



1. A factory produces components. Each component has a unique identity number and it is assumed that 2% of the components are faulty. On a particular day, a quality control manager wishes to take a random sample of 50 components.

(a) Identify a sampling frame.

(1)

The statistic  $F$  represents the number of faulty components in the random sample of size 50.

(b) Specify the sampling distribution of  $F$ .

(2)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---





Leave  
blank

**Question 2 continued**

Lined area for writing the answer to Question 2.



3.

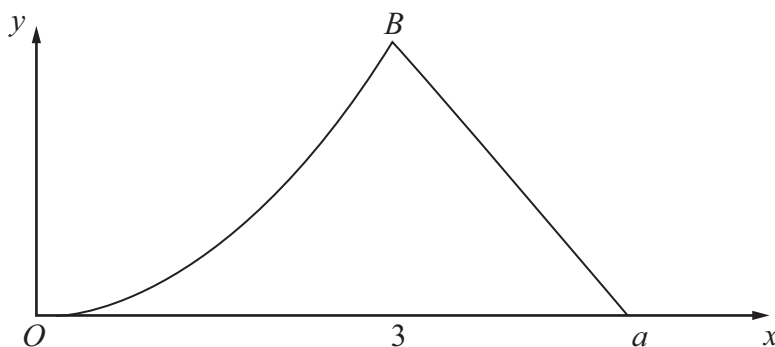


Figure 1

Figure 1 shows a sketch of the probability density function  $f(x)$  of the random variable  $X$ .

For  $0 \leq x \leq 3$ ,  $f(x)$  is represented by a curve  $OB$  with equation  $f(x) = kx^2$ , where  $k$  is a constant.

For  $3 \leq x \leq a$ , where  $a$  is a constant,  $f(x)$  is represented by a straight line passing through  $B$  and the point  $(a, 0)$ .

For all other values of  $x$ ,  $f(x) = 0$ .

Given that the mode of  $X =$  the median of  $X$ , find

(a) the mode, (1)

(b) the value of  $k$ , (4)

(c) the value of  $a$ . (3)

Without calculating  $E(X)$  and with reference to the skewness of the distribution

(d) state, giving your reason, whether  $E(X) < 3$ ,  $E(X) = 3$  or  $E(X) > 3$ . (2)

---

---

---

---

---

---

---

---

---

---





- 4. In a game, players select sticks at random from a box containing a large number of sticks of different lengths. The length, in cm, of a randomly chosen stick has a continuous uniform distribution over the interval  $[7, 10]$ .

A stick is selected at random from the box.

- (a) Find the probability that the stick is shorter than 9.5 cm. (2)

To win a bag of sweets, a player must select 3 sticks and wins if the length of the longest stick is more than 9.5 cm.

- (b) Find the probability of winning a bag of sweets. (2)

To win a soft toy, a player must select 6 sticks and wins the toy if more than four of the sticks are shorter than 7.6 cm.

- (c) Find the probability of winning a soft toy. (4)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---



Leave  
blank

5. Defects occur at random in planks of wood with a constant rate of 0.5 per 10 cm length. Jim buys a plank of length 100 cm.

(a) Find the probability that Jim’s plank contains at most 3 defects. **(2)**

Shivani buys 6 planks each of length 100 cm.

(b) Find the probability that fewer than 2 of Shivani’s planks contain at most 3 defects. **(5)**

(c) Using a suitable approximation, estimate the probability that the total number of defects on Shivani’s 6 planks is less than 18. **(6)**

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---











7. The continuous random variable  $X$  has probability density function given by

$$f(x) = \begin{cases} \frac{3}{32}(x-1)(5-x) & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch  $f(x)$  showing clearly the points where it meets the  $x$ -axis. (2)

(b) Write down the value of the mean,  $\mu$ , of  $X$ . (1)

(c) Show that  $E(X^2) = 9.8$  (4)

(d) Find the standard deviation,  $\sigma$ , of  $X$ . (2)

The cumulative distribution function of  $X$  is given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{32}(a - 15x + 9x^2 - x^3) & 1 \leq x \leq 5 \\ 1 & x > 5 \end{cases}$$

where  $a$  is a constant.

(e) Find the value of  $a$ . (2)

(f) Show that the lower quartile of  $X$ ,  $q_1$ , lies between 2.29 and 2.31 (3)

(g) Hence find the upper quartile of  $X$ , giving your answer to 1 decimal place. (1)

(h) Find, to 2 decimal places, the value of  $k$  so that

$$P(\mu - k\sigma < X < \mu + k\sigma) = 0.5$$
 (2)

---

---

---

---

---

---

---

---

---

---



