

Version 1.0



**General Certificate of Education (A-level)  
June 2011**

**Mathematics**

**MD02**

**(Specification 6360)**

**Decision 2**

**Final**

***Mark Scheme***

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Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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## Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

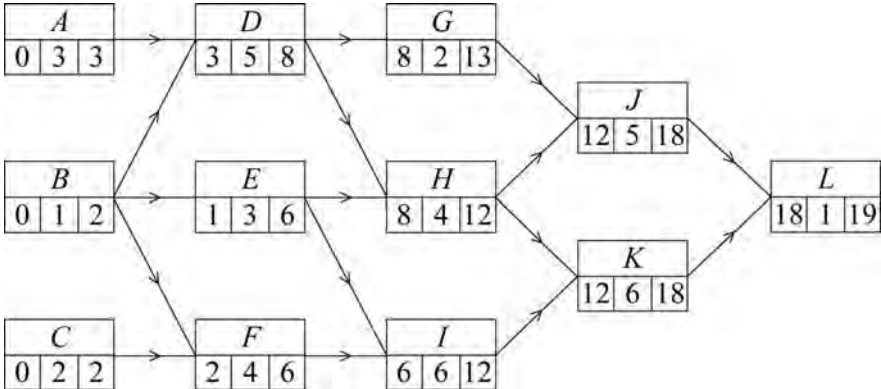
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

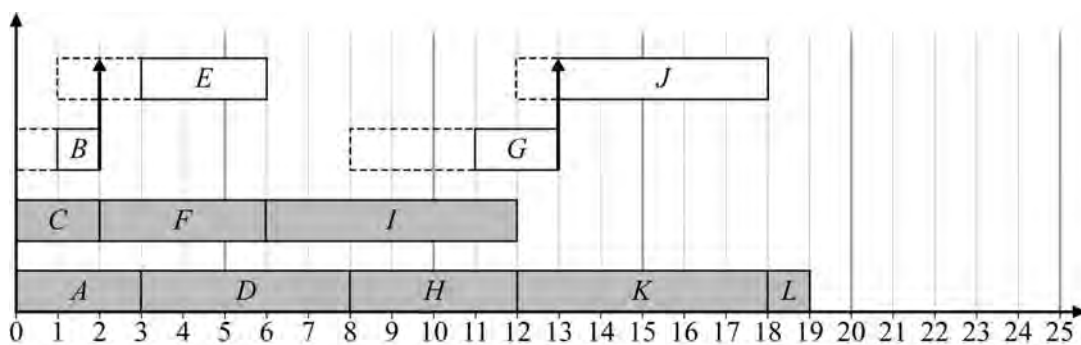
Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

MD02

Q	Solution	Marks	Total	Comments
1(a)	 <p>Earliest start times</p> <p>Latest finish times</p> <p>(b) Critical paths <i>A D H K L</i> <i>C F I K L</i> Minimum time = 19</p> <p>(c) Greatest float time at G (13 – 8 – 2) = 3 (days)</p> <p>(d) <i>A, D, H, K, L</i> and <i>C, F, I</i> <i>B, E, G, J</i> <i>B(1-2); E(3-6); G(11-13); J(13-18)</i></p>	<p>M1 A1</p> <p>M1 A1</p> <p>B1 B1 B1</p> <p>M1 A1cso</p> <p>M1 A1</p> <p>M1 A1cso</p>	<p></p> <p>4</p> <p>3</p> <p>2</p> <p>4</p>	<p>condone one slip + ft all correct</p> <p>condone one slip + ft all correct</p> <p>one path correct second path correct and no others 19 days</p> <p>ft their activity with greatest float for M1 values at G must be correct</p> <p>one of ‘their’ critical paths “correct” all 8 of these activities correct</p> <p>3 of these with correct duration and latest start time (may omit slack) all 4 correct with correct slack shown</p>
	<b>Total</b>		<b>13</b>	

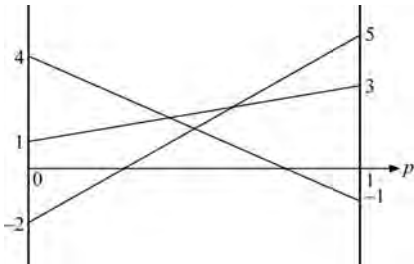


**MD02 (cont)**

Q	Solution	Marks	Total	Comments	
<b>2(a)</b>	$\begin{array}{ccccc} 3 & 1 & 0 & 4 & 1 \\ 1 & 4 & 1 & 2 & 4 \\ 1 & 0 & 3 & 1 & 2 \\ 2 & 3 & 2 & 0 & 0 \\ 0 & 5 & 1 & 2 & 1 \end{array}$	M1		reducing columns first	
	$\begin{array}{ccccc} 3 & 1 & 0 & 4 & 1 \\ 0 & k & 0 & 1 & 3 \\ 1 & 0 & 3 & 1 & 2 \\ 2 & 3 & 2 & 0 & 0 \\ 0 & 5 & 1 & 2 & 1 \end{array}$			then rows $k = 3$ stated or value 3 in table	
		A1cso	2	<b>AG</b>	
	<b>(b)(i)</b> Lines through columns 1, 2, 3 and row 4	B1	1		
	<b>(ii)</b>	$\begin{array}{ccccc} 3 & 1 & 0 & 3 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 1 & 0 & 3 & 0 & 1 \\ 3 & 4 & 3 & 0 & 0 \\ 0 & 5 & 1 & 1 & 0 \end{array}$	M1		subtract 1 from all uncovered and add 1 to all double covered (condone one slip)
			A1	2	all correct ISW
		This now requires 5 lines to cover zeros			
	<b>(c)</b>	$A2 \quad B3 \quad C1 \quad D4 \quad E5$	B1		one of these correct
		$A5 \quad B3 \quad C1 \quad D2 \quad E4$	B1		second way correct
		$A5 \quad B3 \quad C2 \quad D4 \quad E1$	B1	3	third way correct and no others
<b>(d)</b> Minimum total = 68 (mins)	B1	1			
<b>(e)</b> Replace each element $x$ by $N - x$	E1	1	any value of $N$		
	<b>Total</b>		<b>10</b>		

Q	Solution	Marks	Total	Comments
<b>3(a)</b>	Row minima are $-4, -3, -7$	M1		<b>both</b> row minima <b>and</b> column maxima attempted (condone 2 errors) all values correct
	Column maxima are $-3, 6, 8$	A1		
	$\max(\text{row min}) = \min(\text{col max}) = -3$	E1		condone arrows pointing to this element but must <b>state</b> $\max(\text{row min})$ and $\min(\text{col max})$ or equivalent
	Play-safe Tom <b>II</b> and Jerry <b>A</b>	B1	4	

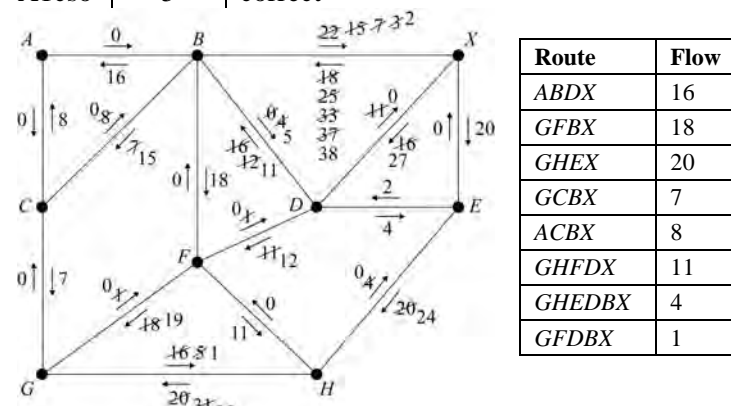
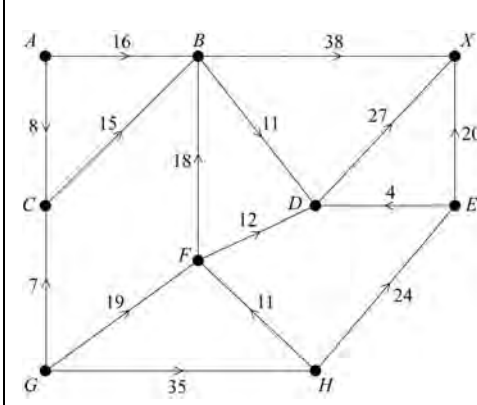
**MD02 (cont)**

Q	Solution	Marks	Total	Comments
<p><b>3(b)(i)</b></p>	<p>Let Rohan play <math>R_1</math> with prob <math>p</math>  <math>\Rightarrow</math> plays <math>R_2</math> with prob <math>1 - p</math></p> <p>When Carla plays <math>C_1</math>,                      Rohan's expected gain <math>= 3p + (1 - p)</math>  <math>= 1 + 2p</math></p> <p><math>C_2 : 5p + (-2)(1 - p) = 7p - 2</math></p> <p><math>C_3 : -p + 4(1 - p) = 4 - 5p</math></p>  <p><math>7p - 2 = 4 - 5p</math>  <math>12p = 6</math>  <math>\Rightarrow p = \frac{1}{2} \Rightarrow</math> Rohan plays <math>R_1</math> 50% of the time and <math>R_2</math> 50% of the time</p> <p>Value of game <math>= 7 \times \frac{1}{2} - 2 = \frac{3}{2}</math> AG</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1cso</p> <p>B1</p>	<p>7</p>	<p>at least 2 expected gains correct unsimplified</p> <p>all 3 correct unsimplified</p> <p>at least 2 lines correct</p> <p>all lines correct for <math>0 \leq p \leq 1</math> and values at 0 and 1 clear</p> <p>choosing highest point or using correct equation</p> <p>or <math>4 - \frac{5}{2} = \frac{3}{2}</math> must see working</p>
<p><b>(b)(ii)</b></p>	<p>When Rohan plays <math>R_1</math>, expected loss for Carla is <math>3p + 5q + (-1)(1 - p - q)</math></p> <p>and when Rohan plays <math>R_2</math>, expected loss for Carla is <math>p + (-2)q + 4(1 - p - q)</math></p> <p><math>4p + 6q = \frac{3}{2} + 1</math></p> <p><math>3p + 6q = 4 - \frac{3}{2}</math></p> <p><math>\Rightarrow p = 0, q = \frac{5}{12}</math></p> <p><math>\Rightarrow</math> Carla never plays <math>C_1</math>,                      plays <math>C_2</math> with prob <math>\frac{5}{12}</math>                      and plays <math>C_3</math> with prob <math>\frac{7}{12}</math></p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>E1cso</p>	<p>4</p>	<p>either expression correct unsimplified</p> <p>correct simultaneous equations unsimplified</p> <p>condone 0.42 or better</p> <p>Must have all 3 correct probabilities</p>
<b>Total</b>			<b>15</b>	

**MD02 (cont)**

Q	Solution	Marks	Total	Comments			
<b>4(a)</b>	$5x + 3y + 10z \leq 15$	M1	2	2 inequalities correct or all 3 LHS & RHS correct but using < all correct			
	$7x + 6y + 4z \leq 28$ $4x + 3y + 6z \leq 12$	A1					
<b>(b)(i)</b>	Choosing 3 from bottom row as pivot	B1	4	identified or used  row operations (even with wrong pivot)  one of rows 1, 2, 3 correct  all correct (condone multiples of rows)			
	$\begin{array}{cccccccc} 1 & 6 & 0 & 12-k & 0 & 0 & 2 & 24 \\ 0 & 1 & 0 & 4 & 1 & 0 & -1 & 3 \\ 0 & -1 & 0 & -8 & 0 & 1 & -2 & 4 \\ 0 & \frac{4}{3} & 1 & 2 & 0 & 0 & \frac{1}{3} & 4 \end{array}$	M1					
		A1					
		A1					
		A1					
<b>(ii)</b>	$12 - k < 0$ $\Rightarrow k > 12$	M1 A1	2	their '12 - k' < 0 SC B1 for $k \geq 13$			
<b>(c)(i)</b>	$\begin{array}{cccccccc} 1 & 6 & 0 & -8 & 0 & 0 & 2 & 24 \\ 0 & 1 & 0 & 4^* & 1 & 0 & -1 & 3 \\ 0 & -1 & 0 & -8 & 0 & 1 & -2 & 4 \\ 0 & \frac{4}{3} & 1 & 2 & 0 & 0 & \frac{1}{3} & 4 \end{array}$	M1	4	correct pivot from z column 4* (identified or used)			
	$\begin{array}{cccccccc} 1 & 8 & 0 & 0 & 2 & 0 & 0 & 30 \\ 0 & \frac{1}{4} & 0 & 1 & \frac{1}{4} & 0 & -\frac{1}{4} & \frac{3}{4} \\ 0 & 1 & 0 & 0 & 2 & 1 & -4 & 10 \\ 0 & \frac{5}{6} & 1 & 0 & -\frac{1}{2} & 0 & \frac{5}{6} & \frac{5}{2} \end{array}$				A1	4	one of rows 1, 3 or 4 correct  another of rows 1, 3 or 4 correct  all correct (condone multiples of rows)
					A1		
					A1		
		A1					
	<b>(ii)</b>	Maximum value of P now reached	E1	3	their tableau must have no negatives in top row  ft their values from their tableau provided at least 2 marks earned in (c)(i)  condone up to 2 slips in their final tableau		
		$P = 30, x = 0, y = \frac{5}{2}, z = \frac{3}{4}$	B1✓				
		$s = 0, t = 10, u = 0$	B1cao				
	<b>Total</b>			<b>15</b>			

MD02 (cont)

Q	Solution	Marks	Total	Comments																		
5(a)	Cut value = $40 + 27 + 0 + 24$ = 91	B1	1																			
(b)	ABDX 16 GFBX 18 GHEX 20	B1 B1 B1	3																			
(c)(i)	One correct route with additional flow  Another 2 routes and flows correct  All routes correct with total flow = 85  Forward and backward flows on diagram (directions must be clear)  Augmenting flows  <i>Consider other possible correct flows</i>  <i>Condone diagram as shown but really should have initial flows in DE, etc</i>	M1  A1  A1cso  M1  A1cso	5	any feasible route and flow  total flow at least 80  at least 8 edges with pairs of values 'correct'  correct																		
				 <table border="1" data-bbox="1276 851 1500 1187"> <thead> <tr> <th>Route</th> <th>Flow</th> </tr> </thead> <tbody> <tr> <td>ABDX</td> <td>16</td> </tr> <tr> <td>GFBX</td> <td>18</td> </tr> <tr> <td>GHEX</td> <td>20</td> </tr> <tr> <td>GCBX</td> <td>7</td> </tr> <tr> <td>ACBX</td> <td>8</td> </tr> <tr> <td>GHFDX</td> <td>11</td> </tr> <tr> <td>GHEDBX</td> <td>4</td> </tr> <tr> <td>GFDBX</td> <td>1</td> </tr> </tbody> </table>	Route	Flow	ABDX	16	GFBX	18	GHEX	20	GCBX	7	ACBX	8	GHFDX	11	GHEDBX	4	GFDBX	1
Route	Flow																					
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GFBX	18																					
GHEX	20																					
GCBX	7																					
ACBX	8																					
GHFDX	11																					
GHEDBX	4																					
GFDBX	1																					
(ii)	Max flow = 85  Correct max flow  <i>Consider other possible correct flows</i>	B1  B1	2																			
(d)	Considering 'their' $AB + CB + FB - 45$ = 4 fewer $\Rightarrow$ max number = 81	M1  A1cao	2																			
<b>Total</b>			<b>13</b>																			



**MD02 (cont)**

Q	Solution	Marks	Total	Comments																																																																																																																																																																					
6	Wednesday profits	M1	9	4 more calculations/ profits correct																																																																																																																																																																					
		A1		6 more profits correct																																																																																																																																																																					
		A1		all profits correct																																																																																																																																																																					
	Tuesday: use of maxima from Wednesday	M1		6 more calculations/profits correct																																																																																																																																																																					
		A1		8 profits correct																																																																																																																																																																					
		A1✓		all profits correct																																																																																																																																																																					
	Monday values correct	A1✓		ft one slip from Wednesday figures																																																																																																																																																																					
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	(Monday builds shed) <i>D</i>  ⇒ order <i>D B A C</i>	M1  A1cso			Choosing largest Monday profit from their table  SC B1 only for order <i>D B A C</i> NMS or without “correct” table																																																																																																																																																																				
<table border="1"> <thead> <tr> <th>Stage (Day)</th> <th>State (Sheds already built)</th> <th>Action (shed to build)</th> <th>Calculation</th> <th>Profit in pounds</th> </tr> </thead> <tbody> <tr><td>Thursday</td><td><i>A, B, C</i></td><td><i>D</i></td><td></td><td>90</td></tr> <tr><td></td><td><i>A, B, D</i></td><td><i>C</i></td><td></td><td>87</td></tr> <tr><td></td><td><i>A, C, D</i></td><td><i>B</i></td><td></td><td>76</td></tr> <tr><td></td><td><i>B, C, D</i></td><td><i>A</i></td><td></td><td>70</td></tr> <tr><td>Wednesday</td><td><i>A, B</i></td><td><i>C</i></td><td>84 + 90</td><td>174</td></tr> <tr><td></td><td></td><td><i>D</i></td><td>88 + 87</td><td>175 →</td></tr> <tr><td></td><td><i>A, C</i></td><td><i>B</i></td><td>71 + 90</td><td>161 →</td></tr> <tr><td></td><td></td><td><i>D</i></td><td>82 + 76</td><td>158</td></tr> <tr><td></td><td><i>A, D</i></td><td><i>B</i></td><td>74 + 87</td><td>161 →</td></tr> <tr><td></td><td></td><td><i>C</i></td><td>83 + 76</td><td>159</td></tr> <tr><td></td><td><i>B, C</i></td><td><i>A</i></td><td>65 + 90</td><td>155</td></tr> <tr><td></td><td></td><td><i>D</i></td><td>86 + 70</td><td>156 →</td></tr> <tr><td></td><td><i>B, D</i></td><td><i>A</i></td><td>69 + 87</td><td>156 →</td></tr> <tr><td></td><td></td><td><i>C</i></td><td>85 + 70</td><td>155</td></tr> <tr><td></td><td><i>C, D</i></td><td><i>A</i></td><td>66 + 76</td><td>142</td></tr> <tr><td></td><td></td><td><i>B</i></td><td>73 + 70</td><td>143 →</td></tr> <tr><td>Tuesday</td><td><i>A</i></td><td><i>B</i></td><td>72 + 175</td><td>247 →</td></tr> <tr><td></td><td></td><td><i>C</i></td><td>83 + 161</td><td>244</td></tr> <tr><td></td><td></td><td><i>D</i></td><td>84 + 161</td><td>245</td></tr> <tr><td></td><td><i>B</i></td><td><i>A</i></td><td>60 + 175</td><td>235</td></tr> <tr><td></td><td></td><td><i>C</i></td><td>80 + 156</td><td>236</td></tr> <tr><td></td><td></td><td><i>D</i></td><td>83 + 156</td><td>239 →</td></tr> <tr><td></td><td><i>C</i></td><td><i>A</i></td><td>57 + 161</td><td>218</td></tr> <tr><td></td><td></td><td><i>B</i></td><td>68 + 156</td><td>224</td></tr> <tr><td></td><td></td><td><i>D</i></td><td>85 + 143</td><td>228 →</td></tr> <tr><td></td><td><i>D</i></td><td><i>A</i></td><td>62 + 161</td><td>223</td></tr> <tr><td></td><td></td><td><i>B</i></td><td>70 + 156</td><td>226 →</td></tr> <tr><td></td><td></td><td><i>C</i></td><td>81 + 143</td><td>224</td></tr> <tr><td>Monday</td><td>-</td><td><i>A</i></td><td>50 + 247</td><td>297</td></tr> <tr><td></td><td></td><td><i>B</i></td><td>65 + 239</td><td>304</td></tr> <tr><td></td><td></td><td><i>C</i></td><td>70 + 228</td><td>298</td></tr> <tr><td></td><td></td><td><i>D</i></td><td>80 + 226</td><td>306 →</td></tr> </tbody> </table>					Stage (Day)	State (Sheds already built)	Action (shed to build)	Calculation	Profit in pounds	Thursday	<i>A, B, C</i>	<i>D</i>		90		<i>A, B, D</i>	<i>C</i>		87		<i>A, C, D</i>	<i>B</i>		76		<i>B, C, D</i>	<i>A</i>		70	Wednesday	<i>A, B</i>	<i>C</i>	84 + 90	174			<i>D</i>	88 + 87	175 →		<i>A, C</i>	<i>B</i>	71 + 90	161 →			<i>D</i>	82 + 76	158		<i>A, D</i>	<i>B</i>	74 + 87	161 →			<i>C</i>	83 + 76	159		<i>B, C</i>	<i>A</i>	65 + 90	155			<i>D</i>	86 + 70	156 →		<i>B, D</i>	<i>A</i>	69 + 87	156 →			<i>C</i>	85 + 70	155		<i>C, D</i>	<i>A</i>	66 + 76	142			<i>B</i>	73 + 70	143 →	Tuesday	<i>A</i>	<i>B</i>	72 + 175	247 →			<i>C</i>	83 + 161	244			<i>D</i>	84 + 161	245		<i>B</i>	<i>A</i>	60 + 175	235			<i>C</i>	80 + 156	236			<i>D</i>	83 + 156	239 →		<i>C</i>	<i>A</i>	57 + 161	218			<i>B</i>	68 + 156	224			<i>D</i>	85 + 143	228 →		<i>D</i>	<i>A</i>	62 + 161	223			<i>B</i>	70 + 156	226 →			<i>C</i>	81 + 143	224	Monday	-	<i>A</i>	50 + 247	297			<i>B</i>	65 + 239	304			<i>C</i>	70 + 228	298			<i>D</i>	80 + 226	306 →
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