in **Question 7**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Figure 1 (for use in Question 7)