

# Mark Scheme (Results)

June 2011

GCE Core Mathematics C4 (6666) Paper 1

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## EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.

### 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol  $\checkmark$  will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- \* The answer is printed on the paper
- $\square$  The second mark is dependent on gaining the first mark

**June 2011 FINAL**  
**Core Mathematics C4 6666**  
**Mark Scheme**

Question Number	Scheme	Marks
<b>1.</b>	$9x^2 = A(x-1)(2x+1) + B(2x+1) + C(x-1)^2$ <p> <math>x \rightarrow 1 \quad 9 = 3B \Rightarrow B = 3</math> </p> <p> <math>x \rightarrow -\frac{1}{2} \quad \frac{9}{4} = \left(-\frac{3}{2}\right)^2 C \Rightarrow C = 1</math> </p> <p> <math>x^2</math> terms <math>9 = 2A + C \Rightarrow A = 4</math> </p> <p><i>Alternatives for finding A.</i></p> <p> <math>x</math> terms <math>0 = -A + 2B - 2C \Rightarrow A = 4</math>            Constant terms <math>0 = -A + B + C \Rightarrow A = 4</math> </p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p style="text-align: right;"><b>(4)</b> <b>[4]</b></p>

Question Number	Scheme	Marks
<p><b>2.</b></p>	$f(x) = (\dots + \dots)^{-\frac{1}{2}}$ $= 9^{-\frac{1}{2}} (\dots + \dots)^{\dots}$ $(1+kx^2)^n = 1+nkx^2 + \dots$ $(1+kx^2)^{-\frac{1}{2}} = \dots + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(kx^2)^2$ $\left(1 + \frac{4}{9}x^2\right)^{-\frac{1}{2}} = 1 - \frac{2}{9}x^2 + \frac{2}{27}x^4$ $f(x) = \frac{1}{3} - \frac{2}{27}x^2 + \frac{2}{81}x^4$	<p>M1</p> <p>B1 <math>3^{-1}, \frac{1}{3}</math> or <math>\frac{1}{9^{\frac{1}{2}}}</math></p> <p>M1 <math>n</math> not a natural number, <math>k \neq 1</math></p> <p>A1 ft ft their <math>k \neq 1</math></p> <p>A1</p> <p>A1 <b>(6)</b> <b>[6]</b></p>

Question Number	Scheme	Marks
3.	(a) $\frac{dV}{dh} = \frac{1}{2}\pi h - \pi h^2$	or equivalent M1 A1
	At $h = 0.1$ , $\frac{dV}{dh} = \frac{1}{2}\pi(0.1) - \pi(0.1)^2 = 0.04\pi$	$\frac{\pi}{25}$ M1 A1 (4)
	(b) $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{\pi}{800} \times \frac{1}{\frac{1}{2}\pi h - \pi h^2}$	or $\frac{\pi}{800} \div$ their (a) M1
	At $h = 0.1$ , $\frac{dh}{dt} = \frac{\pi}{800} \times \frac{25}{\pi} = \frac{1}{32}$	awrt 0.031 A1 (2) [6]

Question Number	Scheme	Marks
<p><b>4.</b></p>	<p>(a) 0.0333, 1.3596</p>	<p>awrt 0.0333, 1.3596 B1 B1 (2)</p>
	<p>(b) <math>\text{Area}(R) \approx \frac{1}{2} \times \frac{\sqrt{2}}{4} [ \dots ]</math>  <math>\approx \dots [ 0 + 2(0.0333 + 0.3240 + 1.3596) + 3.9210 ]</math>  <math>\approx 1.30</math></p>	<p>B1 M1 Accept A1 (3)</p>
	<p>(c) <math>u = x^2 + 2 \Rightarrow \frac{du}{dx} = 2x</math>  <math>\text{Area}(R) = \int_0^{\sqrt{2}} x^3 \ln(x^2 + 2) dx</math>  <math>\int x^3 \ln(x^2 + 2) dx = \int x^2 \ln(x^2 + 2) x dx = \int (u - 2)(\ln u) \frac{1}{2} du</math>                  Hence <math>\text{Area}(R) = \frac{1}{2} \int_2^4 (u - 2) \ln u du</math> *                  cso</p>	<p>B1 B1 M1 A1 (4)</p>
	<p>(d) <math>\int (u - 2) \ln u du = \left( \frac{u^2}{2} - 2u \right) \ln u - \int \left( \frac{u^2}{2} - 2u \right) \frac{1}{u} du</math>  <math>= \left( \frac{u^2}{2} - 2u \right) \ln u - \int \left( \frac{u}{2} - 2 \right) du</math>  <math>= \left( \frac{u^2}{2} - 2u \right) \ln u - \left( \frac{u^2}{4} - 2u \right) (+C)</math></p>	<p>M1 A1 M1 A1</p>
	<p><math>\text{Area}(R) = \frac{1}{2} \left[ \left( \frac{u^2}{2} - 2u \right) \ln u - \left( \frac{u^2}{4} - 2u \right) \right]_2^4</math>  <math>= \frac{1}{2} [ (8 - 8) \ln 4 - 4 + 8 - ((2 - 4) \ln 2 - 1 + 4) ]</math>  <math>= \frac{1}{2} (2 \ln 2 + 1)</math></p>	<p>M1 ln 2 + 1/2 A1 (6) [15]</p>

Question Number	Scheme	Marks
<p>5.</p>	$\frac{1}{y} \frac{dy}{dx} = \dots$ $\dots = 2 \ln x + 2x \left( \frac{1}{x} \right)$ <p>At <math>x = 2</math>, leading to</p> $\ln y = 2(2) \ln 2$ $y = 16$ <p>At <math>(2, 16)</math></p> $\frac{1}{16} \frac{dy}{dx} = 2 \ln 2 + 2$ $\frac{dy}{dx} = 16(2 + 2 \ln 2)$	<p>B1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (7)</p> <p>[7]</p>
	<p><i>Alternative</i></p> $y = e^{2x \ln x}$ $\frac{d}{dx}(2x \ln x) = 2 \ln x + 2x \left( \frac{1}{x} \right)$ $\frac{dy}{dx} = \left( 2 \ln x + 2x \left( \frac{1}{x} \right) \right) e^{2x \ln x}$ <p>At <math>x = 2</math>,</p> $\frac{dy}{dx} = (2 \ln 2 + 2) e^{4 \ln 2}$ $= 16(2 + 2 \ln 2)$	<p>B1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (7)</p>



Question Number	Scheme	Marks
6.	<p>(a) <b>i:</b> <math>6 - \lambda = -5 + 2\mu</math>  <b>j:</b> <math>-3 + 2\lambda = 15 - 3\mu</math>                      leading to <math>\lambda = 3, \mu = 4</math>  <math>\mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + 3 \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}</math> or <math>\mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + 4 \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}</math>  <b>k:</b> LHS = <math>-2 + 3(3) = 7</math>, RHS = <math>3 + 4(1) = 7</math>                      (As LHS = RHS, lines intersect)</p> <p>Alternatively for B1, showing that <math>\lambda = 3</math> and <math>\mu = 4</math> both give <math>\begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}</math></p> <p>(b) <math>\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = -2 - 6 + 3 = \sqrt{14}\sqrt{14}\cos\theta</math> (<math>\theta \approx 110.92^\circ</math>)                      Acute angle is <math>69.1^\circ</math></p> <p>(c) <math>\mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + 1 \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}</math> (<math>\Rightarrow B</math> lies on <math>l_1</math>)</p> <p>(d) Let <math>d</math> be shortest distance from <math>B</math> to <math>l_2</math></p> <div style="display: flex; align-items: center;"> <div style="flex: 1;"> <p><math>\overrightarrow{AB} = \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -6 \end{pmatrix}</math></p> <p><math> \overrightarrow{AB}  = \sqrt{(2)^2 + (-4)^2 + (-6)^2} = \sqrt{56}</math></p> <p><math>\frac{d}{\sqrt{56}} = \sin\theta</math>  <math>d = \sqrt{56}\sin 69.1^\circ \approx 6.99</math></p> </div> <div style="flex: 1; text-align: center;"> </div> </div>	<p>Any two equations</p> <p>M1 M1 A1 M1 A1</p> <p>B1 (6)</p> <p>M1 A1</p> <p>A1 (3)</p> <p>B1 (1)</p> <p>M1 A1 M1 A1 (4) [14]</p>

Question Number	Scheme	Marks
7.	(a) $\tan \theta = \sqrt{3}$ or $\sin \theta = \frac{\sqrt{3}}{2}$ $\theta = \frac{\pi}{3}$	M1 awrt 1.05 A1 (2)
	(b) $\frac{dx}{d\theta} = \sec^2 \theta, \frac{dy}{d\theta} = \cos \theta$ $\frac{dy}{dx} = \frac{\cos \theta}{\sec^2 \theta} (= \cos^3 \theta)$	M1 A1
	At P, $m = \cos^3 \left( \frac{\pi}{3} \right) = \frac{1}{8}$	Can be implied A1
	Using $mm' = -1, m' = -8$ For normal $y - \frac{1}{2}\sqrt{3} = -8(x - \sqrt{3})$	M1 M1
	At Q, $y = 0$ $-\frac{1}{2}\sqrt{3} = -8(x - \sqrt{3})$	
	leading to $x = \frac{17}{16}\sqrt{3} \quad (k = \frac{17}{16})$	1.0625 A1 (6)
	(c) $\int y^2 dx = \int y^2 \frac{dx}{d\theta} d\theta = \int \sin^2 \theta \sec^2 \theta d\theta$ $= \int \tan^2 \theta d\theta$ $= \int (\sec^2 \theta - 1) d\theta$ $= \tan \theta - \theta (+C)$	M1 A1 A1 M1 A1
	$V = \pi \int_0^{\frac{\pi}{3}} y^2 dx = [\tan \theta - \theta]_0^{\frac{\pi}{3}} = \pi \left[ \left( \sqrt{3} - \frac{\pi}{3} \right) - (0 - 0) \right]$	M1
	$= \sqrt{3}\pi - \frac{1}{3}\pi^2 \quad (p = 1, q = -\frac{1}{3})$	A1 (7) [15]

Question Number	Scheme	Marks
8.	<p>(a) <math>\int (4y+3)^{-\frac{1}{2}} dx = \frac{(4y+3)^{\frac{1}{2}}}{(4)(\frac{1}{2})} (+C)</math>  <math>(= \frac{1}{2}(4y+3)^{\frac{1}{2}} + C)</math></p> <p>(b) <math>\int \frac{1}{\sqrt{4y+3}} dy = \int \frac{1}{x^2} dx</math>  <math>\int (4y+3)^{-\frac{1}{2}} dy = \int x^{-2} dx</math>  <math>\frac{1}{2}(4y+3)^{\frac{1}{2}} = -\frac{1}{x} (+C)</math></p> <p>Using <math>(-2, 1.5)</math> <math>\frac{1}{2}(4 \times 1.5 + 3)^{\frac{1}{2}} = -\frac{1}{-2} + C</math>                      leading to <math>C = 1</math>  <math>\frac{1}{2}(4y+3)^{\frac{1}{2}} = -\frac{1}{x} + 1</math>  <math>(4y+3)^{\frac{1}{2}} = 2 - \frac{2}{x}</math>  <math>y = \frac{1}{4} \left( 2 - \frac{2}{x} \right)^2 - \frac{3}{4}</math></p>	<p>M1 A1 (2)</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (6)</p> <p>[8]</p>

or equivalent

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