



ADVANCED GCE
MATHEMATICS
Further Pure Mathematics 3

4727

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

None

Thursday 29 January 2009
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

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1 In this question G is a group of order n , where $3 \leq n < 8$.

(i) In each case, write down the smallest possible value of n :

(a) if G is cyclic, [1]

(b) if G has a proper subgroup of order 3, [1]

(c) if G has at least two elements of order 2. [1]

(ii) Another group has the same order as G , but is not isomorphic to G . Write down the possible value(s) of n . [2]

2 (i) Express $\frac{\sqrt{3} + i}{\sqrt{3} - i}$ in the form $re^{i\theta}$, where $r > 0$ and $0 \leq \theta < 2\pi$. [3]

(ii) Hence find the smallest positive value of n for which $\left(\frac{\sqrt{3} + i}{\sqrt{3} - i}\right)^n$ is real and positive. [2]

3 Two skew lines have equations

$$\frac{x}{2} = \frac{y+3}{1} = \frac{z-6}{3} \quad \text{and} \quad \frac{x-5}{3} = \frac{y+1}{1} = \frac{z-7}{5}.$$

(i) Find the direction of the common perpendicular to the lines. [2]

(ii) Find the shortest distance between the lines. [4]

4 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 65 \sin 2x. \quad [9]$$

5 The variables x and y are related by the differential equation

$$x^3 \frac{dy}{dx} = xy + x + 1. \quad (\text{A})$$

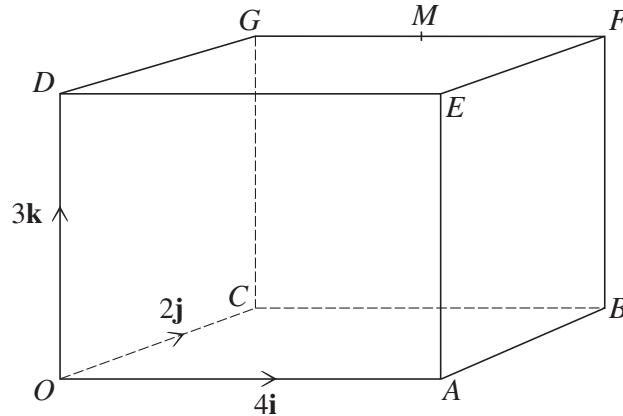
(i) Use the substitution $y = u - \frac{1}{x}$, where u is a function of x , to show that the differential equation may be written as

$$x^2 \frac{du}{dx} = u. \quad [4]$$

(ii) Hence find the general solution of the differential equation (A), giving your answer in the form $y = f(x)$. [5]

3

6



The cuboid $OABCDEFG$ shown in the diagram has $\vec{OA} = 4\mathbf{i}$, $\vec{OC} = 2\mathbf{j}$, $\vec{OD} = 3\mathbf{k}$, and M is the mid-point of GF .

- (i) Find the equation of the plane $ACGE$, giving your answer in the form $\mathbf{r} \cdot \mathbf{n} = p$. [4]
- (ii) The plane $OEFC$ has equation $\mathbf{r} \cdot (3\mathbf{i} - 4\mathbf{k}) = 0$. Find the acute angle between the planes $OEFC$ and $ACGE$. [4]
- (iii) The line AM meets the plane $OEFC$ at the point W . Find the ratio $AW : WM$. [5]

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- (i) The operation $*$ is defined by $x * y = x + y - a$, where x and y are real numbers and a is a real constant.
 - (a) Prove that the set of real numbers, together with the operation $*$, forms a group. [6]
 - (b) State, with a reason, whether the group is commutative. [1]
 - (c) Prove that there are no elements of order 2. [2]
- (ii) The operation \circ is defined by $x \circ y = x + y - 5$, where x and y are **positive** real numbers. By giving a numerical example in each case, show that two of the basic group properties are not necessarily satisfied. [4]

8

- (i) By expressing $\sin \theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$, show that

$$\sin^6 \theta \equiv -\frac{1}{32}(\cos 6\theta - 6 \cos 4\theta + 15 \cos 2\theta - 10). \quad [5]$$

- (ii) Replace θ by $(\frac{1}{2}\pi - \theta)$ in the identity in part (i) to obtain a similar identity for $\cos^6 \theta$. [3]

- (iii) Hence find the exact value of $\int_0^{\frac{1}{4}\pi} (\sin^6 \theta - \cos^6 \theta) d\theta$. [4]

There are no questions printed on this page.



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