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MATHEMATICS

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MARK SCHEME

Maximum Mark: 75

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This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Commence division and reach a partial quotient $x^2 + kx$	M1
	Obtain quotient $x^2 - 2x + 5$	A1
	Obtain remainder $-12x + 5$	A1
		3

Question	Answer	Marks
2	Plot the four points and draw straight line	B1
	State or imply that $\ln y = \ln C + x \ln a$	B1
	Carry out a completely correct method for finding $\ln C$ or $\ln a$	M1
	Obtain answer $C = 3.7$	A1
	Obtain answer $a = 1.5$	A1
		5

Question	Answer	Marks
3(i)	Calculate value of a relevant expression or expressions at $x = 2$ and $x = 3$	M1
	Complete the argument correctly with correct calculated values	A1
		2
3(ii)	Use an iterative formula correctly at least once	M1
	Show that (B) fails to converge	A1
	Using (A) , obtain final answer 2.43	A1
	Show sufficient iterations to justify 2.43 to 2 d.p., or show there is a sign change in (2.425, 2.435)	A1
		4

Question	Answer	Marks
4(i)	Use correct $\tan(A \pm B)$ formula and express the LHS in terms of $\tan x$	M1
	Using $\tan 45^\circ = 1$ express LHS as a single fraction	A1
	Use Pythagoras or correct double angle formula	M1
	Obtain given answer	A1
		4
4(ii)	Show correct sketch for one branch	B1
	Both branches correct and nothing else seen in the interval	B1
	Show asymptote at $x = 45^\circ$	B1
		3

Question	Answer	Marks
5(i)	State or imply $y^3 + 3xy^2 \frac{dy}{dx}$ as derivative of xy^3	B1
	State or imply $4y^3 \frac{dy}{dx}$ as derivative of y^4	B1
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	M1
	Obtain the given answer	A1
		4
5(ii)	Equate numerator to zero	*M1
	Obtain $y = -2x$, or equivalent	A1
	Obtain an equation in x or y	DM1
	Obtain final answer $x = -1, y = 2$ and $x = 1, y = -2$	A1
		4

Question	Answer	Marks
6	Separate variables correctly and attempt integration of one side	B1
	Obtain term $\tan y$, or equivalent	B1
	Obtain term of the form $k \ln \cos x$, or equivalent	M1
	Obtain term $-4 \ln \cos x$, or equivalent	A1
	Use $x = 0$ and $y = \frac{1}{4}\pi$ in solution containing $a \tan y$ and $b \ln \cos x$ to evaluate a constant, or as limits	M1
	Obtain correct solution in any form, e.g. $\tan y = 4 \ln \sec x + 1$	A1
	Substitute $y = \frac{1}{3}\pi$ in solution containing terms $a \tan y$ and $b \ln \cos x$, and use correct method to find x	M1
	Obtain answer $x = 0.587$	A1
		8

Question	Answer	Marks
7(a)	Square $x + iy$ and equate real and imaginary parts to 8 and -15	M1
	Obtain $x^2 - y^2 = 8$ and $2xy = -15$	A1
	Eliminate one unknown and find a horizontal equation in the other	M1
	Obtain $4x^4 - 32x^2 - 225 = 0$ or $4y^4 + 32y^2 - 225 = 0$, or three term equivalent	A1
	Obtain answers $\pm \frac{1}{\sqrt{2}}(5 - 3i)$ or equivalent	A1
		5
7(b)	Show a circle with centre $2 + i$ in a relatively correct position	B1
	Show a circle with radius 2 and centre not at the origin	B1
	Show line through i at an angle of $\frac{1}{4}\pi$ to the real axis	B1
	Shade the correct region	B1
		4

Question	Answer	Marks
8(i)	Use a relevant method to determine a constant	M1
	Obtain one of the values $A = 2, B = 2, C = -1$	A1
	Obtain a second value	A1
	Obtain the third value	A1
		4
8(ii)	Integrate and obtain terms $2x + 2\ln(x+2) - \frac{1}{2}\ln(2x-1)$ (deduct B1 for each error or omission) [The FT is on A, B and C]	B2 FT
	Substitute limits correctly in an integral containing terms $a\ln(x+2)$ and $b\ln(2x-1)$, where $ab \neq 0$	*M1
	Use at least one law of logarithms correctly	DM1
	Obtain the given answer after full and correct working	A1
		5

Question	Answer	Marks
9(i)	Use correct product or quotient rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to zero and obtain a 3 term quadratic equation in x	M1
	Obtain answers $x = 2 \pm \sqrt{3}$	A1
		4
9(ii)	Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l \int xe^{-\frac{1}{2}x} dx$	*M1
	Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4 \int xe^{-\frac{1}{2}x} dx$, or equivalent	A1
	Complete the integration and obtain $(-18 - 8x - 2x^2)e^{-\frac{1}{2}x}$, or equivalent	A1
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	DM1
	Obtain the given answer	A1
		5

Question	Answer	Marks
10(i)	Equate at least two pairs of components of general points on l and m and solve for λ or for μ	M1
	Obtain correct answer for λ or μ , e.g. $\lambda = 3$ or $\mu = -2$; $\lambda = 0$ or $\mu = -\frac{1}{2}$; or $\lambda = \frac{3}{2}$ or $\mu = -\frac{7}{2}$	A1
	Verify that not all three pairs of equations are satisfied and that the lines fail to intersect	A1
		3
10(ii)	Carry out correct process for evaluating scalar product of direction vectors for l and m	*M1
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	DM1
	Obtain answer 45° or $\frac{1}{4}\pi$ (0.785) radians	A1
		3
10(iii)	<i>EITHER:</i> Use scalar product to obtain a relevant equation in a , b and c , e.g. $-a + b + 4c = 0$	B1
	Obtain a second equation, e.g. $2a + b - 2c = 0$ and solve for one ratio, e.g. $a : b$	M1
	Obtain $a : b : c = 2 : -2 : 1$, or equivalent	A1
	Substitute $(3, -2, -1)$ and values of a , b and c in general equation and find d	M1
	Obtain answer $2x - 2y + z = 9$, or equivalent	A1
	<i>OR1:</i> Attempt to calculate vector product of relevant vectors, e.g. $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	(M1
	Obtain two correct components	A1
	Obtain correct answer, e.g. $-6\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	A1
	Substitute $(3, -2, -1)$ in $-6x + 6y - 3z = d$, or equivalent, and find d	M1
	Obtain answer $-2x + 2y - z = -9$, or equivalent	A1)
	<i>OR2:</i> Using the relevant point and relevant vectors, form a 2-parameter equation for the plane	(M1
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	A1
	State three correct equations in x , y , z , λ and μ	A1
	Eliminate λ and μ	M1

Question	Answer	Marks
	Obtain answer $2x - 2y + z = 9$, or equivalent	A1)
	<i>OR3:</i> Using the relevant point and relevant vectors, form a determinant equation for the plane	(M1
	State a correct equation, e.g. $\begin{vmatrix} x-3 & y+2 & z+1 \\ -1 & 1 & 4 \\ 2 & 1 & -2 \end{vmatrix} = 0$	A1
	Attempt to expand the determinant	M1
	Obtain two correct cofactors	A1
	Obtain answer $-2x + 2y - z = -9$, or equivalent	A1)
		5