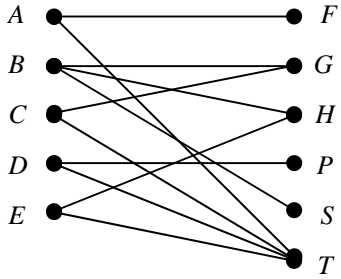
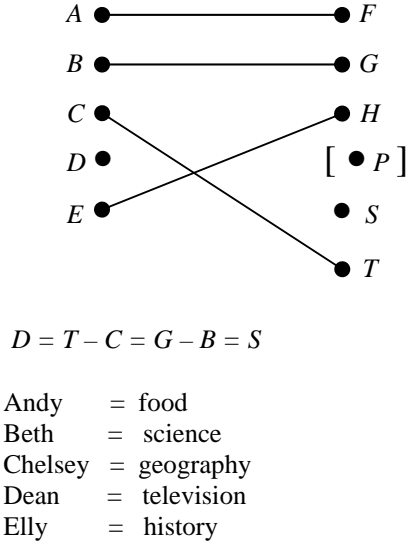
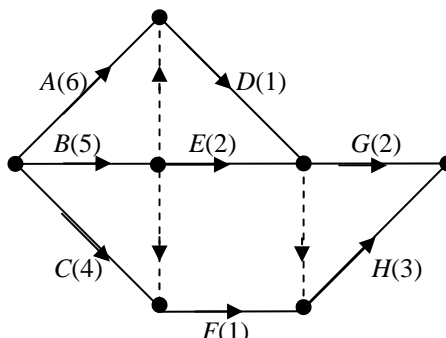
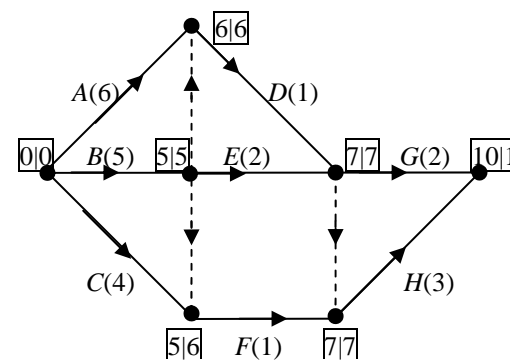
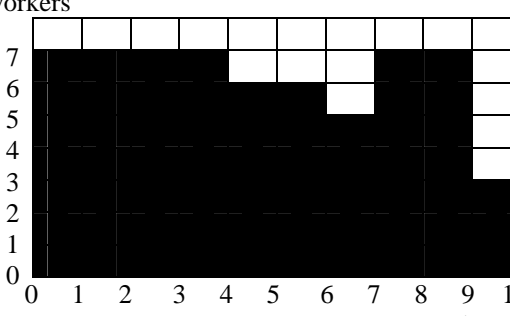


4737 Decision Mathematics 2

1	(i)		B1	Bipartite graph correct	[1]						
	(ii)	 <p style="margin-left: 20px;">$D = T - C = G - B = S$</p> <p style="margin-left: 20px;">Andy = food Beth = science Chelsey = geography Dean = television Elly = history</p>	B1	A new bipartite graph showing the pairings AF, BG, CT and EH but not DS	M1	This alternating path written down, not read off from labels on graph	A1	$B = S, C = G$ and $D = T$ written down	B1	$A = F, E = H$ written down	[4]
	(iii)	<p>Andy = food Beth = television Chelsey = geography Dean = politics Elly = history</p> <p>Science did not arise</p>	B1	$A = F, C = G, D = P$ and $E = H$ (cao) ($B = T$ may be omitted)	B1	S (cao)	[2]				
Total = 7											

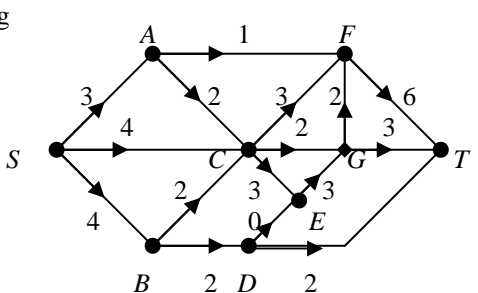
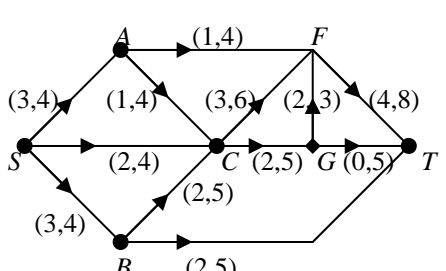
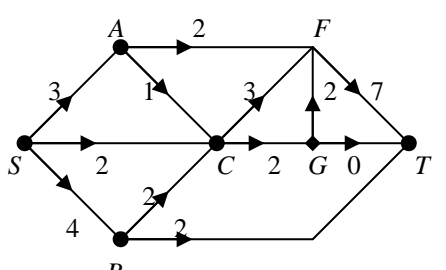
2	<p>Add a dummy row</p> <table border="1" style="margin-left: 20px;"> <thead> <tr> <th></th> <th>P</th> <th>R</th> <th>S</th> <th>T</th> </tr> </thead> <tbody> <tr> <td>April</td> <td>30</td> <td>28</td> <td>32</td> <td>25</td> </tr> <tr> <td>May</td> <td>32</td> <td>34</td> <td>32</td> <td>35</td> </tr> <tr> <td>June</td> <td>40</td> <td>40</td> <td>39</td> <td>38</td> </tr> <tr> <td>Dummy</td> <td>40</td> <td>40</td> <td>40</td> <td>40</td> </tr> </tbody> </table> <p>Reduce rows</p> <table border="1" style="margin-left: 20px;"> <tbody> <tr> <td>5</td> <td>3</td> <td>7</td> <td>0</td> </tr> <tr> <td>0</td> <td>2</td> <td>0</td> <td>3</td> </tr> <tr> <td>2</td> <td>2</td> <td>1</td> <td>0</td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> </tbody> </table> <p>Columns are already reduced</p> <p>Incomplete matching, cross through zeros</p> <table border="1" style="margin-left: 20px;"> <tbody> <tr> <td>5</td> <td>3</td> <td>7</td> <td>0</td> </tr> <tr> <td>0</td> <td>2</td> <td>0</td> <td>3</td> </tr> <tr> <td>2</td> <td>2</td> <td>1</td> <td>0</td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> </tbody> </table> <p>Augment by 1</p> <table border="1" style="margin-left: 20px;"> <tbody> <tr> <td>4</td> <td>2</td> <td>6</td> <td>0</td> </tr> <tr> <td>0</td> <td>2</td> <td>0</td> <td>4</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> </tbody> </table> <p>Complete matching</p> <table border="1" style="margin-left: 20px;"> <thead> <tr> <th></th> <th>P</th> <th>R</th> <th>S</th> <th>T</th> </tr> </thead> <tbody> <tr> <td>April</td> <td>4</td> <td>2</td> <td>6</td> <td>0</td> </tr> <tr> <td>May</td> <td>0</td> <td>2</td> <td>0</td> <td>4</td> </tr> <tr> <td>June</td> <td>1</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>Dummy</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> </tbody> </table> <p>April = Tall Trees £2500 May = Palace £3200 June = Sunnyside £3900</p> <p>Total cost = £9600</p>		P	R	S	T	April	30	28	32	25	May	32	34	32	35	June	40	40	39	38	Dummy	40	40	40	40	5	3	7	0	0	2	0	3	2	2	1	0	0	0	0	0	5	3	7	0	0	2	0	3	2	2	1	0	0	0	0	0	4	2	6	0	0	2	0	4	1	1	0	0	0	0	0	1		P	R	S	T	April	4	2	6	0	May	0	2	0	4	June	1	1	0	0	Dummy	0	0	0	1	<p>B1 Adding a dummy row of all equal values</p> <p>M1 Substantially correct attempt to reduce matrix (condone 1 numerical slip)</p> <p>A1 Correct reduced cost matrix from reducing rows first and statement of how table was formed, including reference to columns (cao)</p> <p>B1 Cross through zeros using minimum number of lines</p> <p>B1 Correct augmented matrix and statement of how table was formed (cao)</p> <p>B1 A = T, M = P, J = S (cao)</p> <p>B1 £9600 (cao) with units</p>	<p>[3]</p> <p>[2]</p> <p>[2]</p>
	P	R	S	T																																																																																																	
April	30	28	32	25																																																																																																	
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1	1	0	0																																																																																																		
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June	1	1	0	0																																																																																																	
Dummy	0	0	0	1																																																																																																	
Total =			7																																																																																																		

<p>3</p>	<p>(i)</p>		<p>M1 Durations not necessary</p> <p>M1 Correct structure, even without directions shown Activities must be labelled</p> <p>A1 Completely correct, with exactly three dummies and all arcs directed</p>	<p>[2]</p>
	<p>(ii)</p>	 <p>Minimum project completion time = 10 hours</p> <p>Critical activities A, B, D, E, H</p>	<p>M1 Follow through their activity network if possible Substantially correct attempt at forward pass (at most 1 independent error)</p> <p>M1 Substantially correct attempt at backward pass (at most 1 independent error)</p> <p>A1ft Both passes wholly correct</p> <p>B1 10 hours (with units) cao</p> <p>M1 Either B, E, H or A, D, H (possibly with other critical activities, but C, F, G not listed). Not follow through.</p> <p>A1 A, B, D, E, H (and no others) cao</p>	<p>[3]</p> <p>[3]</p>
	<p>(iii)</p>	<p>No. of workers</p>  <p>hours</p>	<p>On graph paper</p> <p>M1 A plausible resource histogram with no holes or overhangs</p> <p>A1 Axes scaled and labelled and histogram completely correct, cao</p>	<p>[2]</p>
	<p>(iv)</p>	<p>1 hour</p>	<p>B1 Accept 1 (with units missing) cao</p>	<p>[1]</p>
	<p>(v)</p>	<p>No need to change start times for A, B, C, D and E Activities G and H cannot happen at the same time, so they must follow one another This causes a 2 hour delay</p> <p>F could be delayed until 1 hour before H starts H should be started as late as possible ⇒ a maximum delay of 3 hours</p>	<p>M1 G and H cannot happen together (stated, not just implied from a diagram)</p> <p>A1 2 cao</p> <p>B1 Diagram or explaining that for max delay on F need H to happen as late as possible</p> <p>B1 3 cao</p>	<p>[2]</p> <p>[2]</p>
<p style="text-align: right;">Total = 15</p>				

4	(i)		B1	Correct structure (vertex labels and graph correct)	[3]																																									
			M1	Assigning weights to their graph (no more than 1 error or no more than 2 arcs missing/extra)																																										
			A1	Completely correct network																																										
	(ii)	Maximin	B1	cao	[1]																																									
	(iii)	<table border="1"> <thead> <tr> <th>Stage</th> <th>State</th> <th>Action</th> <th>Working</th> <th>Suboptimal maximin</th> </tr> </thead> <tbody> <tr> <td rowspan="3">2</td> <td rowspan="3">0</td> <td>0</td> <td>10</td> <td>10</td> </tr> <tr> <td>1</td> <td>10</td> <td>10</td> </tr> <tr> <td>2</td> <td>10</td> <td>10</td> </tr> <tr> <td rowspan="6">1</td> <td rowspan="3">0</td> <td>0</td> <td>$\min(6,10) = 6$</td> <td rowspan="3">8</td> </tr> <tr> <td>1</td> <td>$\min(7,10) = 7$</td> </tr> <tr> <td>2</td> <td>$\min(8,10) = 8$</td> </tr> <tr> <td rowspan="3">1</td> <td rowspan="3">0</td> <td>0</td> <td>$\min(6,10) = 6$</td> <td rowspan="3">8</td> </tr> <tr> <td>1</td> <td>$\min(7,10) = 7$</td> </tr> <tr> <td>2</td> <td>$\min(8,10) = 8$</td> </tr> <tr> <td rowspan="2">0</td> <td rowspan="2">0</td> <td>0</td> <td>$\min(9,8) = 8$</td> <td rowspan="2">8</td> </tr> <tr> <td>1</td> <td>$\min(7,8) = 7$</td> </tr> </tbody> </table> <p>Weight of heaviest truck = 8 tonnes Maximin route = (0; 0) – (1; 0) – (2; 2) – (3; 0)</p>	Stage	State	Action	Working	Suboptimal maximin	2	0	0	10	10	1	10	10	2	10	10	1	0	0	$\min(6,10) = 6$	8	1	$\min(7,10) = 7$	2	$\min(8,10) = 8$	1	0	0	$\min(6,10) = 6$	8	1	$\min(7,10) = 7$	2	$\min(8,10) = 8$	0	0	0	$\min(9,8) = 8$	8	1	$\min(7,8) = 7$	B1	Four or five columns, including 'stage', 'state' and 'action'	[3]
Stage	State	Action	Working	Suboptimal maximin																																										
2	0	0	10	10																																										
		1	10	10																																										
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		1	$\min(7,8) = 7$																																											
		B1	Stage and state columns completed correctly																																											
		B1	Action column completed correctly																																											
			M1	Min values correct for stage 1	[2]																																									
		A1	Suboptimal maximin values correct for stages 2 and 1 (follow through their network if possible, no more than 2 arcs missing/extra)																																											
			M1	Min values correct for stage 0	[2]																																									
		A1	Maximin value for stage 0 (follow through their network if possible, no more than 2 arcs missing/extra)																																											
			B1	8, cao	[2]																																									
			B1	Correct route, or in reverse																																										
Total = 13																																														

SR		Special ruling for working forwards																																																		
	(iii)	<table border="1"> <thead> <tr> <th>Stage</th> <th>State</th> <th>Action</th> <th>Working</th> <th>Suboptimal maximin</th> </tr> </thead> <tbody> <tr> <td rowspan="2">1</td> <td rowspan="2">0</td> <td>0</td> <td>9</td> <td>9</td> </tr> <tr> <td>1</td> <td>7</td> <td>7</td> </tr> <tr> <td rowspan="6">2</td> <td rowspan="3">0</td> <td>0</td> <td>$\min(9, 6) = 6$</td> <td rowspan="3">6</td> </tr> <tr> <td>1</td> <td>$\min(7, 6) = 6$</td> </tr> <tr> <td>2</td> <td>$\min(9, 7) = 7$</td> </tr> <tr> <td rowspan="3">1</td> <td rowspan="3">0</td> <td>0</td> <td>$\min(9, 7) = 7$</td> <td rowspan="3">7</td> </tr> <tr> <td>1</td> <td>$\min(7, 7) = 7$</td> </tr> <tr> <td>2</td> <td>$\min(9, 8) = 8$</td> </tr> <tr> <td rowspan="2">2</td> <td rowspan="2">0</td> <td>0</td> <td>$\min(9, 8) = 8$</td> <td rowspan="2">8</td> </tr> <tr> <td>1</td> <td>$\min(7, 8) = 7$</td> </tr> <tr> <td rowspan="3">3</td> <td rowspan="3">0</td> <td>0</td> <td>$\min(6,10) = 6$</td> <td rowspan="3">8</td> </tr> <tr> <td>1</td> <td>$\min(7,10) = 7$</td> </tr> <tr> <td>2</td> <td>$\min(8,10) = 8$</td> </tr> </tbody> </table> <p>Weight of heaviest truck = 8 tonnes Maximin route = (0; 0) – (1; 0) – (2; 2) – (3; 0)</p>	Stage	State	Action	Working	Suboptimal maximin	1	0	0	9	9	1	7	7	2	0	0	$\min(9, 6) = 6$	6	1	$\min(7, 6) = 6$	2	$\min(9, 7) = 7$	1	0	0	$\min(9, 7) = 7$	7	1	$\min(7, 7) = 7$	2	$\min(9, 8) = 8$	2	0	0	$\min(9, 8) = 8$	8	1	$\min(7, 8) = 7$	3	0	0	$\min(6,10) = 6$	8	1	$\min(7,10) = 7$	2	$\min(8,10) = 8$	B1	Four or five columns, including 'stage', 'state' and 'action'	[3]
Stage	State	Action	Working	Suboptimal maximin																																																
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		B0	No follow through from incorrect networks																																																	
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			M1	Min values correct for stage 2 and suboptimal maximin values correct for stages 1 and 2 (cao)	[2]																																															
			A0																																																	
			M1	No follow through from incorrect networks	[2]																																															
			A0	Correct min values for stage 3 and maximin value for stage 3 (cao)																																																
			B1	8, cao	[2]																																															
			B1	Correct route, or in reverse																																																
Maximum = B1 M1 M1 B1 B1 = 5 marks out of 9																																																				

5	(i)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td colspan="3" style="text-align: center;">Conan</td> <td></td> </tr> <tr> <td></td> <td style="text-align: center;">G</td> <td style="text-align: center;">H</td> <td style="text-align: center;">I</td> <td style="text-align: center;">row min</td> </tr> <tr> <td rowspan="3" style="vertical-align: middle;">Robbie</td> <td style="text-align: center;">D</td> <td style="text-align: center;">-1</td> <td style="text-align: center;">-4</td> <td style="text-align: center;">2</td> <td style="text-align: center;">-4</td> </tr> <tr> <td style="text-align: center;">E</td> <td style="text-align: center;">3</td> <td style="text-align: center;">1</td> <td style="text-align: center;">-4</td> <td style="text-align: center;">-4</td> </tr> <tr> <td style="text-align: center;">F</td> <td style="text-align: center;">1</td> <td style="text-align: center;">-1</td> <td style="text-align: center;">1</td> <td style="text-align: center;">-1</td> </tr> <tr> <td style="text-align: center;">col max</td> <td style="text-align: center;">3</td> <td style="text-align: center;">1</td> <td style="text-align: center;">2</td> <td></td> <td></td> </tr> </table> <p style="text-align: right; margin-right: 20px;">*</p>		Conan					G	H	I	row min	Robbie	D	-1	-4	2	-4	E	3	1	-4	-4	F	1	-1	1	-1	col max	3	1	2			<p>M1 Calculating row minima (cao)</p> <p>M1 Calculating column maxima (or their negatives) (cao)</p> <p>A1 Fairy or <i>F</i> (not just -1 or identifying row)</p> <p>A1 Hag or <i>H</i> (not just ± 1 or identifying column)</p> <p>B1 Follow through their play-safe for Conan Elf or <i>E</i></p>	[5]
		Conan																																		
		G	H	I	row min																															
	Robbie	D	-1	-4	2	-4																														
		E	3	1	-4	-4																														
F		1	-1	1	-1																															
col max	3	1	2																																	
(ii)	<p>Dwarf: $\frac{1}{3} [(-1) + (-4) + (2)] = -1$</p> <p>Elf: $\frac{1}{3} [(3) + (1) + (-4)] = 0$</p> <p>Fairy: $\frac{1}{3} [(1) + (-1) + (1)] = \frac{1}{3}$</p>	<p>M1 $D = -1$ or $F = \frac{1}{3}$ or -3, 0, 1</p> <p>A1 All three correct</p>	[2]																																	
(iii)	<p>Goblin: $3p + (1-p) = 1 + 2p$</p> <p>Hag: $p - (1-p) = 2p - 1$</p> <p>Imp: $-4p + (1-p) = 1 - 5p$</p> <p>$2p - 1 = 1 - 5p$</p> <p>$\Rightarrow p = \frac{2}{7}$</p>	<p>M1 Any one correct (in any form)</p> <p>A1 All three correct (in any form)</p> <p>M1 Appropriate equation seen for their expressions</p> <p>A1 $\frac{2}{7}$ or 0.286 (or better) from method seen</p>	[2]																																	
(iv)	<p>4 is added throughout the table to make all the entries non-negative</p> <p>If Conan chooses the goblin, this gives an expected value (in the new table) of $3x + 7y + 5z$</p>	<p>B1 Add 4 to remove negative values</p> <p>B1 Expected value when Conan chooses the goblin</p>	[2]																																	
(v)	<p>$z = \frac{5}{7} \Rightarrow m \leq 5.571, m \leq 3.571, m \leq 3.571$</p> <p>$\Rightarrow m \leq 3.571 (3 \frac{4}{7}) (\frac{25}{7})$</p> <p>Hence, maximum value for <i>M</i> is $3.571 - 4 = -0.429$ or $-\frac{3}{7}$</p>	<p>M1 Using $z = \frac{5}{7}$ to find a value for <i>m</i> (or implied)</p> <p>M1 Subtracting 4 from their <i>m</i> value</p> <p>A1 cao</p>	[3]																																	
Total =			16																																	

6	(i)	$\alpha = 12$ litres per second $\beta = 15$ litres per second	B1 B1	12 15	[2]
	(ii)	At least 3 litres per second must flow into A, so AC and AF cannot both have flows of 1	B1	At least 3 flows along SA	[1]
	(iii)	At most 4 litres per second can flow into B, and at least 4 must flow out, so BC and BD must have flows of 2 Hence, only 2 litres per second flows into D and at least 2 litres per second must flow out, so DE and DT must both be at their lower capacities	B1 B1	At B: flow in ≤ 4 (and flow out ≥ 4) hence given flows in BC and BD Stating that flow into D is 2 and hence given flows in DE and DT	[2]
	(iv)	Flow across $\{S, A, B, C\}, \{D, E, F, G, T\} \geq 11$ (so 10 litres per second is impossible) Minimum = 11 eg  Maximum = 12 No more than 12 can cross cut α and 12 is possible, eg augment flow shown above by 1 litre per second along SAFT	M1 A1 M1 A1 M1 A1	Or any equivalent reasoning (eg flow through C) Wholly convincing argument 11 Showing that 11 is possible (check C) 12 Showing that 12 is possible but 13 is not	[2] [2] [2]
	(v)	 e.g. 	B1 M1 A1	A correct reduced network (vertex E and all arcs incident on E deleted), including arc capacities Or putting E_{in} and E_{out} with a capacity of 0 between them Or giving CE, EG and DE upper and lower capacities of 0 On same diagram or a new diagram SA = 3, SC = 2, SB = 4, BC = 2 and BT = 2 (and nothing through E, if shown) A valid flow of 9 litres per second through the network	[3]
Total =					14