

Edexcel Maths Core 3

Mark Scheme Pack

2006-2013

GCE

Edexcel GCE

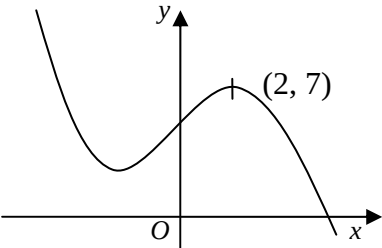
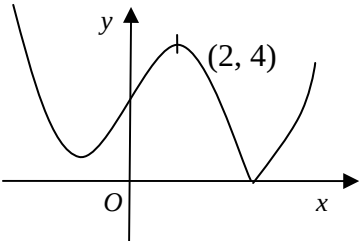
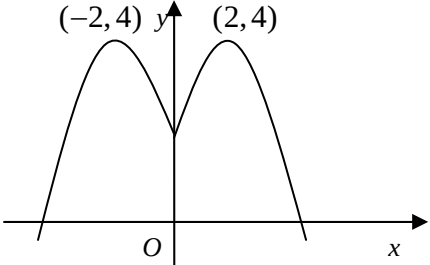
Core Mathematics C3 (6665)

January 2006

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Mark Scheme (Results)

January 2006
6665 Core Mathematics C3
Mark Scheme

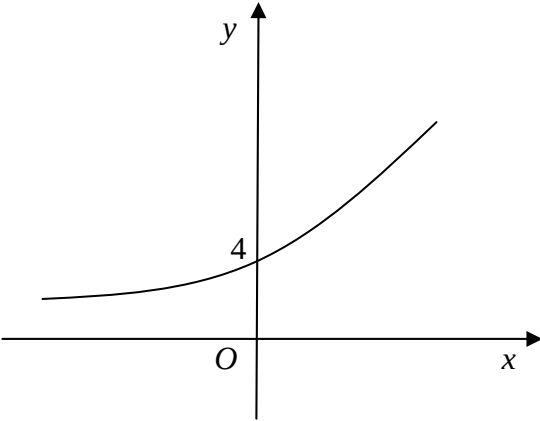
Question Number	Scheme	Marks
1.	(a)  (b)  (c) 	Shape unchanged Point B1 B1 (2) Shape Point B1 B1 (2) Shape (2, 4) (-2, 4) B1 B1 (3) [7]

Question Number	Scheme	Marks
2.	$x^2 - x - 2 = (x - 2)(x + 1)$ At any stage $\frac{2x^2 + 3x}{(2x + 3)(x - 2)} = \frac{x(2x + 3)}{(2x + 3)(x - 2)} = \frac{x}{x - 2}$ $\frac{2x^2 + 3x}{(2x + 3)(x - 2)} - \frac{6}{x^2 - x - 2} = \frac{x(x + 1) - 6}{(x - 2)(x + 1)}$ $= \frac{x^2 + x - 6}{(x - 2)(x + 1)}$ $= \frac{(x + 3)(x - 2)}{(x - 2)(x + 1)}$ $= \frac{x + 3}{x + 1}$ Alternative method $x^2 - x - 2 = (x - 2)(x + 1)$ At any stage (2x + 3) appearing as a factor of the numerator at any stage $\frac{2x^2 + 3x}{(2x + 3)(x - 2)} - \frac{6}{(x - 2)(x + 1)} = \frac{(2x^2 + 3x)(x + 1) - 6(2x + 3)}{(2x + 3)(x - 2)(x + 1)}$ $= \frac{2x^3 + 5x^2 - 9x - 18}{(2x + 3)(x - 2)(x + 1)}$ can be implied $= \frac{(x - 2)(2x^2 + 9x + 9)}{(2x + 3)(x - 2)(x + 1)} \quad \text{or} \quad \frac{(2x + 3)(x^2 + x - 6)}{(2x + 3)(x - 2)(x + 1)} \quad \text{or} \quad \frac{(x + 3)(2x^2 - x - 6)}{(2x + 3)(x - 2)(x + 1)}$ Any one linear factor × quadratic $= \frac{(2x + 3)(x - 2)(x + 3)}{(2x + 3)(x - 2)(x + 1)}$ Complete factors $= \frac{x + 3}{x + 1}$	B1 B1 M1 A1 M1 A1 A1 (7) [7] B1 B1 M1 A1 M1 A1 A1 (7)

Question Number	Scheme	Marks
3.	$\frac{dy}{dx} = \frac{1}{x} \quad \text{accept } \frac{3}{3x}$ $\text{At } x = 3, \frac{dy}{dx} = \frac{1}{3} \Rightarrow m' = -3 \quad \text{Use of } mm' = -1$ $y - \ln 1 = -3(x - 3)$ $y = -3x + 9 \quad \text{Accept } y = 9 - 3x$ $\frac{dy}{dx} = \frac{1}{3x} \text{ leading to } y = -9x + 27 \text{ is a maximum of } M1 \ A0 \ M1 \ M1 \ A0 = 3/5$	M1 A1 M1 M1 A1 (5) [5]
4.	(a) (i) $\frac{d}{dx}(e^{3x+2}) = 3e^{3x+2} \quad (\text{or } 3e^2 e^{3x}) \quad \text{At any stage}$ $\frac{dy}{dx} = 3x^2 e^{3x+2} + 2x e^{3x+2} \quad \text{Or equivalent}$ (ii) $\frac{d}{dx}(\cos(2x^3)) = -6x^2 \sin(2x^3) \quad \text{At any stage}$ $\frac{dy}{dx} = \frac{-18x^3 \sin(2x^3) - 3\cos(2x^3)}{9x^2}$ Alternatively using the product rule for second M1 A1 $y = (3x)^{-1} \cos(2x^3)$ $\frac{dy}{dx} = -3(3x)^{-2} \cos(2x^3) - 6x^2 (3x)^{-1} \sin(2x^3)$ Accept equivalent unsimplified forms (b) $1 = 8 \cos(2y + 6) \frac{dy}{dx} \quad \text{or} \quad \frac{dx}{dy} = 8 \cos(2y + 6)$ $\frac{dy}{dx} = \frac{1}{8 \cos(2y + 6)}$ $\frac{dy}{dx} = \frac{1}{8 \cos\left(\arcsin\left(\frac{x}{4}\right)\right)} \quad \left(= (\pm) \frac{1}{2\sqrt{(16 - x^2)}} \right)$	B1 M1 A1+A1 (4) M1 A1 M1 A1 (4) M1 M1 A1 M1 A1 (5) [13]

Question Number	Scheme	Marks
5.	(a) $2x^2 - 1 - \frac{4}{x} = 0$ Dividing equation by x M1 $x^2 = \frac{1}{2} + \frac{4}{2x}$ Obtaining $x^2 = \dots$ M1 $x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)} *$ cso A1 (3) (b) $x_1 = 1.41, x_2 = 1.39, x_3 = 1.39$ B1, B1, B1 If answers given to more than 2 dp, penalise first time then accept awrt above. (3) (c) Choosing (1.3915, 1.3925) or a tighter interval M1 $f(1.3915) \approx -3 \times 10^{-3}, f(1.3925) \approx 7 \times 10^{-3}$ Both, awrt A1 Change of sign (and continuity) $\Rightarrow \alpha \in (1.3915, 1.3925)$ $\Rightarrow \alpha = 1.392$ to 3 decimal places * cso A1 (3) [9]	
6.	(a) $R \cos \alpha = 12, R \sin \alpha = 4$ $R = \sqrt{(12^2 + 4^2)} = \sqrt{160}$ Accept if just written down, awrt 12.6 M1 A1 $\tan \alpha = \frac{4}{12}, \Rightarrow \alpha \approx 18.43^\circ$ awrt 18.4° M1, A1 (4) (b) $\cos(x + \text{their } \alpha) = \frac{7}{\text{their } R} (\approx 0.5534)$ M1 $x + \text{their } \alpha = 56.4^\circ$ awrt 56° A1 $= \dots, 303.6^\circ$ $360^\circ - \text{their principal value}$ M1 $x = 38.0^\circ, 285.2^\circ$ Ignore solutions out of range A1, A1 (5) If answers given to more than 1 dp, penalise first time then accept awrt above. (c)(i) minimum value is $-\sqrt{160}$ ft their R B1ft (ii) $\cos(x + \text{their } \alpha) = -1$ M1 $x \approx 161.57^\circ$ cao A1 (3) [12]	

Question Number	Scheme	Marks
7.	(a) (i) Use of $\cos 2x = \cos^2 x - \sin^2 x$ in an attempt to prove the identity. $\frac{\cos 2x}{\cos x + \sin x} = \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x} = \cos x - \sin x \quad *$ cso	M1 A1 (2)
	(ii) Use of $\cos 2x = 2\cos^2 x - 1$ in an attempt to prove the identity. Use of $\sin 2x = 2\sin x \cos x$ in an attempt to prove the identity. $\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(2\cos^2 x - 1 - 2\sin x \cos x) = \cos^2 x - \cos x \sin x - \frac{1}{2} \quad *$ cso	M1 M1 A1 (3)
	(b) $\cos \theta (\cos \theta - \sin \theta) = \frac{1}{2}$ Using (a)(i)	M1
	$\cos^2 \theta - \cos \theta \sin \theta - \frac{1}{2} = 0$	
	$\frac{1}{2}(\cos 2\theta - \sin 2\theta) = 0$ Using (a)(ii)	M1
	$\cos 2\theta = \sin 2\theta \quad *$	A1 (3)
	(c) $\tan 2\theta = 1$	M1
	$2\theta = \frac{\pi}{4}, \left(\frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}\right)$ any one correct value of 2θ	A1
	$\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$ Obtaining at least 2 solutions in range	M1
	The 4 correct solutions If decimals (0.393, 1.963, 3.534, 5.105) or degrees (22.5°, 112.5°, 202.5°, 292.5°) are given, but all 4 solutions are found, penalise one A mark only. Ignore solutions out of range.	A1 (4) [12]

Question Number	Scheme	Marks
8.	(a) $\begin{aligned} gf(x) &= e^{2(2x+\ln 2)} \\ &= e^{4x} e^{2\ln 2} \\ &= e^{4x} e^{\ln 4} \\ &= 4e^{4x} \end{aligned}$ (Hence $gf : x \mapsto 4e^{4x}, \quad x \in \mathbb{R}$)	M1 M1 M1 A1 (4) Give mark at this point, cso
	(b)  Shape and point	B1 (1)
	(c) Range is $\mathbb{R}_+$	Accept $gf(x) > 0, y > 0$ B1 (1)
	(d) $\begin{aligned} \frac{d}{dx}[gf(x)] &= 16e^{4x} \\ e^{4x} &= \frac{3}{16} \\ 4x &= \ln \frac{3}{16} \\ x &\approx -0.418 \end{aligned}$	M1 A1 M1 A1 (4) [10]

Mark Scheme (Results)

January 2007

GCE

GCE Mathematics

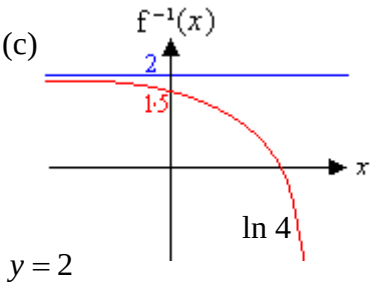
Core Mathematics C3 (6665)

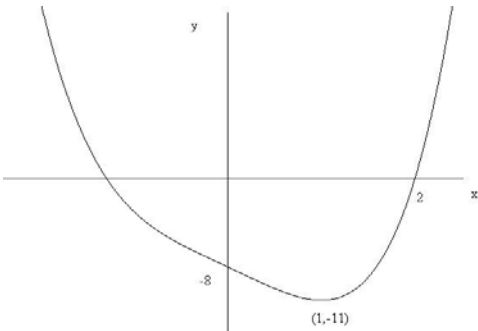
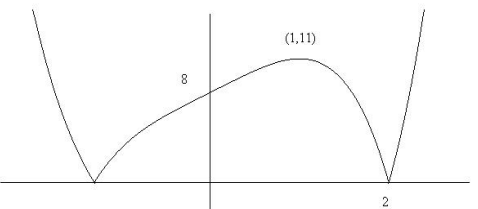
January 2007
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $\sin 3\theta = \sin(2\theta + \theta) = \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$ $= 2 \sin \theta \cos^2 \theta + (1 - 2 \sin^2 \theta) \sin \theta$ $= 2 \sin \theta - 2 \sin^3 \theta + \sin \theta - 2 \sin^3 \theta$ $= 3 \sin \theta - 4 \sin^3 \theta$ * (b) $\sin 3\theta = 3 \times \frac{\sqrt{3}}{4} - 4 \left(\frac{\sqrt{3}}{4} \right)^3 = \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \frac{9\sqrt{3}}{16}$ equivalent	B1 B1 B1 M1 A1 (5) or exact M1 A1 (2) [7]
2.	(a) $f(x) = \frac{(x+2)^2, -3(x+2)+3}{(x+2)^2}$ $= \frac{x^2 + 4x + 4 - 3x - 6 + 3}{(x+2)^2} = \frac{x^2 + x + 1}{(x+2)^2}$ * (b) $x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}, > 0$ for all values of x. (c) $f(x) = \frac{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}{(x+2)^2}$ Numerator is positive from (b) $x \neq -2 \Rightarrow (x+2)^2 > 0$ (Denominator is positive) Hence $f(x) > 0$	M1 A1, A1 A1 (4) M1 A1, A1 (3) B1 (1) [8]
	Alternative to (b) $\frac{d}{dx}(x^2 + x + 1) = 2x + 1 = 0 \Rightarrow x = -\frac{1}{2} \Rightarrow x^2 + x + 1 = \frac{3}{4}$ A parabola with positive coefficient of x^2 has a minimum $\Rightarrow x^2 + x + 1 > 0$ Accept equivalent arguments	M1 A1 A1 (3)

Question Number	Scheme	Marks
3.	(a) $y = \frac{\pi}{4} \Rightarrow x = 2 \sin \frac{\pi}{4} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} \Rightarrow P \in C$ Accept equivalent (reversed) arguments. In any method it must be clear that $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ or exact equivalent is used. (b) $\frac{dx}{dy} = 2 \cos y \quad \text{or} \quad 1 = 2 \cos y \frac{dy}{dx}$ $\frac{dy}{dx} = \frac{1}{2 \cos y}$ May be awarded after substitution $y = \frac{\pi}{4} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{2}} \quad *$ CSO (c) $m' = -\sqrt{2}$ $y - \frac{\pi}{4} = -\sqrt{2}(x - \sqrt{2})$ $y = -\sqrt{2}x + 2 + \frac{\pi}{4}$	B1 (1) M1 A1 M1 A1 (4) B1 M1 A1 A1 (4) [9]
4.	(i) $\frac{dy}{dx} = \frac{(9+x^2)-x(2x)}{(9+x^2)^2} \left(= \frac{9-x^2}{(9+x^2)^2} \right)$ $\frac{dy}{dx} = 0 \Rightarrow 9-x^2 = 0 \Rightarrow x = \pm 3$ $\left(3, \frac{1}{6}\right), \left(-3, -\frac{1}{6}\right)$ Final two A marks depend on second M only (ii) $\frac{dy}{dx} = \frac{3}{2}(1+e^{2x})^{\frac{1}{2}} \times 2e^{2x}$ $x = \frac{1}{2} \ln 3 \Rightarrow \frac{dy}{dx} = \frac{3}{2}(1+e^{\ln 3})^{\frac{1}{2}} \times 2e^{\ln 3} = 3 \times 4^{\frac{1}{2}} \times 3 = 18$	M1 A1 M1 A1 A1, A1 (6) M1 A1 A1 M1 A1 (5) [11]

Question Number	Scheme	Marks
5.	(a) $R^2 = (\sqrt{3})^2 + 1^2 \Rightarrow R = 2$ $\tan \alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$ accept awrt 1.05 (b) $\sin(x + \text{their } \alpha) = \frac{1}{2}$ $x + \text{their } \alpha = \frac{\pi}{6} \left(\frac{5\pi}{6}, \frac{13\pi}{6} \right)$ $x = \frac{\pi}{2}, \frac{11\pi}{6}$ accept awrt 1.57, 5.76 The use of degrees loses only one mark in this question. Penalise the first time it occurs in an answer and then ignore.	M1 A1 M1 A1 (4) M1 A1 M1 A1 (4) [8]

Question Number	Scheme	Marks
6.	(a) $y = \ln(4 - 2x)$ $e^y = 4 - 2x$ leading to $x = 2 - \frac{1}{2}e^y$ Changing subject and removing $\ln$ $y = 2 - \frac{1}{2}e^x \Rightarrow f^{-1} \mapsto 2 - \frac{1}{2}e^x$ * Domain of f^{-1} is (b) Range of f^{-1} is $f^{-1}(x) < 2$ (and $f^{-1}(x) \in \mathbb{R}$) (c)  (d) $x_1 \approx -0.3704, x_2 \approx -0.3452$ If more than 4 dp given in this part a maximum on one mark is lost. Penalise on the first occasion. (e) $x_3 = -0.354\ 030\ 19 \dots$ $x_4 = -0.350\ 926\ 88 \dots$ $x_5 = -0.352\ 017\ 61 \dots$ $x_6 = -0.351\ 633\ 86 \dots$ $k \approx -0.352$ Alternative to (e) $k \approx -0.352$ Let $g(x) = x + \frac{1}{2}e^x$ $g(-0.3515) \approx +0.0003, g(-0.3525) \approx -0.001$ Change of sign (and continuity) $\Rightarrow k \in (-0.3525, -0.3515)$ $\Rightarrow k = -0.352$ (to 3 dp)	M1 A1 A1 B1 (4) B1 (1) Shape 1.5 $\ln 4$ B1 B1 B1 (4) cao B1, B1 (2) M1 A1 (2) [13] Found in any way M1 A1 (2)

Question Number	Scheme	Marks
7.	(a) $f(-2) = 16 + 8 - 8 (=16) > 0$ $f(-1) = 1 + 4 - 8 (= -3) < 0$ Change of sign (and continuity) $\Rightarrow$ root in interval $(-2, -1)$ ft their calculation as long as there is a sign change (b) $\frac{dy}{dx} = 4x^3 - 4 = 0 \Rightarrow x = 1$ Turning point is $(1, -11)$ (c) $a = 2, b = 4, c = 4$ (d)  (e) 	B1 B1 B1ft (3) M1 A1 A1 (3) B1 B1 B1 (3) Shape ft their turning point in correct quadrant only 2 and -8 B1 (3) Shape B1 (1) [13]

Question Number	Scheme	Marks
8.	(i) $\sec^2 x - \operatorname{cosec}^2 x = (1 + \tan^2 x) - (1 + \cot^2 x)$ $= \tan^2 x - \cot^2 x$ * (ii)(a) $y = \arccos x \Rightarrow x = \cos y$ $x = \sin\left(\frac{\pi}{2} - y\right) \Rightarrow \arcsin x = \frac{\pi}{2} - y$ Accept $\arcsin x = \arcsin \cos y$ (b) $\arccos x + \arcsin x = y + \frac{\pi}{2} - y = \frac{\pi}{2}$	M1 A1 A1 (3) B1 B1 (2) B1 (1) [6]
	<i>Alternatives for (i)</i> $\sec^2 x - \tan^2 x = 1 = \operatorname{cosec}^2 x - \cot^2 x$ Rearranging $\sec^2 x - \operatorname{cosec}^2 x = \tan^2 x - \cot^2 x$ * CSO $\left(\text{LHS} = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x} \right)$ $\text{RHS} = \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x} = \frac{\sin^4 x - \cos^4 x}{\cos^2 x \sin^2 x} = \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{\cos^2 x \sin^2 x}$ $= \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}$ $= \text{LHS} *$ or equivalent	M1 A1 A1 (3) M1 A1 A1 (3)

Mark Scheme (Results)

January 2008

GCE

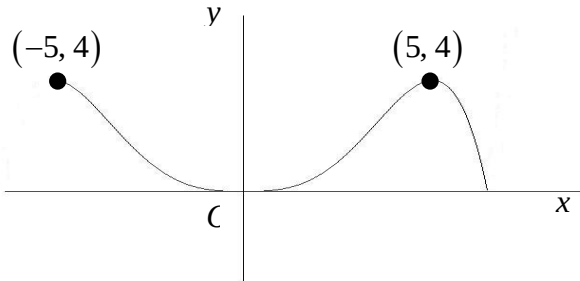
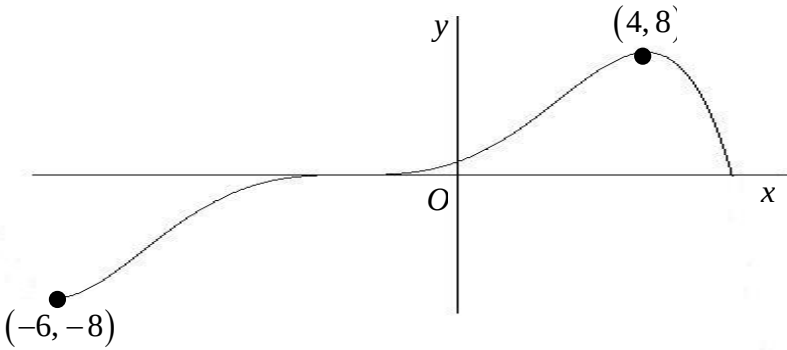
GCE Mathematics (6665/01)

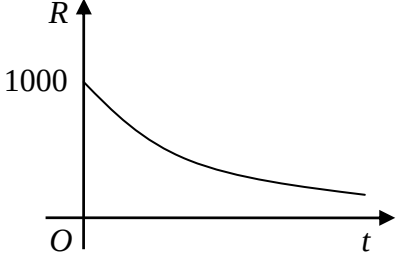


January 2008
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	$ \begin{array}{r} x^2 - 1 \quad \begin{array}{r} \begin{array}{r} 2x^2 \quad -1 \\ \hline 2x^4 \quad -3x^2 + x + 1 \\ 2x^4 \quad -2x^2 \\ \hline -x^2 + x + 1 \\ -x^2 \quad +1 \\ \hline x \end{array} \end{array} \end{array} $ $a = 2$ stated or implied $c = -1$ stated or implied $2x^2 - 1 + \frac{x}{x^2 - 1}$ $a = 2, b = 0, c = -1, d = 1, e = 0$ $d = 1$ and $b = 0, e = 0$ stated or implied	M1 A1 A1 A1 [4]
2.	(a) $\frac{dy}{dx} = 2e^{2x} \tan x + e^{2x} \sec^2 x$ $\frac{dy}{dx} = 0 \Rightarrow 2e^{2x} \tan x + e^{2x} \sec^2 x = 0$ $2 \tan x + 1 + \tan^2 x = 0$ $(\tan x + 1)^2 = 0$ $\tan x = -1 \quad *$ cso (b) $\left(\frac{dy}{dx} \right)_0 = 1$ Equation of tangent at $(0, 0)$ is $y = x$	M1 A1+A1 M1 A1 A1 (6) M1 A1 (2) [8]

Question Number	Scheme	Marks
3.	(a) $f(2) = 0.38 \dots$ $f(3) = -0.39 \dots$ Change of sign (and continuity) $\Rightarrow$ root in $(2, 3)$ *	M1 A1 (2)
	(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$ $x_2 \approx 2.50498$ $x_3 \approx 2.50518$	M1 A1 A1 (3)
	(c) Selecting $[2.5045, 2.5055]$, or appropriate tighter range, and evaluating at both ends. $f(2.5045) \approx 6 \times 10^{-4}$ $f(2.5055) \approx -2 \times 10^{-4}$ Change of sign (and continuity) $\Rightarrow$ root $\in (2.5045, 2.5055)$ $\Rightarrow$ root = 2.505 to 3 dp *	M1 A1 (2)
	Note: The root, correct to 5 dp, is 2.50524	[7]

Question Number	Scheme	Marks
4.	(a)  Shape (5, 4) B1 (-5, 4) B1 (3) (b) For the purpose of marking this paper, the graph is identical to (a) Shape (5, 4) B1 (-5, 4) B1 (3) (c)  General shape – unchanged B1 Translation to left B1 (4, 8) B1 (-6, -8) B1 (4) In all parts of this question ignore any drawing outside the domains shown in the diagrams above.	[10]

Question Number	Scheme	Marks
5.	(a) 1000 (b) $1000e^{-5730c} = 500$ $e^{-5730c} = \frac{1}{2}$ $-5730c = \ln \frac{1}{2}$ $c = 0.000121$ (c) $R = 1000e^{-22920c} = 62.5$ (d) 	B1 (1) M1 A1 M1 cao A1 (4) Accept 62-63 M1 A1 (2) Shape 1000 B1 B1 (2) [9]

Question Number	Scheme	Marks
6.	(a) $\cos(2x+x) = \cos 2x \cos x - \sin 2x \sin x$ $= (2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x$ $= (2\cos^2 x - 1)\cos x - 2(1 - \cos^2 x)\cos x$ any correct expression $= 4\cos^3 x - 3\cos x$ (b)(i) $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = \frac{\cos^2 x + (1+\sin x)^2}{(1+\sin x)\cos x}$ $= \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)\cos x}$ $= \frac{2(1+\sin x)}{(1+\sin x)\cos x}$ $= \frac{2}{\cos x} = 2\sec x$ * (c) $\sec x = 2$ or $\cos x = \frac{1}{2}$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$ accept awrt 1.05, 5.24	M1 M1 A1 A1 (4) M1 A1 M1 A1 (4) M1 A1, A1 (3) [11]
7.	(a) $\frac{dy}{dx} = 6\cos 2x - 8\sin 2x$ $\left(\frac{dy}{dx}\right)_0 = 6$ $y - 4 = -\frac{1}{6}x$ or equivalent (b) $R = \sqrt{3^2 + 4^2} = 5$ $\tan \alpha = \frac{4}{3}, \alpha \approx 0.927$ awrt 0.927 (c) $\sin(2x + \text{their } \alpha) = 0$ $x = -2.03, -0.46, 1.11, 2.68$ First A1 any correct solution; second A1 a second correct solution; third A1 all four correct and to the specified accuracy or better. Ignore the y-coordinate.	M1 A1 B1 M1 A1 (5) M1 A1 M1 A1 (4) M1 A1 A1 A1 (4) [13]

Question Number	Scheme	Marks
8.	$(a) \quad x = 1 - 2y^3 \Rightarrow y = \left(\frac{1-x}{2}\right)^{\frac{1}{3}} \text{ or } \sqrt[3]{\frac{1-x}{2}}$	M1 A1 (2)
	$f^{-1} : x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$	Ignore domain
	$(b) \quad gf(x) = \frac{3}{1-2x^3} - 4$	M1 A1
	$= \frac{3-4(1-2x^3)}{1-2x^3}$	M1
	$= \frac{8x^3-1}{1-2x^3} *$	CSO A1 (4)
	$gf : x \mapsto \frac{8x^3-1}{1-2x^3}$	Ignore domain
	$(c) \quad 8x^3 - 1 = 0$	Attempting solution of numerator = 0 M1
	$x = \frac{1}{2}$	Correct answer and no additional answers A1 (2)
	$(d) \quad \frac{dy}{dx} = \frac{(1-2x^3) \times 24x^2 + (8x^3-1) \times 6x^2}{(1-2x^3)^2}$	M1 A1
	$= \frac{18x^2}{(1-2x^3)^2}$	A1
	Solving their numerator = 0 and substituting to find y.	M1
	$x = 0, y = -1$	A1 (5)
		[13]

Mark Scheme (Final)

January 2009

GCE

GCE Core Mathematics C3 (6665/01)



General Marking Guidance

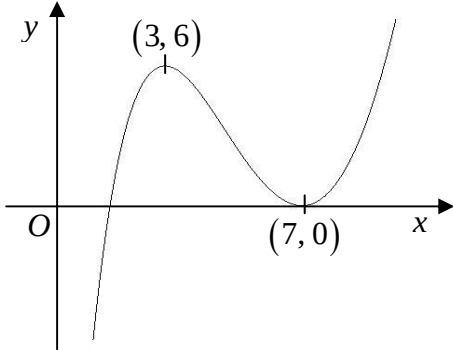
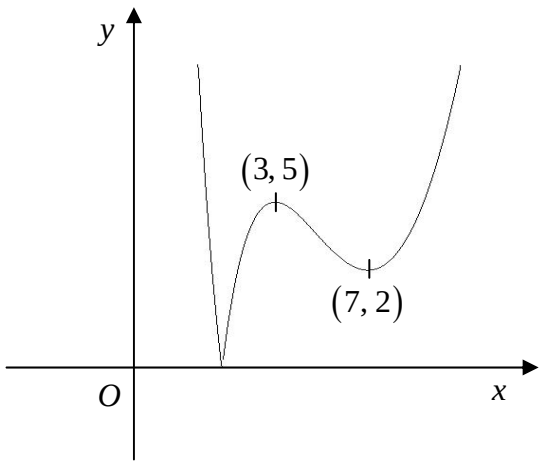
- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

January 2009
6665 Core C3
Mark Scheme

Version for Online Standardisation

Question Number	Scheme	Marks
1.	(a) $\frac{d}{dx}(\sqrt{(5x-1)}) = \frac{d}{dx}((5x-1)^{\frac{1}{2}})$ $= 5 \times \frac{1}{2} (5x-1)^{-\frac{1}{2}}$ $\frac{dy}{dx} = 2x\sqrt{(5x-1)} + \frac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$ At $x = 2$, $\frac{dy}{dx} = 4\sqrt{9} + \frac{10}{\sqrt{9}} = 12 + \frac{10}{3}$ $= \frac{46}{3}$ Accept awrt 15.3	M1 A1 M1 A1ft M1 A1 (6)
	(b) $\frac{d}{dx}\left(\frac{\sin 2x}{x^2}\right) = \frac{2x^2 \cos 2x - 2x \sin 2x}{x^4}$	M1 $\frac{A1+A1}{A1}$ (4) [10]
	<i>Alternative to (b)</i> $\frac{d}{dx}(\sin 2x \times x^{-2}) = 2 \cos 2x \times x^{-2} + \sin 2x \times (-2)x^{-3}$ $= 2x^{-2} \cos 2x - 2x^{-3} \sin 2x \quad \left(= \frac{2 \cos 2x}{x^2} - \frac{2 \sin 2x}{x^3} \right)$	M1 A1 + A1 A1 (4)

Question Number	Scheme	Marks
2.	(a) $\frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3} = \frac{2x+2}{(x-3)(x+1)} - \frac{x+1}{x-3}$ $= \frac{2x+2-(x+1)(x+1)}{(x-3)(x+1)}$ $= \frac{(x+1)(1-x)}{(x-3)(x+1)}$ $= \frac{1-x}{x-3}$ Accept $-\frac{x-1}{x-3}, \frac{x-1}{3-x}$ M1 A1 M1 A1 (4) (b) $\frac{d}{dx}\left(\frac{1-x}{x-3}\right) = \frac{(x-3)(-1) - (1-x)1}{(x-3)^2}$ $= \frac{-x+3-1+x}{(x-3)^2} = \frac{2}{(x-3)^2} *$ CSO M1 A1 A1 (3) [7]	
	Alternative to (a) $\frac{2x+2}{x^2-2x-3} = \frac{2(x+1)}{(x-3)(x+1)} = \frac{2}{x-3}$ $\frac{2}{x-3} - \frac{x+1}{x-3} = \frac{2-(x+1)}{x-3}$ $= \frac{1-x}{x-3}$ Alternatives to (b) ① $f(x) = \frac{1-x}{x-3} = -1 - \frac{2}{x-3} = -1 - 2(x-3)^{-1}$ $f'(x) = (-1)(-2)(x-3)^{-2}$ $= \frac{2}{(x-3)^2} *$ CSO M1 A1 A1 (3) ② $f(x) = (1-x)(x-3)^{-1}$ $f'(x) = (-1)(x-3) + (1-x)(-1)(x-3)^{-2}$ $= -\frac{1}{x-3} - \frac{1-x}{(x-3)^2} = \frac{-(x-3)-(1-x)}{(x-3)^2}$ $= \frac{2}{(x-3)^2} *$ M1 A1 A1 (3)	

Question Number	Scheme		Marks
3.	(a)		Shape (3, 6) (7, 0) B1 B1 B1 (3)
	(b)		Shape (3, 5) (7, 2) B1 B1 B1 (3) [6]

Question Number	Scheme	Marks
4.	$x = \cos(2y + \pi)$ $\frac{dx}{dy} = -2 \sin(2y + \pi)$ $\frac{dy}{dx} = -\frac{1}{2 \sin(2y + \pi)}$ At $y = \frac{\pi}{4}$, $\frac{dy}{dx} = -\frac{1}{2 \sin \frac{3\pi}{2}} = \frac{1}{2}$ $y - \frac{\pi}{4} = \frac{1}{2}x$ $y = \frac{1}{2}x + \frac{\pi}{4}$ Follow through their $\frac{dx}{dy}$ before or after substitution	M1 A1 A1ft B1 M1 A1 (6) [6]

Question Number	Scheme	Marks
5.	(a) $g(x) \geq 1$	B1 (1)
	(b) $fg(x) = f(e^{x^2}) = 3e^{x^2} + \ln e^{x^2}$ $= x^2 + 3e^{x^2} \quad *$ $(fg: x \mapsto x^2 + 3e^{x^2})$	M1 A1 (2)
	(c) $fg(x) \geq 3$	B1 (1)
	(d) $\frac{d}{dx}(x^2 + 3e^{x^2}) = 2x + 6xe^{x^2}$ $2x + 6xe^{x^2} = x^2 e^{x^2} + 2x$ $e^{x^2}(6x - x^2) = 0$ $e^{x^2} \neq 0, \quad 6x - x^2 = 0$ $x = 0, 6$	M1 A1 M1 A1 A1 A1 (6) [10]

Question Number	Scheme	Marks
6.	(a)(i) $\sin 3\theta = \sin(2\theta + \theta)$ $= \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$ $= 2 \sin \theta \cos \theta \cdot \cos \theta + (1 - 2 \sin^2 \theta) \sin \theta$ $= 2 \sin \theta (1 - \sin^2 \theta) + \sin \theta - 2 \sin^3 \theta$ $= 3 \sin \theta - 4 \sin^3 \theta$ * (ii) $8 \sin^3 \theta - 6 \sin \theta + 1 = 0$ $-2 \sin 3\theta + 1 = 0$ $\sin 3\theta = \frac{1}{2}$ $3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\theta = \frac{\pi}{18}, \frac{5\pi}{18}$ (b) $\sin 15^\circ = \sin(60^\circ - 45^\circ) = \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ$ $= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$ $= \frac{1}{4} \sqrt{6} - \frac{1}{4} \sqrt{2} = \frac{1}{4} (\sqrt{6} - \sqrt{2})$ *	M1 A1 M1 A1 (4) M1 A1 M1 A1 A1 (5) M1 M1 A1 A1 (4) [13]
	<i>Alternatives to (b)</i> ① $\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$ $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$ $= \frac{1}{4} \sqrt{6} - \frac{1}{4} \sqrt{2} = \frac{1}{4} (\sqrt{6} - \sqrt{2})$ * ② Using $\cos 2\theta = 1 - 2 \sin^2 \theta$, $\cos 30^\circ = 1 - 2 \sin^2 15^\circ$ $2 \sin^2 15^\circ = 1 - \cos 30^\circ = 1 - \frac{\sqrt{3}}{2}$ $\sin^2 15^\circ = \frac{2 - \sqrt{3}}{4}$ $\left(\frac{1}{4} (\sqrt{6} - \sqrt{2}) \right)^2 = \frac{1}{16} (6 + 2 - 2\sqrt{12}) = \frac{2 - \sqrt{3}}{4}$ Hence $\sin 15^\circ = \frac{1}{4} (\sqrt{6} - \sqrt{2})$ *	M1 M1 A1 A1 (4) M1 A1 M1 A1 (4)

Question Number	Scheme	Marks
7.	(a) $f'(x) = 3e^x + 3xe^x$ $3e^x + 3xe^x = 3e^x(1+x) = 0$ $x = -1$ $f(-1) = -3e^{-1} - 1$	M1 A1 M1 A1 B1 (5)
	(b) $x_1 = 0.2596$ $x_2 = 0.2571$ $x_3 = 0.2578$	B1 B1 B1 (3)
	(c) Choosing $(0.257\ 55, 0.257\ 65)$ or an appropriate tighter interval. $f(0.257\ 55) = -0.000\ 379 \dots$ $f(0.257\ 65) = 0.000\ 109 \dots$	M1 A1 A1
	Change of sign (and continuity) $\Rightarrow$ root $\in (0.257\ 55, 0.257\ 65)$ * ($\Rightarrow x = 0.2576$, is correct to 4 decimal places)	cso A1
	<i>Note:</i> $x = 0.257\ 627\ 65 \dots$ is accurate	(3) [11]

Question Number	Scheme	Marks
8.	(a) $R^2 = 3^2 + 4^2$ $R = 5$ $\tan \alpha = \frac{4}{3}$ $\alpha = 53 \dots^\circ$ awrt 53°	M1 A1 M1 A1 (4)
	(b) Maximum value is 5 At the maximum, $\cos(\theta - \alpha) = 1$ or $\theta - \alpha = 0$ $\theta = \alpha = 53 \dots^\circ$	ft their R B1 ft M1 ft their α A1 ft (3)
	(c) $f(t) = 10 + 5 \cos(15t - \alpha)^\circ$ Minimum occurs when $\cos(15t - \alpha)^\circ = -1$ The minimum temperature is $(10 - 5)^\circ = 5^\circ$	M1 A1 ft (2)
	(d) $15t - \alpha = 180$ $t = 15.5$ awrt 15.5	M1 M1 A1 (3) [12]

Mark Scheme (Results) January 2010

GCE

Core Mathematics C3 (6665)

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January 2010

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January 2010
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
Q1	$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$ $= \frac{x+1}{3(x^2-1)} - \frac{1}{3x+1}$ $= \frac{x+1}{3(x+1)(x-1)} - \frac{1}{3x+1}$ $= \frac{1}{3(x-1)} - \frac{1}{3x+1}$ $= \frac{3x+1-3(x-1)}{3(x-1)(3x+1)}$ or $\frac{3x+1}{3(x-1)(3x+1)} - \frac{3(x-1)}{3(x-1)(3x+1)}$ $= \frac{4}{3(x-1)(3x+1)}$	Award below $x^2 - 1 \rightarrow (x+1)(x-1)$ or $3x^2 - 3 \rightarrow (x+1)(3x-3)$ or $3x^2 - 3 \rightarrow (3x+3)(x-1)$ seen or implied anywhere in candidate's working. Attempt to combine. M1 Correct result. A1 Decide to award M1 here!! M1 Either $\frac{4}{3(x-1)(3x+1)}$ or $\frac{\frac{4}{3}}{(x-1)(3x+1)}$ or $\frac{4}{(3x-3)(3x+1)}$ or $\frac{4}{9x^2-6x-3}$ A1 aef [4]

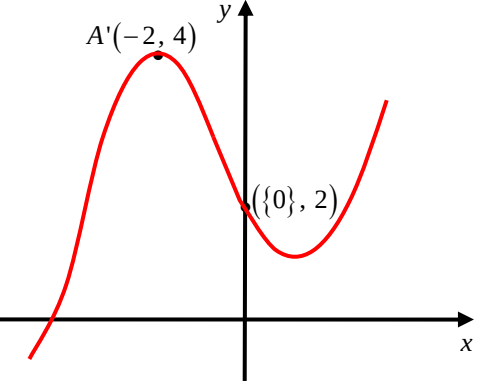
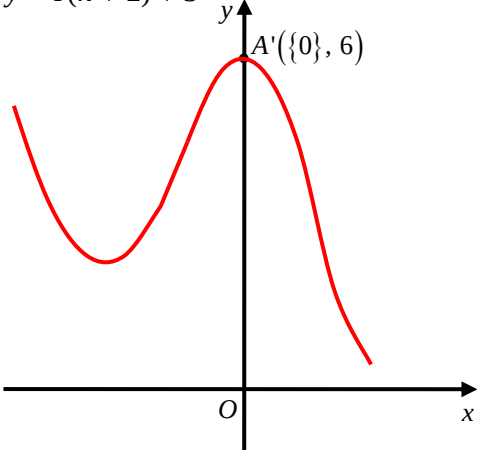
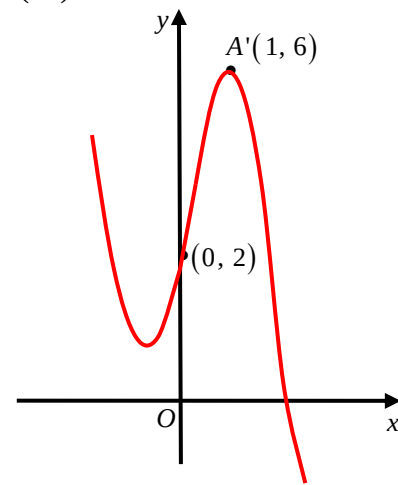
Question Number	Scheme	Marks
Q2	$f(x) = x^3 + 2x^2 - 3x - 11$	
(a)	$f(x) = 0 \Rightarrow x^3 + 2x^2 - 3x - 11 = 0$ $\Rightarrow x^2(x + 2) - 3x - 11 = 0$ $\Rightarrow x^2(x + 2) = 3x + 11$ $\Rightarrow x^2 = \frac{3x + 11}{x + 2}$ $\Rightarrow x = \sqrt{\left(\frac{3x + 11}{x + 2}\right)}$	Sets $f(x) = 0$ (can be implied) and takes out a factor of x^2 from $x^3 + 2x^2$, or x from $x^3 + 2x$ (slip). M1 then rearranges to give the quoted result on the question paper. A1 AG (2)
(b)	Iterative formula: $x_{n+1} = \sqrt{\left(\frac{3x_n + 11}{x_n + 2}\right)}$, $x_1 = 0$ $x_2 = \sqrt{\left(\frac{3(0) + 11}{(0) + 2}\right)}$ $x_2 = 2.34520788...$ $x_3 = 2.037324945...$ $x_4 = 2.058748112...$	An attempt to substitute $x_1 = 0$ into the iterative formula. Can be implied by $x_2 = \sqrt{5.5}$ or 2.35 or awrt 2.345 M1 Both $x_2 =$ awrt 2.345 and $x_3 =$ awrt 2.037 $x_4 =$ awrt 2.059 A1 A1 (3)
(c)	Let $f(x) = x^3 + 2x^2 - 3x - 11 = 0$ $f(2.0565) = -0.013781637...$ $f(2.0575) = 0.0041401094...$ Sign change (and $f(x)$ is continuous) therefore a root α is such that $\alpha \in (2.0565, 2.0575) \Rightarrow \alpha = 2.057$ (3 dp)	Choose suitable interval for x, e.g. [2.0565, 2.0575] or tighter any one value awrt 1 sf both values correct awrt 1sf, sign change and conclusion As a minimum, both values must be correct to 1 sf, candidate states "change of sign, hence root". M1 dM1 A1 (3)
		[8]

Question Number	Scheme	Marks
Q3 (a)	$5\cos x - 3\sin x = R\cos(x + \alpha), \quad R > 0, \quad 0 < x < \frac{\pi}{2}$ $5\cos x - 3\sin x = R\cos x \cos \alpha - R\sin x \sin \alpha$ Equate $\cos x$: $5 = R\cos \alpha$ Equate $\sin x$: $3 = R\sin \alpha$ $R = \sqrt{5^2 + 3^2}; = \sqrt{34} \{= 5.83095..\}$ $\tan \alpha = \frac{3}{5} \Rightarrow \alpha = 0.5404195003...^{\circ}$ Hence, $5\cos x - 3\sin x = \sqrt{34}\cos(x + 0.5404)$	M1; A1 M1 A1 (4)
(b)	$5\cos x - 3\sin x = 4$ $\sqrt{34}\cos(x + 0.5404) = 4$ $\cos(x + 0.5404) = \frac{4}{\sqrt{34}} \{= 0.68599...\}$ $(x + 0.5404) = 0.814826916...^{\circ}$ $x = 0.2744...^{\circ}$ $(x + 0.5404) = 2\pi - 0.814826916...^{\circ} \{= 5.468358...^{\circ}\}$ $x = 4.9279...^{\circ}$ Hence, $x = \{0.27, 4.93\}$	M1 M1 A1 ddM1 A1 (5)
		[9]

Part (b): If there are any EXTRA solutions inside the range $0 \leq x < 2\pi$, then withhold the final accuracy mark if the candidate would otherwise score all 5 marks. Also ignore EXTRA solutions outside the range $0 \leq x < 2\pi$.

Question Number	Scheme	Marks
Q4 (i)	$y = \frac{\ln(x^2 + 1)}{x}$ $u = \ln(x^2 + 1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2 + 1}$ Apply quotient rule: $\left\{ \begin{array}{l} u = \ln(x^2 + 1) \\ \frac{du}{dx} = \frac{2x}{x^2 + 1} \end{array} \right. \quad \left\{ \begin{array}{l} v = x \\ \frac{dv}{dx} = 1 \end{array} \right.$ $\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2 + 1} \right)(x) - \ln(x^2 + 1)}{x^2}$ $\left\{ \frac{dy}{dx} = \frac{2}{(x^2 + 1)} - \frac{1}{x^2} \ln(x^2 + 1) \right\}$	M1 A1 M1 A1 (4)
(ii)	$x = \tan y$ $\frac{dx}{dy} = \sec^2 y$ $\frac{dy}{dx} = \frac{1}{\sec^2 y} \{ = \cos^2 y \}$ $\frac{dy}{dx} = \frac{1}{1 + \tan^2 y}$ Hence, $\frac{dy}{dx} = \frac{1}{1 + x^2}$, (as required)	$\tan y \rightarrow \sec^2 y$ or an attempt to differentiate $\frac{\sin y}{\cos y}$ using either the quotient rule or product rule. M1* A1 Finding $\frac{dy}{dx}$ by reciprocating $\frac{dx}{dy}$. dM1* For writing down or applying the identity $\sec^2 y = 1 + \tan^2 y$, which must be applied/stated completely in y. dM1* For the correct proof, leading on from the previous line of working. A1 AG (5)

Question Number	Scheme	Marks
Q5	$y = \ln x$ Right-hand branch in quadrants 4 and 1. Correct shape. Left-hand branch in quadrants 2 and 3. Correct shape. Completely correct sketch and both $(-1, \{0\})$ and $(1, \{0\})$	B1 B1 B1 (3) [3]

Question Number	Scheme	Marks
Q6 (i)	$y = f(-x) + 1$  Shape of  and must have a maximum in quadrant 2 and a minimum in quadrant 1 or on the positive y-axis. Either $(\{0\}, 2)$ or $A'(-2, 4)$ Both $(\{0\}, 2)$ and $A'(-2, 4)$	B1 B1 B1 (3)
(ii)	$y = f(x + 2) + 3$  Any translation of the original curve. The translated maximum has either x-coordinate of 0 (can be implied) or y-coordinate of 6. The translated curve has maximum $(\{0\}, 6)$ and is in the correct position on the Cartesian axes.	B1 B1 B1 (3)
(iii)	$y = 2f(2x)$  Shape of  with a minimum in quadrant 2 and a maximum in quadrant 1. Either $(\{0\}, 2)$ or $A'(1, 6)$ Both $(\{0\}, 2)$ and $A'(1, 6)$	B1 B1 B1 (3) [9]

Question Number	Scheme	Marks
Q7 (a)	$y = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$ $\frac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$ $\frac{dy}{dx} = \left\{ \frac{\sin x}{\cos^2 x} \right\} = \left(\frac{1}{\cos x} \right) \left(\frac{\sin x}{\cos x} \right) = \underline{\underline{\sec x \tan x}}$	$\frac{dy}{dx} = \pm ((\cos x)^{-2}(\sin x))$ M1 $-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$ A1 Convincing proof. Must see both <u>underlined steps</u>. A1 AG (3)
(b)	$y = e^{2x} \sec 3x$ $\left\{ \begin{array}{ll} u = e^{2x} & v = \sec 3x \\ \frac{du}{dx} = 2e^{2x} & \frac{dv}{dx} = 3\sec 3x \tan 3x \end{array} \right\}$ Seen or implied Either $e^{2x} \rightarrow 2e^{2x}$ or $\sec 3x \rightarrow 3\sec 3x \tan 3x$ M1 Both $e^{2x} \rightarrow 2e^{2x}$ and $\sec 3x \rightarrow 3\sec 3x \tan 3x$ A1 $\frac{dy}{dx} = 2e^{2x} \sec 3x + 3e^{2x} \sec 3x \tan 3x$ Applies $vu' + uv'$ correctly for their u, u', v, v' M1 $2e^{2x} \sec 3x + 3e^{2x} \sec 3x \tan 3x$ A1 isw (4)	
(c)	Turning point $\Rightarrow \frac{dy}{dx} = 0$ Hence, $e^{2x} \sec 3x (2 + 3 \tan 3x) = 0$ {Note $e^{2x} \neq 0$, $\sec 3x \neq 0$, so $2 + 3 \tan 3x = 0$, } giving $\tan 3x = -\frac{2}{3}$ $\Rightarrow 3x = -0.58800 \Rightarrow x = \{a\} = -0.19600\dots$ Hence, $y = \{b\} = e^{2(-0.196)} \sec(3 \times -0.196)$ $= 0.812093\dots = 0.812$ (3sf)	Sets their $\frac{dy}{dx} = 0$ and factorises (or cancels) out at least e^{2x} from at least two terms. M1 $\tan 3x = \pm k$; $k \neq 0$ M1 Either awrt -0.196° or awrt -11.2° A1 0.812 A1 cao (4)

[11]

Part (c): If there are any EXTRA solutions for x (or a) inside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$ or ANY EXTRA solutions for y (or b), (for these values of x) then withhold the final accuracy mark.
Also ignore EXTRA solutions outside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$.

Question Number	Scheme	Marks
Q8	$\operatorname{cosec}^2 2x - \cot 2x = 1, \text{ (eqn *) } 0 \leq x \leq 180^\circ$ Using $\operatorname{cosec}^2 2x = 1 + \cot^2 2x$ gives $1 + \cot^2 2x - \cot 2x = 1$ $\cot^2 2x - \cot 2x = 0 \quad \text{or} \quad \cot^2 2x = \cot 2x$ $\cot 2x(\cot 2x - 1) = 0 \quad \text{or} \quad \cot 2x = 1$ $\cot 2x = 0 \quad \text{or} \quad \cot 2x = 1$ $\cot 2x = 0 \Rightarrow (\tan 2x \rightarrow \infty) \Rightarrow 2x = 90, 270$ $\Rightarrow x = 45, 135$ $\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$ $\Rightarrow x = 22.5, 112.5$ Overall, $x = \{22.5, 45, 112.5, 135\}$	M1 A1 dm1 A1 ddM1 A1 B1 [7]

If there are any EXTRA solutions inside the range $0 \leq x \leq 180^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final accuracy mark (the sixth mark in this question). Also ignore EXTRA solutions outside the range $0 \leq x \leq 180^\circ$.

Question Number	Scheme	Marks
Q9 (i)(a)	$\ln(3x - 7) = 5$ $e^{\ln(3x - 7)} = e^5$ $3x - 7 = e^5 \Rightarrow x = \frac{e^5 + 7}{3} \{= 51.804...\}$	Takes e of both sides of the equation. This can be implied by $3x - 7 = e^5$. Then rearranges to make x the subject. Exact answer of $\frac{e^5 + 7}{3}$. M1 dM1 A1 (3)
(b)	$3^x e^{7x+2} = 15$ $\ln(3^x e^{7x+2}) = \ln 15$ $\ln 3^x + \ln e^{7x+2} = \ln 15$ $x \ln 3 + 7x + 2 = \ln 15$ $x(\ln 3 + 7) = -2 + \ln 15$ $x = \frac{-2 + \ln 15}{7 + \ln 3} \{= 0.0874...\}$	Takes ln (or logs) of both sides of the equation. Applies the addition law of logarithms. $x \ln 3 + 7x + 2 = \ln 15$ Factorising out at least two x terms on one side and collecting number terms on the other side. Exact answer of $\frac{-2 + \ln 15}{7 + \ln 3}$ M1 M1 A1 oe ddM1 A1 oe (5)
(ii) (a)	$f(x) = e^{2x} + 3, x \in \mathbb{R}$ $y = e^{2x} + 3 \Rightarrow y - 3 = e^{2x}$ $\Rightarrow \ln(y - 3) = 2x$ $\Rightarrow \frac{1}{2} \ln(y - 3) = x$ Hence $f^{-1}(x) = \frac{1}{2} \ln(x - 3)$ $f^{-1}(x)$: Domain: $x > 3$ or $(3, \infty)$	Attempt to make x (or swapped y) the subject Makes e^{2x} the subject and takes ln of both sides $\frac{1}{2} \ln(x - 3)$ or $\ln \sqrt{x - 3}$ or $f^{-1}(y) = \frac{1}{2} \ln(y - 3)$ (see appendix) Either $x > 3$ or $(3, \infty)$ or <u>Domain</u> > 3. M1 M1 <u>A1</u> cao B1 (4)
(b)	$g(x) = \ln(x - 1), x \in \mathbb{R}, x > 1$ $fg(x) = e^{2 \ln(x-1)} + 3 \{= (x - 1)^2 + 3\}$ $fg(x)$: Range: $y > 3$ or $(3, \infty)$	An attempt to put function g into function f. $e^{2 \ln(x-1)} + 3$ or $(x - 1)^2 + 3$ or $x^2 - 2x + 4$. Either $y > 3$ or $(3, \infty)$ or <u>Range</u> > 3 or <u>$fg(x) > 3$</u>. M1 A1 isw B1 (3)
		[15]

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GCE

GCE Core Mathematics C3 (6665) Paper 1

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1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\checkmark$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- $\square$ The second mark is dependent on gaining the first mark

Question Number	Scheme	Marks
2.		
(a)	$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$ $= \frac{(4x-1)(2x-1) - 3}{2(x-1)(2x-1)}$ $= \frac{8x^2 - 6x - 2}{\{2(x-1)(2x-1)\}}$ $= \frac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$ $= \frac{4x+1}{2x-1}$ An attempt to form a single fraction Simplifies to give a correct quadratic numerator over a correct quadratic denominator An attempt to factorise a 3 term quadratic numerator	M1 A1 aef M1 A1 (4)
(b)	$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \quad x > 1$ $f(x) = \frac{(4x+1)}{(2x-1)} - 2$ $= \frac{(4x+1) - 2(2x-1)}{(2x-1)}$ $= \frac{4x+1-4x+2}{(2x-1)}$ $= \frac{3}{(2x-1)}$ An attempt to form a single fraction Correct result	M1 A1 * (2)
(c)	$f(x) = \frac{3}{(2x-1)} = 3(2x-1)^{-1}$ $f'(x) = 3(-1)(2x-1)^{-2}(2)$ $f'(2) = \frac{-6}{9} = -\frac{2}{3}$ $\pm k(2x-1)^{-2}$ Either $\frac{-6}{9}$ or $-\frac{2}{3}$	M1 A1 aef A1 (3) [9]

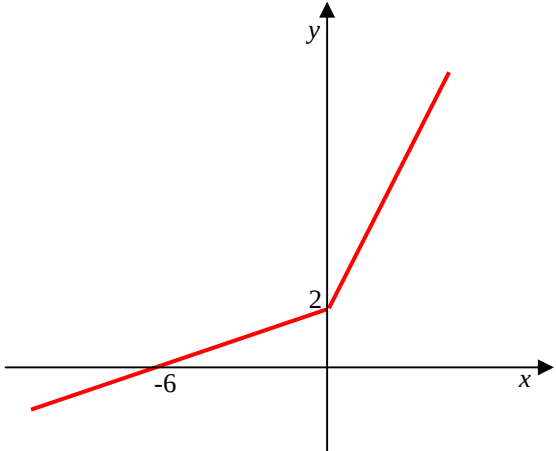
Question Number	Scheme	Marks
3.	$2 \cos 2\theta = 1 - 2 \sin \theta$ $2(1 - 2 \sin^2 \theta) = 1 - 2 \sin \theta$ $2 - 4 \sin^2 \theta = 1 - 2 \sin \theta$ $4 \sin^2 \theta - 2 \sin \theta - 1 = 0$ $\sin \theta = \frac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$ PVs: $\alpha_1 = 54^\circ$ or $\alpha_2 = -18^\circ$ $\theta = \{54, 126, 198, 342\}$	Substitutes either $1 - 2 \sin^2 \theta$ or $2 \cos^2 \theta - 1$ or $\cos^2 \theta - \sin^2 \theta$ for $\cos 2\theta$. M1 Forms a “quadratic in sine” = 0 M1(*) Applies the quadratic formula See notes for alternative methods. M1 Any one correct answer A1 180-their pv dM1(*) All four solutions correct. A1 [6]

Question Number	Scheme	Marks
4.		
(a)	$\theta = 20 + Ae^{-kt}$ (eqn *) $\{t = 0, \theta = 90 \Rightarrow\} \quad 90 = 20 + Ae^{-k(0)} \quad \text{Substitutes } t = 0 \text{ and } \theta = 90 \text{ into eqn } *$ $90 = 20 + A \Rightarrow \underline{A = 70} \quad \underline{A = 70}$	M1 A1 (2)
(b)	$\theta = 20 + 70e^{-kt}$ $\{t = 5, \theta = 55 \Rightarrow\} \quad 55 = 20 + 70e^{-k(5)} \quad \text{Substitutes } t = 5 \text{ and } \theta = 55 \text{ into eqn } *$ $\frac{35}{70} = e^{-5k} \quad \text{and rearranges eqn } * \text{ to make } e^{\pm 5k} \text{ the subject.}$ $\ln\left(\frac{35}{70}\right) = -5k \quad \text{Takes 'lns' and proceeds to make '}\pm 5k\text{' the subject.}$ $-5k = \ln\left(\frac{1}{2}\right)$ $-5k = \ln 1 - \ln 2 \Rightarrow -5k = -\ln 2 \Rightarrow \underline{k = \frac{1}{5}\ln 2} \quad \text{Convincing proof that } k = \frac{1}{5}\ln 2$	M1 dM1 A1 * (3)
(c)	$\theta = 20 + 70e^{-\frac{1}{5}t\ln 2}$ $\frac{d\theta}{dt} = -\frac{1}{5}\ln 2 \cdot (70)e^{-\frac{1}{5}t\ln 2} \quad \pm \alpha e^{-kt} \text{ where } k = \frac{1}{5}\ln 2$ $-14\ln 2 e^{-\frac{1}{5}t\ln 2}$ When $t = 10$, $\frac{d\theta}{dt} = -14\ln 2 e^{-2\ln 2}$ $\frac{d\theta}{dt} = -\frac{7}{2}\ln 2 = -2.426015132...$ Rate of decrease of $\theta = 2.426^\circ \text{C/min}$ (3 dp.) awrt ± 2.426	M1 A1 oe A1 (3) [8]

Question Number	Scheme	Marks
5.		
(a)	Crosses x-axis $\Rightarrow f(x) = 0 \Rightarrow (8 - x)\ln x = 0$ Either $(8 - x) = 0$ or $\ln x = 0 \Rightarrow x = 8, 1$ Either one of $\{x\}=1$ OR $x=\{8\}$ Coordinates are $A(1, 0)$ and $B(8, 0)$. Both $A(1, \{0\})$ and $B(8, \{0\})$	B1 B1 (2)
(b)	Apply product rule: $\left\{ \begin{array}{l} u = (8 - x) \quad v = \ln x \\ \frac{du}{dx} = -1 \quad \frac{dv}{dx} = \frac{1}{x} \end{array} \right\}$ $vu' + uv'$ $f'(x) = -\ln x + \frac{8-x}{x}$ Any one term correct Both terms correct	M1 A1 A1 (3)
(c)	$f'(3.5) = 0.032951317...$ $f'(3.6) = -0.058711623...$ Sign change (and as $f'(x)$ is continuous) therefore the x-coordinate of Q lies between 3.5 and 3.6.	Attempts to evaluate both $f'(3.5)$ and $f'(3.6)$ M1 both values correct to at least 1 sf, sign change and conclusion A1 (2)
(d)	At Q, $f'(x) = 0 \Rightarrow -\ln x + \frac{8-x}{x} = 0$ Setting $f'(x) = 0$. $\Rightarrow -\ln x + \frac{8}{x} - 1 = 0$ Splitting up the numerator and proceeding to $x=$ $\Rightarrow \frac{8}{x} = \ln x + 1 \Rightarrow 8 = x(\ln x + 1)$ $\Rightarrow x = \frac{8}{\ln x + 1}$ (as required) For correct proof. No errors seen in working.	M1 M1 A1 (3)

Question Number	Scheme	Marks
(e)	Iterative formula: $x_{n+1} = \frac{8}{\ln x_n + 1}$ $x_1 = \frac{8}{\ln(3.55) + 1}$ $x_1 = 3.528974374...$ $x_2 = 3.538246011...$ $x_3 = 3.534144722...$ $x_1 = 3.529, x_2 = 3.538, x_3 = 3.534, \text{ to 3 dp.}$	An attempt to substitute $x_0 = 3.55$ into the iterative formula. Can be implied by $x_1 = 3.528(97)...$ Both $x_1 = \text{awrt } 3.529$ and $x_2 = \text{awrt } 3.538$ M1 A1 x_1, x_2, x_3 all stated correctly to 3 dp A1 (3) [13]

Question Number	Scheme	Marks
6.		
(a)	$y = \frac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$ Attempt to make x (or swapped y) the subject $xy - 5y = 3 - 2x$ $\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3 + 5y$ Collect x terms together and factorise. $\Rightarrow x = \frac{3+5y}{y+2} \quad \therefore f^{-1}(x) = \frac{3+5x}{x+2}$ $\frac{3+5x}{x+2}$	M1 M1 A1 oe (3)
(b)	Range of g is $-9 \leq g(x) \leq 4$ or $-9 \leq y \leq 4$ <u>Correct Range</u>	B1 (1)
(c)	$g(g(2)) = g(0) = -6$, from sketch.	Deduces that g(2) is 0. Seen or implied. M1 -6 A1 (2)
(d)	$fg(8) = f(4)$ $= \frac{3-4(2)}{4-5} = \frac{-5}{-1} = 5$	Correct order g followed by f M1 5 A1 (2)

Question Number	Scheme	Marks
(e)(ii)	 Correct shape Graph goes through $(\{0\}, 2)$ and $(-6, \{0\})$ which are marked.	B1 B1 (4)
(f)	Domain of g^{-1} is $-9 \leq x \leq 4$ Either correct answer or a follow through from part (b) answer	B1 $\sqrt{\quad}$ (1) [13]

Question Number	Scheme	Marks
7	(a) $y = \frac{3 + \sin 2x}{2 + \cos 2x}$ Apply quotient rule: $\left\{ \begin{array}{l} u = 3 + \sin 2x \quad v = 2 + \cos 2x \\ \frac{du}{dx} = 2 \cos 2x \quad \frac{dv}{dx} = -2 \sin 2x \end{array} \right\}$ $\frac{dy}{dx} = \frac{2 \cos 2x(2 + \cos 2x) - -2 \sin 2x(3 + \sin 2x)}{(2 + \cos 2x)^2}$ $= \frac{4 \cos 2x + 2 \cos^2 2x + 6 \sin 2x + 2 \sin^2 2x}{(2 + \cos 2x)^2}$ $= \frac{4 \cos 2x + 6 \sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2 + \cos 2x)^2}$ $= \frac{4 \cos 2x + 6 \sin 2x + 2}{(2 + \cos 2x)^2} \quad (\text{as required})$	Applying $\frac{u'v - uv'}{v^2}$ M1 Any one term correct on the numerator A1 Fully correct (unsimplified). A1 For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$. A1* No errors seen in working. A1* (4)
(b)	When $x = \frac{\pi}{2}$, $y = \frac{3 + \sin \pi}{2 + \cos \pi} = \frac{3}{1} = 3$ At $(\frac{\pi}{2}, 3)$, $m(\mathbf{T}) = \frac{6 \sin \pi + 4 \cos \pi + 2}{(2 + \cos \pi)^2} = \frac{-4 + 2}{1^2} = -2$ Either $\mathbf{T}$: $y - 3 = -2(x - \frac{\pi}{2})$ or $y = -2x + c$ and $3 = -2(\frac{\pi}{2}) + c \Rightarrow c = 3 + \pi$; $\mathbf{T}$: $y = -2x + (\pi + 3)$	$y = 3$ B1 $m(\mathbf{T}) = -2$ B1 $y - y_1 = m(x - \frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; M1 or uses $y = mx + c$ with 'their TANGENT gradient'; $y = -2x + \pi + 3$ A1 (4) [8]

Question Number	Scheme	Marks
8.		
(a)	$y = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$ Writes $\sec x$ as $(\cos x)^{-1}$ and gives $\frac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$ $\frac{dy}{dx} = \pm((\cos x)^{-2}(\sin x))$ $-1(\cos x)^{-2}(-\sin x) \text{ or } (\cos x)^{-2}(\sin x)$ $\frac{dy}{dx} = \left\{ \frac{\sin x}{\cos^2 x} \right\} = \left(\frac{1}{\cos x} \right) \left(\frac{\sin x}{\cos x} \right) = \underline{\underline{\sec x \tan x}}$ Convincing proof. Must see both <u>underlined steps.</u>	M1 A1 A1 AG (3)
(b)	$x = \sec 2y, \quad y \neq (2n+1)\frac{\pi}{4}, n \in \mathbb{Z}.$ $\frac{dx}{dy} = 2 \sec 2y \tan 2y$ $K \sec 2y \tan 2y$ $2 \sec 2y \tan 2y$	M1 A1 (2)
(c)	$\frac{dy}{dx} = \frac{1}{2 \sec 2y \tan 2y}$ Applies $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$ $\frac{dy}{dx} = \frac{1}{2x \tan 2y}$ Substitutes x for $\sec 2y$. $1 + \tan^2 A = \sec^2 A \Rightarrow \tan^2 2y = \sec^2 2y - 1$ Attempts to use the identity $1 + \tan^2 A = \sec^2 A$ So $\tan^2 2y = x^2 - 1$ $\frac{dy}{dx} = \frac{1}{2x\sqrt{(x^2-1)}}$ $\frac{dy}{dx} = \frac{1}{2x\sqrt{(x^2-1)}}$	M1 M1 M1 A1 (4) [9]

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Mark Scheme (Results)

January 2012

GCE Core Mathematics C3 (6665) Paper 1

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January 2012

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS**General Instructions for Marking**

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 - Marks should not be subdivided.
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- bod – benefit of doubt
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 - the symbol $\uparrow$ will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - $\square$ The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

General Principals for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \dots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \dots$$

2. Formula

Attempt to use correct formula (with values for a , b and c), leading to $x = \dots$

3. Completing the square

$$\text{Solving } x^2 + bx + c = 0 : \quad \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0, \quad \text{leading to } x = \dots$$

Method marks for differentiation and integration:1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

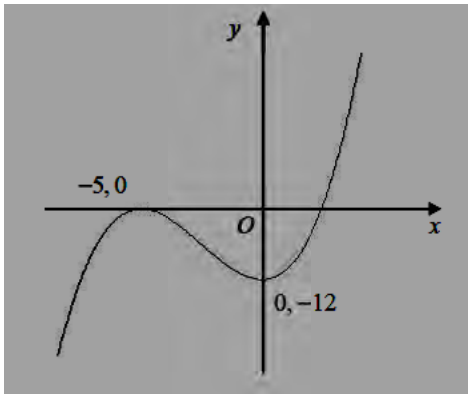
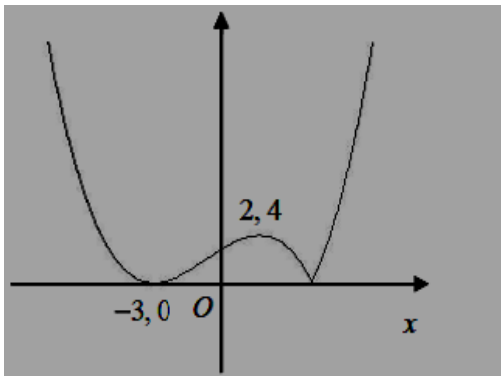
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Question No	Scheme	Marks
1	(a) $\frac{d}{dx}(\ln(3x)) \rightarrow \frac{B}{x} \text{ for any constant } B$ Applying $vu' + uv'$, $\ln(3x) \times 2x + x$ (b) Applying $\frac{vu' - uv'}{v^2}$ $\frac{x^3 \times 4\cos(4x) - \sin(4x) \times 3x^2}{x^6}$ $= \frac{4x\cos(4x) - 3\sin(4x)}{x^4}$	M1 M1, A1 A1 (4) M1 <u>A1+A1</u> A1 A1 (5) (9 MARKS)

- (a) M1 Differentiates the $\ln(3x)$ term to $\frac{B}{x}$. Note that $\frac{1}{3x}$ is fine for this mark.
- M1 Applies the product rule to $x^2 \ln(3x)$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is **not quoted (or implied by their working)** only accept answers of the form $\ln(3x) \times Ax + x^2 \times \frac{B}{x}$ where A and B are non- zero constants
- A1 One term correct and simplified, either $2x \ln(3x)$ or $x \ln 3x^{2x}$ and $\ln(3x) 2x$ are acceptable forms
- A1 Both terms correct and simplified on the same line. $2x \ln(3x) + x$, $\ln(3x) \times 2x + x$, $x(2 \ln 3x + 1)$ oe
- (b) M1 Applies the quotient rule. A version of this appears in the formula booklet. If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the formula is **not quoted (nor implied by their working)** only accept answers of the form $\frac{x^3 \times \pm 4\cos(4x) - \sin(4x) \times Bx^2}{(x^3)^2 \text{ or } x^6 \text{ or } x^5 \text{ or } x^9}$ with $B > 0$
- A1 Correct first term on numerator $x^3 \times 4\cos(4x)$
- A1 Correct second term on numerator $-\sin(4x) \times 3x^2$
- A1 Correct denominator x^6 , the $(x^3)^2$ needs to be simplified
- A1 Fully correct simplified expression $\frac{4x\cos(4x) - 3\sin(4x)}{x^4}$, $\frac{\cos(4x)4x - \sin(4x)3}{x^4}$ oe .
- Accept $4x^{-3} \cos(4x) - 3x^{-4} \sin(4x)$ oe

Alternative method using the product rule.

- M1,A1** Writes $\frac{\sin(4x)}{x^3}$ as $\sin(4x) \times x^{-3}$ and applies the product rule. They will score both of these marks or neither of them. If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the formula is **not quoted (nor implied by their working)** only accept answers of the form $x^{-3} \times A\cos(4x) + \sin(4x) \times \pm Bx^{-4}$
- A1 One term correct, either $x^{-3} \times 4\cos(4x)$ or $\sin(4x) \times -3x^{-4}$
- A1 Both terms correct, Eg. $x^{-3} \times 4\cos(4x) + \sin(4x) \times -3x^{-4}$.
- A1 Fully correct expression. $4x^{-3} \cos(4x) - 3x^{-4} \sin(4x)$ or $4\cos(4x)x^{-3} - 3\sin(4x)x^{-4}$ oe
- The negative must have been dealt with for the final mark.

Question No	Scheme	Marks
2	(a)  Shape B1 x coordinates correct B1 y coordinates correct B1 (3) (b)  Shape B1 Max at (2,4) B1 Min at (-3,0) B1 (3) 6 marks	

(a)

- B1 Shape unchanged. The positioning of the curve is not significant for this mark. The right hand section of the curve does not have to cross x axis.
- B1 The x- coordinates of P' and Q' are -5 and 0 respectively. This is for translating the curve 2 units left. The minimum point Q' must be on the y axis. Accept if -5 is marked on the x axis for P' with Q' on the y axis (marked -12).
- B1 The y- coordinates of P' and Q' are 0 and -12 respectively. This is for the stretch $\times 3$ parallel to the y axis. The maximum P' must be on the x axis. Accept if -12 is marked on the y axis for Q' with P' on the x axis (marked -5)

(b)

- B1 The curve below the x axis reflected in the x axis and the curve above the x axis is unchanged. Do not accept if the curve is clearly rounded off with a zero gradient at the x axis but allow small curvature issues. Use the same principles on the lhs- do not accept if this is a cusp.
- B1 Both the x- and y- coordinates of Q', (2,4) given correctly and associated with the maximum point in the first quadrant. To gain this mark there must be a graph and it must only have one maximum. Accept as 2 and 4 marked on the correct axes or in the script as long as there is no ambiguity.
- B1 Both the x- and y- coordinates of P', (-3,0) given correctly and associated with the minimum point in the second quadrant. To gain this mark there must be a graph. Tolerate two cusps if this mark has been lost earlier in the question. Accept (0, -3) marked on the correct axis.

Question No	Scheme	Marks
3	(a) $20 \text{ (mm}^2\text{)}$	B1 M1 (1)

	(b) $'40' = 20 e^{1.5t} \rightarrow e^{1.5t} = c$ $e^{1.5t} = \frac{40}{20} = (2)$ Correct order $1.5t = \ln'2' \rightarrow t = \frac{\ln c}{1.5}$ $t = \frac{\ln 2}{1.5} = (\text{awrt } 0.46)$ 12.28 or 28 (minutes}	A1 M1 A1 A1 (5) (6 marks)
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(a)

B1 Sight of 20 relating to the value of A at t=0. Do not worry about (incorrect) units. Accept its sight in (b)

(b)

M1 Substitutes A=40 or twice their answer to (a) **and** proceeds to $e^{1.5t} = \text{constant}$. Accept non numerical answers.

A1 $e^{1.5t} = \frac{40}{20}$ or 2

M1 Correct ln work to find t. Eg $e^{1.5t} = \text{constant} \rightarrow 1.5t = \ln(\text{constant}) \rightarrow t =$
The order must be correct. Accept non numerical answers. **See below for correct alternatives**

A1 Achieves either $\frac{\ln(2)}{1.5}$ or awrt 0.46 2sf

A1 Either 12:28 or 28 (minutes). Cao

Alternatives in (b)

Alt 1- taking ln's of both sides on line 1

M1 Substitutes A=40, or twice (a) takes ln's of both sides **and** proceeds to $\ln('40') = \ln 20 + \ln e^{1.5t}$

A1 $\ln(40) = \ln 20 + 1.5t$

M1 Make t the subject with correct ln work.

$$\ln('40') - \ln 20 = 1.5t \text{ or } \ln\left(\frac{'40'}{20}\right) = 1.5t \rightarrow t =$$

A1,A1 are the same

Alt 2- trial and improvement-hopefully seen rarely

M1 Substitutes t= 0.46 and t=0.47 into $20e^{1.5t}$ to obtain A at both values. Must be to at least 2dp but you may accept tighter interval but the interval must span the correct value of 0.46209812

A1 Obtains A(0.46)=39.87 AND A(0.47)=40.47 or equivalent

M1 Substitutes t=0.462 and t=0.4625 into $40e^{1.5t}$

A1 Obtains A(0.462)=39.99 AND A(0.4625)=40.02 or equivalent and states t=0.462 (3sf)

A1 AS ABOVE

No working leading to fully correct accurate answer (3sf or better) send/escalate to team leader

Question No	Scheme	Marks
4	$\left(\frac{dx}{dy}\right) = 2\sec^2\left(y + \frac{\pi}{12}\right)$ substitute $y = \frac{\pi}{4}$ into their $\frac{dx}{dy} = 2\sec^2\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = 8$ When $y = \frac{\pi}{4}$, $x = 2\sqrt{3}$ awrt 3.46 $\left(y - \frac{\pi}{4}\right) = \text{their } m(x - \text{their } 2\sqrt{3})$ $\left(y - \frac{\pi}{4}\right) = -8(x - 2\sqrt{3}) \text{ oe}$	M1,A1 M1, A1 B1 M1 A1 (7 marks)

M1 For differentiation of $2\tan\left(y + \frac{\pi}{12}\right) \rightarrow 2\sec^2\left(y + \frac{\pi}{12}\right)$. There is no need to identify this with $\frac{dx}{dy}$

A1 For correctly writing $\frac{dx}{dy} = 2\sec^2\left(y + \frac{\pi}{12}\right)$ or $\frac{dy}{dx} = \frac{1}{2\sec^2\left(y + \frac{\pi}{12}\right)}$

M1 Substitute $y = \frac{\pi}{4}$ into their $\frac{dx}{dy}$. Accept if $\frac{dx}{dy}$ is inverted and $y = \frac{\pi}{4}$ substituted into $\frac{dy}{dx}$.

A1 $\frac{dx}{dy} = 8$ or $\frac{dy}{dx} = \frac{1}{8}$ oe

B1 Obtains the value of $x = 2\sqrt{3}$ corresponding to $y = \frac{\pi}{4}$. Accept awrt 3.46

M1 This mark requires **all of the necessary elements for finding a numerical equation of the normal**.

Either Invert their value of $\frac{dx}{dy}$, to find $\frac{dy}{dx}$, then use $m_1 \times m_2 = -1$ to find the numerical gradient of the normal

Or use their numerical value of $-\frac{dx}{dy}$

Having done this then use $\left(y - \frac{\pi}{4}\right) = \text{their } m(x - \text{their } 2\sqrt{3})$

The $2\sqrt{3}$ could appear as awrt 3.46, the $\frac{\pi}{4}$ as awrt 0.79,

This cannot be awarded for finding the equation of a tangent.

Watch for candidates who correctly use $\left(x - \text{their } 2\sqrt{3}\right) = -\text{their numerical } \frac{dy}{dx} \left(y - \frac{\pi}{4}\right)$

If they use 'y=mx+c' it must be a full method to find c.

A1 Any correct form of the answer. It does not need to be simplified and the question does not ask for an exact answer.

$$\left(y - \frac{\pi}{4}\right) = -8(x - 2\sqrt{3}) , \quad \frac{y - \frac{\pi}{4}}{x - 2\sqrt{3}} = -8 , \quad y = -8x + \frac{\pi}{4} + 16\sqrt{3} , \quad y = -8x + (\text{awrt } 28.5)$$

Alternatives using arctan (first 3 marks)

M1 Differentiates $y = \arctan\left(\frac{x}{2}\right) - \frac{\pi}{12}$ to get $\frac{1}{1+(\frac{x}{2})^2} \times \text{constant}$. Don't worry about the lhs

A1 Achieves $\frac{dy}{dx} = \frac{1}{1+(\frac{x}{2})^2} \times \frac{1}{2}$

M1 This method mark requires x to be found, which then needs to be substituted into $\frac{dy}{dx}$

The rest of the marks are then the same.

Or implicitly (first 2 marks)

M1 Differentiates implicitly to get $1 = 2 \sec^2\left(y + \frac{\pi}{12}\right) \times \frac{dy}{dx}$

A1 Rearranges to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of y

The rest of the marks are the same

Or by compound angle identities

$$x = 2 \tan\left(y + \frac{\pi}{12}\right) = \frac{2 \tan y + 2 \tan\left(\frac{\pi}{12}\right)}{1 - \tan y \tan\frac{\pi}{12}} \text{ or}$$

M1 Differentiates using quotient rule-see question 1 in applying this. Additionally the tany **must** have been differentiated to $\sec^2 y$. There is no need to assign to $\frac{dx}{dy}$

A1 The correct answer for $\frac{dx}{dy} = \frac{(1 - \tan y \tan\frac{\pi}{12}) \times 2 \sec^2 y - (2 \tan y + 2 \tan(\frac{\pi}{12})) \times -\sec^2 y \tan\frac{\pi}{12}}{(1 - \tan y \tan\frac{\pi}{12})^2}$

The rest of the marks are as the main scheme

Question No	Scheme	Marks
5.	Uses the identity $\cot^2(3\theta) = \operatorname{cosec}^2(3\theta) - 1$ in $2\cot^2(3\theta) = 7\operatorname{cosec}(3\theta) - 5$	M1

	$2\operatorname{cosec}^2(3\theta) - 7\operatorname{cosec}(3\theta) + 3 = 0$	A1
	$(2\operatorname{cosec}3\theta - 1)(\operatorname{cosec}3\theta - 3) = 0$	dM1
	$\operatorname{cosec}3\theta = 3$	A1
	$\theta = \frac{\operatorname{invsin}(\frac{1}{3})}{3}, \frac{19.5^\circ}{3} = \text{awrt } 6.5^\circ$	ddM1, A1
value	$\theta = \frac{180^\circ - \operatorname{invsin}(\frac{1}{3})}{3}, 53.5^\circ$	Correct 2 nd ddM1,A1
	$\theta = \frac{360^\circ + \operatorname{invsin}(\frac{1}{3})}{3}$	Correct 3 rd value ddM1
	All 4 correct answers awrt $6.5^0, 53.5^0, 126.5^0$ or 173.5^0	A1 (10 marks)

- M1 Uses the substitution $\cot^2(3\theta) = \pm 1 \pm \operatorname{cosec}^2(3\theta)$ to produce a quadratic equation in $\operatorname{cosec}(3\theta)$
 Accept 'invisible' brackets in which $2\cot^2(3\theta)$ is replaced by $2\operatorname{cosec}^2(3\theta) - 1$
 A (longer) but acceptable alternative is to convert everything to $\sin(3\theta)$.
 For this to be scored $\cot^2 3\theta$ must be replaced by $\frac{\cos^2(3\theta)}{\sin^2(3\theta)}$, $\operatorname{cosec}(3\theta)$ must be replaced by $\frac{1}{\sin 3\theta}$.
 An attempt must be made to multiply by $\sin^2(3\theta)$ and finally $\cos^2(3\theta)$ replaced by $= \pm 1 \pm \sin^2(3\theta)$
- A1 A correct equation ($=0$) written or implied by working is obtained. Terms must be collected together on one side of the equation. The usual alternatives are
 $2\operatorname{cosec}^2(3\theta) - 7\operatorname{cosec}(3\theta) + 3 = 0$ or $3\sin^2(3\theta) - 7\sin(3\theta) + 2 = 0$
- dM1 Either an attempt to factorise a 3 term quadratic **in cosec(3θ) or sin(3θ)** with the usual rules
 Or use of a correct formula to produce a solution in $\operatorname{cosec}(3\theta)$ or $\sin(3\theta)$
- A1 Obtaining the correct value of $\operatorname{cosec}(3\theta) = 3$ or $\sin(3\theta) = \frac{1}{3}$. Ignore other values
- ddM1 Correct method to produce the principal value of θ . It is dependent upon the two M's being scored.
 Look for $\theta = \frac{\operatorname{invsin}(\text{their } \frac{1}{3})}{3}$
- A1 Awrt 6.5
- ddM1 Correct method to produce a secondary value. This is dependent upon the candidate having scored the first 2 M's. Usually you look for $\frac{180 - \text{their } 19.5}{3}$ or $\frac{360 + \text{their } 19.5}{3}$ or $\frac{540 - \text{their } 19.5}{3}$
Note 180-their 6.5 must be marked correct BUT 360+their 6.5 is incorrect
- A1 Any other correct answer. Awrt 6.5, 53.5, 126.5 or 173.5
- ddM1 Correct method to produce a THIRD value. This is dependent upon the candidate having scored the first 2 M's. See above for alternatives
- A1 All 4 correct answers awrt 6.5, 53.5, 126.5 or 173.5 and no extras inside the range. Ignore any answers outside the range.

Radian answers: awrt 0.11, 0.93, 2.21, 3.03. Accuracy must be to 2dp.

Lose the first mark that could have been scored. Fully correct radian answer scores 1,1,1,1,0,1,1,1,1=9 marks

Candidates cannot mix degrees and radians for method marks.

Special case: Some candidates solve the equation in $\operatorname{cosec}(\theta \text{ or } x)$, $\sin(\theta \text{ or } x)$ to produce $\operatorname{cosec}(\theta \text{ or } x) = 3$

$$\sin(\theta \text{ or } x) = \frac{1}{3}.$$

Question No		Marks
6	(a) $f(0.8) = 0.082$, $f(0.9) = -0.089$ Change of sign $\Rightarrow$ root (0.8,0.9)	M1 A1 (2)
	(b) $f'(x) = 2x - 3 - \sin\left(\frac{1}{2}x\right)$ Sets $f'(x) = 0 \Rightarrow x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}$	M1 A1 M1A1* (4)
	(c) Sub $x_0=2$ into $x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}$ $x_1=\text{awrt } 1.921$, $x_2=\text{awrt } 1.91(0)$ and $x_3=\text{awrt } 1.908$	M1 A1,A1 (3)
	(d) [1.90775,1.90785] $f'(1.90775)=-0.00016..$ AND $f'(1.90785)=0.0000076..$ Change of sign $\Rightarrow x=1.9078$	M1 M1 A1 (3)
		(12 marks)

(a)

M1 Calculates both $f(0.8)$ and $f(0.9)$. Evidence of this mark could be, either, seeing both 'x' substitutions written out in the expression, or, one value correct to 1 sig fig, or the appearance of incorrect values of $f(0.8)=\text{awrt } 0.2$ or $f(0.9)=\text{awrt } 0.1$ from use of degrees

A1 This requires both values to be correct as well as a reason and a conclusion.

Accept $f(0.8) = 0.08$ truncated or rounded (2dp) or 0.1 rounded (1dp) and $f(0.9) = -0.08$ truncated or rounded as -0.09 (2dp) or -0.1(1dp)

Acceptable reasons are change of sign, $<0 >0$, +ve -ve, $f(0.8)f(0.9)<0$. Acceptable conclusion is hence root or □

(b)

M1 Attempts to differentiate $f(x)$. Seeing any of $2x, 3$ or $\pm A\sin(\frac{1}{2}x)$ is sufficient evidence.

A1 $f'(x)$ correct. Accept $\frac{dy}{dx} = 2x - 3 - \sin\left(\frac{1}{2}x\right)$

M1 Sets their $f'(x)=0$ and proceeds to $x=....$. You must be sure that they are setting what they think is $f'(x)=0$.

Accept $2x = 3 + \sin\left(\frac{1}{2}x\right)$ going to $x=..$ only if $f'(x)=0$ is stated first

A1 * $x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}$. This is a given answer so don't accept just the sight of this answer. It is cso

(c) M1 Substitutes $x_0=2$ into $x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}$. Evidence of this mark could be awrt 1.9 or 1.5 (from degrees)

A1 $x_1=\text{awrt } 1.921$

A1 $x_2=\text{awrt } 1.91(0)$ and $x_3=\text{awrt } 1.908$

(d) **Continued iteration is not acceptable for this part. Question states 'By choosing a suitable interval...'**

M1 Chooses the interval [1.90775,1.90785] or tighter containing the root= 1.907845522

M1 Calculates $f'(1.90775)$ and $f'(1.90785)$ or tighter with at least one correct, rounded or truncated

$f'(1.90775)=-0.0001$ truncated or awrt -0.0002 rounded

$f'(1.90785)=0.000007$ truncated or awrt 0.000008 rounded

Accept versions of $g(x)-x$ where $g(x) = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}$.

When $x = 1.90775$, $g(x) - x = 8 \times 10^{-5}$ rounded and truncated

When $x = 1.90785$, $g(x) - x = -3 \times 10^{-6}$ truncated or $= -4 \times 10^{-6}$ rounded

A1 Both values correct, rounded or truncated, a valid reason (see part a) and a minimal conclusion (see part a). Saying hence root is acceptable. There is no need to refer to the 'turning point'.

Question No	Scheme	Marks
7	(a) $2x^2 + 7x - 4 = (2x - 1)(x + 4)$ $\frac{3(x+1)}{(2x-1)(x+4)} - \frac{1}{(x+4)} = \frac{3(x+1) - (2x-1)}{(2x-1)(x+4)}$ $= \frac{x+4}{(2x-1)(x+4)}$ $= \frac{1}{2x-1}$ (b) $y = \frac{1}{2x-1} \Rightarrow y(2x-1) = 1 \Rightarrow 2xy - y = 1$ $2xy = 1 + y \Rightarrow x = \frac{1+y}{2y}$ $y \text{ OR } f^{-1}(x) = \frac{1+x}{2x}$ (c) $x > 0$ (d) $\frac{1}{2\ln(x+1)-1} = \frac{1}{7}$ $\ln(x+1) = 4$ $x = e^4 - 1$	B1 M1 M1 A1* (4) M1M1 A1 (3) B1 (1) M1 A1 M1A1 (4) 12 Marks

(a)

B1 Factorises the expression $2x^2 + 7x - 4 = (2x - 1)(x + 4)$. This may not be on line 1

- M1 Combines the two fractions to form a single fraction with a common denominator. Cubic denominators are fine for this mark. Allow slips on the numerator but one must have been adapted. Allow 'invisible' brackets. Accept two separate fractions with the same denominator. Amongst many possible options are

Correct $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$, Invisible bracket $\frac{3x+1-2x-1}{(2x-1)(x+4)}$,

Cubic and separate $\frac{3(x+1)(x+4)}{(2x^2+7x-4)(x+4)} - \frac{2x^2+7x-4}{(2x^2+7x-4)(x+4)}$

- M1 Simplifies the (now) single fraction to one with a linear numerator divided by a quadratic factorised denominator. Any cubic denominator must have been fully factorised (check first and last terms) and cancelled with terms on a fully factorised numerator (check first and last terms).

- A1* Cso. This is a given solution and it must be fully correct. All bracketing/algebra must have been correct.

You can however accept $\frac{x+4}{(2x-1)(x+4)}$ going to $\frac{1}{2x-1}$ without the need for 'seeing' the cancelling

For example $\frac{3(x+1)-2x-1}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A0. Incorrect line leading to solution.

Whereas $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A1

(b)

- M1 This is awarded for an attempt to make x or a swapped y the subject of the formula. The minimum criteria is that they start by multiplying by (2x-1) and finish with x= or swapped y=. Allow 'invisible' brackets.

- M1 For applying the order of operations correctly. Allow maximum of one 'slip'. Examples of this are

$$y = \frac{1}{2x-1} \rightarrow y(2x-1) = 1 \rightarrow 2x-1 = \frac{1}{y} \rightarrow x = \frac{\frac{1}{y}+1}{2} \text{ (allow slip on sign)}$$

$$y = \frac{1}{2x-1} \rightarrow y(2x-1) = 1 \rightarrow 2xy - y = 1 \rightarrow 2xy = 1 \pm y \rightarrow x = \frac{1 \pm y}{2y} \text{ (allow slip on sign)}$$

$$y = \frac{1}{2x-1} \rightarrow 2x-1 = \frac{1}{y} \rightarrow 2x = \frac{1}{y} + 1 \rightarrow x = \frac{1}{2y} + 1 \text{ (allow slip on } \div 2)$$

- A1 Must be written in terms of x but can be y = $\frac{1+x}{2x}$ or equivalent inc y = $\frac{\frac{1}{x}+1}{2}$, y = $\frac{x^{-1}+1}{2}$, y = $\frac{1}{2x} + \frac{1}{2}$

(c)

- B1 Accept x>0, (0,∞), domain is all values more than 0. **Do not accept** x≥ 0 , y>0, [0, ∞], $f^{-1}(x) > 0$

(d)

- M1 Attempt to write down fg(x) and set it equal to 1/7.

The order must be correct but accept incorrect or lack of bracketing. Eg $\frac{1}{2\ln x + 1 - 1} = \frac{1}{7}$

- A1 Achieving correctly the line $\ln(x+1) = 4$. Accept also $\ln(x+1)^2 = 8$

- M1 Moving from $\ln(x \pm A) = c$ $A \neq 0$ to $x =$ The ln work must be correct

Alternatively moving from $\ln(x+1)^2 = c$ to $x = \dots$

Full solutions to calculate x leading from $gf(x) = \frac{1}{7}$, that is $\ln\left(\frac{1}{2x-1} + 1\right) = \frac{1}{7}$ can score this mark.

- A1 Correct answer only = $e^4 - 1$. Accept $e^4 - e^0$

Question No	Scheme	Marks
8	(a) $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$ $= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1 - \frac{\sin A \sin B}{\cos A \cos B}} \quad (\div \cos A \cos B)$	M1A1
		M1

		$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$	A1 *	
(b)	$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{\tan\theta + \tan\frac{\pi}{6}}{1 - \tan\theta \tan\frac{\pi}{6}}$ $= \frac{\tan\theta + \frac{1}{\sqrt{3}}}{1 - \tan\theta \frac{1}{\sqrt{3}}}$ $= \frac{\sqrt{3}\tan\theta + 1}{\sqrt{3} - \tan\theta}$		M1	(4)
			M1	
			A1 *	
(c)	$\tan\left(\theta + \frac{\pi}{6}\right) = \tan(\pi - \theta).$ $\left(\theta + \frac{\pi}{6}\right) = (\pi - \theta)$ $\theta = \frac{5}{12}\pi$ $\tan\left(\theta + \frac{\pi}{6}\right) = \tan(2\pi - \theta)$ $\theta = \frac{11}{12}\pi$		M1	(3)
			dM1	
			ddM1 A1	
			dddM1	
			A1	(6)
			(13 MARKS)	

(a)

M1 Uses the identity $\left\{ \tan(A + B) = \frac{\sin(A+B)}{\cos(A+B)} \right\} = \frac{\sin A \cos B \pm \cos A \sin B}{\cos A \cos B \mp \sin A \sin B}$. Accept incorrect signs for this.
Just the right hand side is acceptable.

A1 Fully correct statement in terms of cos and sin $\left\{ \tan(A + B) \right\} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$

- M1 Divide **both** numerator and denominator by $\cos A \cos B$. This can be stated or implied by working. If implied you must have seen at least one term modified on both the numerator and denominator.
- A1* This is a given solution. The last two principal's reports have highlighted lack of evidence in such questions. Both sides of the identity must be seen or implied. Eg lhs=
The minimum expectation for full marks is

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} = \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1 - \frac{\sin A \sin B}{\cos A \cos B}} = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

The solution $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ scores M1A1M0A0

The solution $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} (\div \cos A \cos B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ scores M1A1M1A0

(b)

- M1 An attempt to use part (a) with $A=0$ and $B=\frac{\pi}{6}$. Seeing $\frac{\tan \theta + \tan \frac{\pi}{6}}{1 - \tan \theta \tan \frac{\pi}{6}}$ is enough evidence. Accept sign slips

- M1 Uses the identity $\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ in the rhs of the identity on both numerator and denominator

- A1* cso. This is a given solution. Both sides of the identity must be seen. All steps must be correct with no unreasonable jumps. Accept

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{\tan \theta + \tan \frac{\pi}{6}}{1 - \tan \theta \tan \frac{\pi}{6}} = \frac{\tan \theta + \frac{1}{\sqrt{3}}}{1 - \tan \theta \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} \tan \theta + 1}{\sqrt{3} - \tan \theta}$$

However the following is only worth 2 out of 3 as the last step is an unreasonable jump without further explanation

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{\tan \theta + \tan \frac{\pi}{6}}{1 - \tan \theta \tan \frac{\pi}{6}} = \frac{\tan \theta + \frac{\sqrt{3}}{3}}{1 - \tan \theta \frac{\sqrt{3}}{3}} = \frac{\sqrt{3} \tan \theta + 1}{\sqrt{3} - \tan \theta}$$

(c)

- M1 Use the given identity in (b) to obtain $\tan\left(\theta + \frac{\pi}{6}\right) = \tan(\pi - \theta)$. Accept sign slips

- dM1 Writes down an equation that will give one value of θ , usually $\theta + \frac{\pi}{6} = \pi - \theta$. This is dependent upon the first M mark. Follow through on slips

- ddM1 Attempts to solve their equation in θ . It must end $\theta =$ and the first two marks must have been scored.

- A1 Cso $\theta = \frac{5}{12}\pi$ or $\frac{11}{12}\pi$

- dddM1 Writes down an equation that would produce a second value of θ . Usually $\theta + \frac{\pi}{6} = 2\pi - \theta$

- A1 cso $\theta = \frac{5}{12}\pi$ (accept $\frac{\pi}{2.4}$) and $\frac{11}{12}\pi$ with no extra solutions in the range. Ignore extra solutions outside the range.

Note that under this method one correct solution would score 4 marks. A small number of candidates find the second solution only. They would score 1,1,1,1,0,0

Alternative to (a) starting from rhs

- M1 Uses correct identities for both $\tan A$ and $\tan B$ in the rhs expression. Accept only errors in signs

A1
$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1 - \frac{\sin A \sin B}{\cos A \cos B}}$$

- M1 Multiplies both numerator and denominator by $\cos A \cos B$. This can be stated or implied by working. If implied you must have seen at least one term modified on both the numerator and denominator

A1 This is a given answer. Correctly completes proof. All three expressions must be seen or implied.

$$\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} = \frac{\sin(A+B)}{\cos(A+B)} = \tan(A+B)$$

Alternative to (a) starting from both sides

The usual method can be marked like this

M1 Uses correct identities for both $\tan A$ and $\tan B$ in the rhs expression. Accept only errors in signs

$$A1 \quad \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1 - \frac{\sin A \sin B}{\cos A \cos B}}$$

M1 Multiplies both numerator and denominator by $\cos A \cos B$. This can be stated or implied by working. If implied you must have seen at least one term modified on both the numerator and denominator

$$A1 \quad \text{Completes proof. Starting now from the lhs writes } \tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

And then states that the lhs is equal to the rhs **Or** hence proven. There must be a statement of closure

Alternative to (b) from sin and cos

$$M1 \quad \text{Writes } \tan\left(\theta + \frac{\pi}{6}\right) = \frac{\sin\left(\theta + \frac{\pi}{6}\right)}{\cos\left(\theta + \frac{\pi}{6}\right)} = \frac{\sin\theta \cos\frac{\pi}{6} + \cos\theta \sin\frac{\pi}{6}}{\cos\theta \cos\frac{\pi}{6} - \sin\theta \sin\frac{\pi}{6}}$$

M1 Uses the identities $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ or in the rhs of the identity on both numerator and denominator and divides both numerator and denominator by $\cos\theta$ to produce an identity in $\tan\theta$

A1 As in original scheme

Alternative solution for c. Starting with $1 + \sqrt{3}\tan\theta = (\sqrt{3} - \tan\theta) \tan(\pi - \theta)$

Let $\tan\theta = t$

$$\begin{aligned} 1 + \sqrt{3}t &= (\sqrt{3} - t)(-t) \\ t^2 - 2\sqrt{3}t - 1 &= 0 \\ t &= \frac{2\sqrt{3} \pm \sqrt{(12+4)}}{2} \\ &= \sqrt{3} \pm 2 \end{aligned}$$

Must find an exact surd

$$\theta = \frac{5\pi}{12}, \frac{11\pi}{12}$$

Accept the use of a calculator for the A marks as long as there is an exact surd for the solution of the quadratic and exact answers are given.

- M1 Starting with $1 + \sqrt{3}\tan\theta = (\sqrt{3} - \tan\theta)\tan(\pi - \theta)$ expand $\tan(\pi - \theta)$ by the correct compound angle identity (or otherwise) and substitute $\tan\pi=0$ to produce an equation in $\tan\theta$
- dM1 Collect terms and produce a 3 term quadratic in $\tan\theta$
- ddM1 Correct use of quadratic formula to produce exact solutions to $\tan\theta$. All previous marks must have been scored.
- dddM1 All 3 previous marks must have been scored. This is for producing two exact values for θ
- A1 One solution $\frac{5}{12}\pi$ (accept $\frac{\pi}{2.4}$) or $\frac{11}{12}\pi$
- A1 Both solutions $\frac{5}{12}\pi$ (accept $\frac{\pi}{2.4}$) and $\frac{11}{12}\pi$ and no extra solutions inside the range. Ignore extra solutions outside the range.

Special case: Watch for candidates who write $\tan(\pi - \theta) = \tan(\pi) - \tan(\theta) = -\tan(\theta)$ and proceed correctly. They will lose the first mark but potentially can score the others.

Solutions in degrees

Apply as before. Lose the first correct mark that would have been scored-usually 75⁰

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Mark Scheme (Results)

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GCE Core Mathematics – C3 (6665/01)

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January 2013

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- Unless indicated in the mark scheme a correct answer with no working should gain full marks for that part of the question.

EDEXCEL GCE MATHEMATICS**General Instructions for Marking**

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used.

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - $\square$ The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.
8. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		•
aA	•	
bM1		•
bA1	•	
bB	•	
bM2		•
bA2		•

January 2013
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $-32 = (2w - 3)^5 \Rightarrow w = \frac{1}{2}$ oe	M1A1 (2)
	(b) $\frac{dy}{dx} = 5 \times (2x - 3)^4 \times 2$ or $10(2x - 3)^4$	M1A1
	When $x = \frac{1}{2}$, Gradient = 160	M1
	Equation of tangent is '160' = $\frac{y - (-32)}{x - \frac{1}{2}}$ oe	dM1
	$y = 160x - 112$ cso	A1 (5) (7 marks)

- (a) M1 Substitute $y = -32$ into $y = (2w - 3)^5$ **and** proceed to $w = \dots$. [Accept positive sign used of y , ie $y = +32$]
A1 Obtains w or $x = \frac{1}{2}$ oe with no incorrect working seen. Accept alternatives such as 0.5.
Sight of just the answer would score both marks as long as no incorrect working is seen.

- (b) M1 Attempts to differentiate $y = (2x - 3)^5$ using the chain rule.
Sight of $\pm A(2x - 3)^4$ where A is a non-zero constant is sufficient for the method mark.
A1 A correct (un simplified) form of the differential.
Accept $\frac{dy}{dx} = 5 \times (2x - 3)^4 \times 2$ or $\frac{dy}{dx} = 10(2x - 3)^4$
M1 This is awarded for an attempt to find the gradient of the tangent to the curve at P
Award for substituting their numerical value to part (a) into their differential to find the numerical gradient of the tangent
dM1 Award for a correct method to find an equation of the tangent to the curve at P . It is dependent upon the previous M mark being awarded.

Award for 'their 160' = $\frac{y - (-32)}{x - \text{their } \frac{1}{2}}$

If they use $y = mx + c$ it must be a full method, using $m =$ 'their 160', their ' $\frac{1}{2}$ ', and -32.

An attempt must be seen to find $c = \dots$

A1 cso $y = 160x - 112$. The question is specific and requires the answer in this form.
You may isw in this question after a correct answer.

Question Number	Scheme	Marks
2.	(a) $0 = e^{x-1} + x - 6 \Rightarrow x = \ln(6 - x) + 1$ (b) Sub $x_0 = 2$ into $x_{n+1} = \ln(6 - x_n) + 1 \Rightarrow x_1 = 2.3863$ AWRT 4 dp. $x_2 = 2.2847$ $x_3 = 2.3125$ (c) Chooses interval [2.3065, 2.3075] $g(2.3065) = -0.0002(7)$, $g(2.3075) = 0.004(4)$ Sign change, hence root (correct to 3dp)	M1A1* (2) M1, A1 (3) M1 dM1 A1 (3) (8 marks)

- (a) M1 Sets $g(x)=0$, and using correct $\ln$ work, makes the x of the e^{x-1} term the subject of the formula.
Look for $e^{x-1} + x - 6 = 0 \Rightarrow e^{x-1} = \pm 6 \pm x \Rightarrow x = \ln(\pm 6 \pm x) \pm 1$
Do not accept $e^{x-1} = 6 - x$ without firstly seeing $e^{x-1} + x - 6 = 0$ or a statement that $g(x)=0 \Rightarrow$
A1* cso. $x = \ln(6 - x) + 1$ **Note that this is a given answer (and a proof).**
‘Invisible’ brackets are allowed for the M but not the A
Do not accept recovery from earlier errors for the A mark. The solution below scores 0 marks.
 $0 = e^{x-1} + x - 6 \Rightarrow 0 = x - 1 + \ln(x - 6) \Rightarrow x = \ln(6 - x) + 1$
- (b) M1 Sub $x_0 = 2$ into $x_{n+1} = \ln(6 - x_n) + 1$ to produce a numerical value for x_1 .
Evidence for the award could be any of $\ln(6 - 2) + 1$, $\ln 4 + 1$, 2.3..... or awrt 2.4
A1 Answer correct to 4 dp $x_1 = 2.3863$.
The subscript is not important. Mark as the first value given/found.
A1 Awrt 4 dp. $x_2 = 2.2847$ and $x_3 = 2.3125$
The subscripts are not important. Mark as the second and third values given/found
- (c) M1 Chooses the interval [2.3065, 2.3075] or smaller containing the root 2.306558641
dM1 Calculates $g(2.3065)$ and $g(2.3075)$ with at least one of these correct to 1sf.
The answers can be rounded or truncated
 $g(2.3065) = -0.0003$ rounded, $g(2.3065) = -0.0002$ truncated
 $g(2.3075) = (+) 0.004$ rounded and truncated
A1 Both values correct (rounded or truncated),
A reason which could include change of sign, $>0 <0$, $g(2.3065) \times g(2.3075) < 0$
AND a minimal conclusion such as hence root, $\alpha = 2.307$ or $\square$
Do not accept continued iteration as question demands an interval to be chosen.

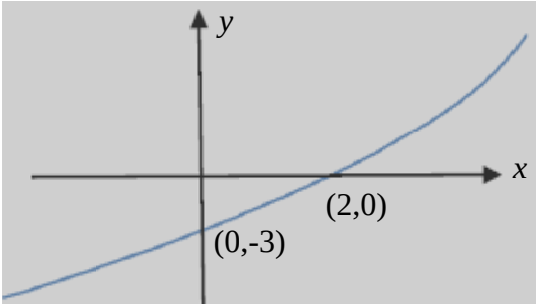
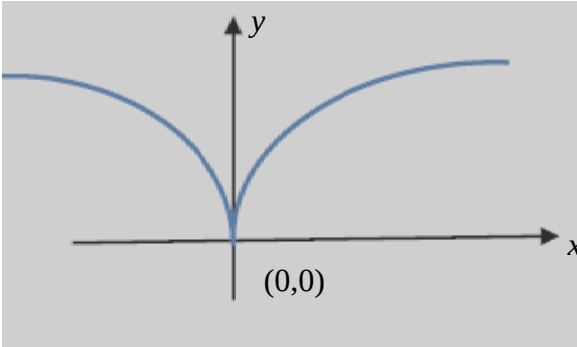
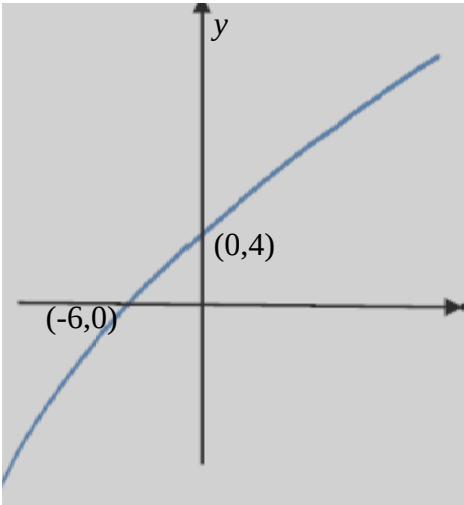
Alternative solution to (a) working backwards

- M1 Proceeds from $x = \ln(6 - x) + 1$ using correct exp work to=0
A1 **Arrives correctly** at $e^{x-1} + x - 6 = 0$ **and** makes a statement to the effect that this is $g(x)=0$

Alternative solution to (c) using $f(x) = \ln(6 - x) + 1 - x$ {Similarly $h(x) = x - 1 - \ln(6 - x)$ }

- M1 Chooses the interval [2.3065, 2.3075] or smaller containing the root 2.306558641
dM1 Calculates $f(2.3065)$ and $f(2.3075)$ with at least 1 correct rounded or truncated
 $f(2.3065) = 0.000074$. Accept 0.00007 rounded or truncated. Also accept 0.0001

$f(2.3075) = -0.0011..$ Accept -0.001 rounded or truncated

Question Number	Scheme	Marks
3.	(a) $ff(-3) = f(0) = 2$	M1, A1 (2)
	(b)  $y = f^{-1}(x)$ Shape (0, -3) and (2, 0)	B1 B1 (2)
	(c)  $y = f(x) - 2$ Shape (0, 0)	B1 B1 (2)
	(d)  $(-6, 0)$ and $(0, 4)$ Shape (-6, 0) or (0, 4) (-6, 0) and (0, 4)	B1 B1 B1 (3)
		(9 marks)

- (a) M1 A full method of finding $ff(-3)$. $f(0)$ is acceptable but $f(-3)=0$ is not.
Accept a solution obtained from two substitutions into the equation $y = \frac{2}{3}x + 2$ as the line passes through both points. Do not allow for $y = \ln(x+4)$, which only passes through one of the points.
- A1 Cao $ff(-3)=2$. Writing down 2 on its own is enough for both marks provided no incorrect working is seen.
- (b) B1 For the correct shape. Award this mark for an increasing function in quadrants 3, 4 and 1 only. Do not award if the curve bends back on itself or has a clear minimum
- B1 This is independent to the first mark and for the graph passing through (0, -3) and (2, 0)

Accept -3 and 2 marked on the correct axes.

Accept (-3,0) and (0,2) instead of (0,-3) and (2,0) as long as they are on the correct axes

Accept $P'=(0,-3)$, $Q'=(2,0)$ stated elsewhere as long as P' and Q' are marked in the correct place on the graph

There must be a graph for this to be awarded

- (c)
- B1 Award for a correct shape 'roughly' symmetrical about the y- axis. It must have a cusp and a gradient that 'decreases' either side of the cusp. Do not award if the graph has a clear maximum
- B1 (0,0) lies on their graph. Accept the graph passing through the origin without seeing (0, 0) marked
- (d)
- B1 Shape. The position is not important. The gradient should be always positive but decreasing
There should not be a clear maximum point.
- B1 The graph passes through (0,4) **or** (-6,0). See part (b) for allowed variations
- B1 The graph passes through (0,4) **and** (-6,0). See part (b) for allowed variations

Question Number	Scheme	Marks
4.	(a) $R^2 = 6^2 + 8^2 \Rightarrow R = 10$	M1A1
	$\tan \alpha = \frac{8}{6} \Rightarrow \alpha = \text{awrt } 0.927$	M1A1
		(4)
	(b)(i) $p(x) = \frac{4}{12 + 10 \cos(\theta - 0.927)}$	
	$p(x) = \frac{4}{12 - 10}$	M1
	Maximum = 2	A1
		(2)
	(b)(ii) $\theta - \text{'their } \alpha' = \pi$	M1
	$\theta = \text{awrt } 4.07$	A1
		(2)
		(8 marks)

- (a) M1 Using Pythagoras' Theorem with 6 and 8 to find R . Accept $R^2 = 6^2 + 8^2$
 If α has been found first accept $R = \pm \frac{8}{\sin' \alpha'}$ or $R = \pm \frac{6}{\cos' \alpha'}$
 A1 $R = 10$. Many candidates will just write this down which is fine for the 2 marks.
 Accept ± 10 but not -10
 M1 For $\tan \alpha = \pm \frac{8}{6}$ or $\tan \alpha = \pm \frac{6}{8}$
 If R is used then only accept $\sin \alpha = \pm \frac{8}{R}$ or $\cos \alpha = \pm \frac{6}{R}$
 A1 $\alpha = \text{awrt } 0.927$. Note that 53.1° is A0
- (b) Note that (b)(i) and (b)(ii) can be marked together
- (i) M1 Award for $p(x) = \frac{4}{12 - 'R'}$.
 A1 Cao $p(x)_{\max} = 2$.
 The answer is acceptable for both marks as long as no incorrect working is seen
- (ii) M1 For setting $\theta - \text{'their } \alpha' = \pi$ and proceeding to $\theta = ..$
 If working exclusively in degrees accept $\theta - \text{'their } \alpha' = 180$
 Do not accept mixed units
 A1 $\theta = \text{awrt } 4.07$. If the final A mark in part (a) is lost for 53.1 , then accept awrt 233.1

Question Number	Scheme	Marks
5.	(i)(a) $\frac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$ $= 3x^2 \ln 2x + x^2$	M1A1A1 (3)
	(i)(b) $\frac{dy}{dx} = 3(x + \sin 2x)^2 \times (1 + 2 \cos 2x)$	B1M1A1 (3)
	(ii) $\frac{dx}{dy} = -\operatorname{cosec}^2 y$ $\frac{dy}{dx} = -\frac{1}{\operatorname{cosec}^2 y}$	M1A1 M1
	Uses $\operatorname{cosec}^2 y = 1 + \cot^2 y$ and $x = \cot y$ in $\frac{dy}{dx}$ or $\frac{dx}{dy}$ to get an expression in x	
	$\frac{dy}{dx} = -\frac{1}{\operatorname{cosec}^2 y} = -\frac{1}{1 + \cot^2 y} = -\frac{1}{1 + x^2}$ cs0	M1, A1* (5) (11 marks)

- (i)(a) M1 Applies the product rule $vu' + uv'$ to $x^3 \ln 2x$.
If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out $u=..., u'=..., v=..., v'=...$ followed by their $vu' + uv'$) then only accept answers of the form
- $$Ax^2 \times \ln 2x + x^3 \times \frac{B}{x} \quad \text{where } A, B \text{ are constants} \neq 0$$
- A1 One term correct, either $3x^2 \times \ln 2x$ or $x^3 \times \frac{1}{2x} \times 2$
- A1 Cao. $\frac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$. The answer does not need to be simplified.
For reference the simplified answer is $\frac{dy}{dx} = 3x^2 \ln 2x + x^2 = x^2(3 \ln 2x + 1)$
- (i)(b) B1 Sight of $(x + \sin 2x)^2$
- M1 For applying the chain rule to $(x + \sin 2x)^3$. If the rule is quoted it must be correct. If it is not quoted possible forms of evidence could be sight of $C(x + \sin 2x)^2 \times (1 \pm D \cos 2x)$ where C and D are non-zero constants.
Alternatively accept $u = x + \sin 2x$, $u' =$ followed by $Cu^2 \times \text{their } u'$
Do not accept $C(x + \sin 2x)^2 \times 2 \cos 2x$ unless you have evidence that this is their u'
Allow 'invisible' brackets for this mark, ie. $C(x + \sin 2x)^2 \times 1 \pm D \cos 2x$
- A1 Cao $\frac{dy}{dx} = 3(x + \sin 2x)^2 \times (1 + 2 \cos 2x)$. There is no requirement to simplify this.

You may ignore subsequent working (isw) after a correct answer in part (i)(a) and (b)

- (ii)
- M1 Writing the derivative of $\cot y$ as $-\operatorname{cosec}^2 y$. It must be in terms of y
- A1 $\frac{dx}{dy} = -\operatorname{cosec}^2 y$ or $1 = -\operatorname{cosec}^2 y \frac{dy}{dx}$. Both lhs and rhs must be correct.
- M1 Using $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$
- M1 Using $\operatorname{cosec}^2 y = 1 + \cot^2 y$ **and** $x = \cot y$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ just in terms of x .
- A1 $\operatorname{csc} \frac{dy}{dx} = -\frac{1}{1+x^2}$

Alternative to (a)(i) when $\ln(2x)$ is written $\ln x + \ln 2$

- M1 Writes $x^3 \ln 2x$ as $x^3 \ln 2 + x^3 \ln x$.
Achieves Ax^2 for differential of $x^3 \ln 2$ and applies the product rule $vu' + uv'$ to $x^3 \ln x$.
- A1 Either $3x^2 \times \ln 2 + 3x^2 \ln x$ or $x^3 \times \frac{1}{x}$
- A1 A correct (un simplified) answer. Eg $3x^2 \times \ln 2 + 3x^2 \ln x + x^3 \times \frac{1}{x}$

Alternative to 5(ii) using quotient rule

- M1 Writes $\cot y$ as $\frac{\cos y}{\sin y}$ and applies the quotient rule, a form of which appears in the formula book. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out $u=..., u'=..., v=..., v'=...$ followed by their $\frac{vu' - uv'}{v^2}$)
- only accept answers of the form $\frac{\sin y \times \pm \sin y - \cos y \times \pm \cos y}{(\sin y)^2}$
- A1 Correct un simplified answer with both lhs and rhs correct.
 $\frac{dx}{dy} = \frac{\sin y \times -\sin y - \cos y \times \cos y}{(\sin y)^2} = \{-1 - \cot^2 y\}$
- M1 Using $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$
- M1 Using $\sin^2 y + \cos^2 y = 1$, $\frac{1}{\sin^2 y} = \operatorname{cosec}^2 y$ and $\operatorname{cosec}^2 y = 1 + \cot^2 y$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in x
- A1 $\operatorname{csc} \frac{dy}{dx} = -\frac{1}{1+x^2}$

Alternative to 5(ii) using the chain rule, first two marks

M1 Writes $\cot y$ as $(\tan y)^{-1}$ and applies the chain rule (or quotient rule).

Accept answers of the form $-(\tan y)^{-2} \times \sec^2 y$

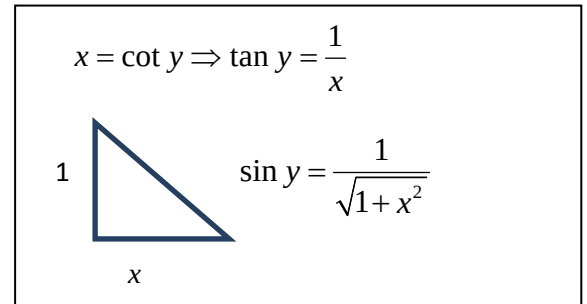
A1 Correct un simplified answer with both lhs and rhs correct.

$$\frac{dx}{dy} = -(\tan y)^{-2} \times \sec^2 y$$

Alternative to 5(ii) using a triangle – last M1

M1 Uses triangle with $\tan y = \frac{1}{x}$ to find $\sin y$

and get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ just in terms of x



Question Number	Scheme	Marks
6.	(i) $(\sin 22.5 + \cos 22.5)^2 = \sin^2 22.5 + \cos^2 22.5 + \dots$ $= \sin^2 22.5 + \cos^2 22.5 + 2\sin 22.5 \cos 22.5$ States or uses $\sin^2 22.5 + \cos^2 22.5 = 1$ Uses $2\sin x \cos x = \sin 2x \Rightarrow 2\sin 22.5 \cos 22.5 = \sin 45$ $(\sin 22.5 + \cos 22.5)^2 = 1 + \sin 45$ $= 1 + \frac{\sqrt{2}}{2}$ or $1 + \frac{1}{\sqrt{2}}$ cso (ii) (a) $\cos 2\theta + \sin \theta = 1 \Rightarrow 1 - 2\sin^2 \theta + \sin \theta = 1$ $\sin \theta - 2\sin^2 \theta = 0$ $2\sin^2 \theta - \sin \theta = 0$ or $k = 2$ (b) $\sin \theta(2\sin \theta - 1) = 0$ $\sin \theta = 0, \sin \theta = \frac{1}{2}$ Any two of 0, 30, 150, 180 All four answers 0, 30, 150, 180	M1 B1 M1 A1 A1 M1 A1* M1 A1 B1 A1 (5) (2) (4) (11 marks)

- | | | |
|-------|-----|--|
| (i) | M1 | Attempts to expand $(\sin 22.5 + \cos 22.5)^2$. Award if you see $\sin^2 22.5 + \cos^2 22.5 + \dots$
There must be $>$ two terms. Condone missing brackets ie $\sin 22.5^2 + \cos 22.5^2 + \dots$ |
| | B1 | Stating or using $\sin^2 22.5 + \cos^2 22.5 = 1$. Accept $\sin 22.5^2 + \cos 22.5^2 = 1$ as the intention is clear.
Note that this may also come from using the double angle formula
$\sin^2 22.5 + \cos^2 22.5 = \left(\frac{1 - \cos 45}{2}\right) + \left(\frac{1 + \cos 45}{2}\right) = 1$ |
| | M1 | Uses $2\sin x \cos x = \sin 2x$ to write $2\sin 22.5 \cos 22.5$ as $\sin 45$ or $\sin(2 \times 22.5)$ |
| | A1 | Reaching the intermediate answer $1 + \sin 45$ |
| | A1 | $\cos 1 + \frac{\sqrt{2}}{2}$ or $1 + \frac{1}{\sqrt{2}}$. Be aware that both 1.707 and $\frac{2 + \sqrt{2}}{2}$ can be found by using a calculator
for $1 + \sin 45$. Neither can be accepted on their own without firstly seeing one of the two answers given above. Each stage should be shown as required by the mark scheme.
Note that if the candidates use $(\sin \theta + \cos \theta)^2$ they can pick up the first M and B marks, but no others until they use $\theta = 22.5$. All other marks then become available. |
| (ii) | M1 | Substitutes $\cos 2\theta = 1 - 2\sin^2 \theta$ in $\cos 2\theta + \sin \theta = 1$ to produce an equation in $\sin \theta$ only.
It is acceptable to use $\cos 2\theta = 2\cos^2 \theta - 1$ or $\cos^2 \theta - \sin^2 \theta$ as long as the $\cos^2 \theta$ is subsequently replaced by $1 - \sin^2 \theta$ |
| | A1* | Obtains the correct simplified equation in $\sin \theta$.
$\sin \theta - 2\sin^2 \theta = 0$ or $\sin \theta = 2\sin^2 \theta$ must be written in the form $2\sin^2 \theta - \sin \theta = 0$ as required by the question. Also accept $k = 2$ as long as no incorrect working is seen. |
| (iib) | M1 | Factorises or divides by $\sin \theta$. For this mark $1 = 'k' \sin \theta$ is acceptable. If they have a 3 TQ in $\sin \theta$ this can be scored for correct factorisation |
| | A1 | Both $\sin \theta = 0$, and $\sin \theta = \frac{1}{2}$ |
| | B1 | Any two answers from 0, 30, 150, 180. |
| | A1 | All four answers 0, 30, 150, 180 with no extra solutions inside the range. Ignore solutions outside the range. |

Question Number	Scheme	Marks
6.alt 1	(i) $(\sin 22.5 + \cos 22.5)^2 = \sin^2 22.5 + \cos^2 22.5 + \dots$ $= \sin^2 22.5 + \cos^2 22.5 + 2 \sin 22.5 \cos 22.5$ States or uses $\sin^2 22.5 + \cos^2 22.5 = 1$ Uses $2 \sin x \cos x = 2 \sqrt{\frac{1 - \cos 2x}{2}} \sqrt{\frac{\cos 2x + 1}{2}} \Rightarrow \sqrt{1 - \cos 45} \sqrt{1 + \cos 45}$ $= \sqrt{1 - \cos^2 45}$ Hence $(\sin 22.5 + \cos 22.5)^2 = 1 + \frac{\sqrt{2}}{2}$ or $1 + \frac{1}{\sqrt{2}}$	M1 B1 M1 A1 A1 (5)

Question Number	Scheme	Marks
6.alt 2	(i) Uses Factor Formula $(\sin 22.5 + \sin 67.5)^2 = (2 \sin 45 \cos 22.5)^2$ Reaching the stage $= 2 \cos^2 22.5$ Uses the double angle formula $= 2 \cos^2 22.5 = 1 + \cos 45$ $= 1 + \frac{\sqrt{2}}{2}$ or $1 + \frac{1}{\sqrt{2}}$	M1,A1 B1 M1 A1 (5)

Question Number	Scheme	Marks
6.alt 3	(i) Uses Factor Formula $(\cos 67.5 + \cos 22.5)^2 = (2 \cos 45 \cos 22.5)^2$ Reaching the stage $= 2 \cos^2 22.5$ Uses the double angle formula $= 2 \cos^2 22.5 = 1 + \cos 45$ $= 1 + \frac{\sqrt{2}}{2}$ or $1 + \frac{1}{\sqrt{2}}$	M1,A1 B1 M1 A1 (5)

Question Number	Scheme	Marks
7.	(a) $\frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x+2)(x^2+5)} = \frac{2(x^2+5) + 4(x+2) - 18}{(x+2)(x^2+5)}$ $= \frac{2x(x+2)}{(x+2)(x^2+5)}$ $= \frac{2x}{(x^2+5)}$ (b) $h'(x) = \frac{(x^2+5) \times 2 - 2x \times 2x}{(x^2+5)^2}$ $h'(x) = \frac{10-2x^2}{(x^2+5)^2}$ cso (c) Maximum occurs when $h'(x) = 0 \Rightarrow 10 - 2x^2 = 0 \Rightarrow x = \dots$ $\Rightarrow x = \sqrt{5}$ When $x = \sqrt{5} \Rightarrow h(x) = \frac{\sqrt{5}}{5}$ Range of $h(x)$ is $0 \leq h(x) \leq \frac{\sqrt{5}}{5}$	M1A1 M1 A1* (4) M1A1 A1 (3) M1 A1 M1,A1 A1ft (5) (12 marks)

(a) M1 Combines the three fractions to form a single fraction with a common denominator.

Allow errors on the numerator but at least one must have been adapted.

Condone 'invisible' brackets for this mark.

Accept three separate fractions with the same denominator.

Amongst possible options allowed for this method are

$$\frac{2x^2+5+4x+2-18}{(x+2)(x^2+5)} \quad \text{Eg 1 An example of 'invisible' brackets}$$

$$\frac{2(x^2+5)}{(x+2)(x^2+5)} + \frac{4}{(x+2)(x^2+5)} - \frac{18}{(x+2)(x^2+5)} \quad \text{Eg 2 An example of an error (on middle term), 1st term has been adapted}$$

$$\frac{2(x^2+5)^2(x+2) + 4(x+2)^2(x^2+5) - 18(x^2+5)(x+2)}{(x+2)^2(x^2+5)^2} \quad \text{Eg 3 An example of a correct fraction with a different denominator}$$

A1 Award for a correct un simplified fraction with the correct (lowest) common denominator.

$$\frac{2(x^2+5) + 4(x+2) - 18}{(x+2)(x^2+5)}$$

Accept if there are three separate fractions with the correct (lowest) common denominator.

$$\text{Eg } \frac{2(x^2+5)}{(x+2)(x^2+5)} + \frac{4(x+2)}{(x+2)(x^2+5)} - \frac{18}{(x+2)(x^2+5)}$$

Note, Example 3 would score M1A0 as it does not have the correct lowest common denominator
 M1 There must be a single denominator. Terms must be collected on the numerator.
 A factor of $(x+2)$ must be taken out of the numerator and then cancelled with one in the denominator. The cancelling may be assumed if the term 'disappears'

A1* Cso $\frac{2x}{(x^2+5)}$ This is a given solution and this mark should be withheld if there are any errors

(b) M1 Applies the quotient rule to $\frac{2x}{(x^2+5)}$, a form of which appears in the formula book.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out $u=..., u'=..., v=..., v'=...$ followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{(x^2+5) \times A - 2x \times Bx}{(x^2+5)^2} \quad \text{where } A, B > 0$$

A1 Correct unsimplified answer $h'(x) = \frac{(x^2+5) \times 2 - 2x \times 2x}{(x^2+5)^2}$

A1 $h'(x) = \frac{10-2x^2}{(x^2+5)^2}$ The correct simplified answer. Accept $\frac{2(5-x^2)}{(x^2+5)^2}$, $\frac{-2(x^2-5)}{(x^2+5)^2}$, $\frac{10-2x^2}{(x^4+10x^2+25)}$

DO NOT ISW FOR PART (b). INCORRECT SIMPLIFICATION IS A0

(c) M1 Sets their $h'(x)=0$ and proceeds with a correct method to find x . There must have been an attempt to differentiate. Allow numerical errors but do not allow solutions from 'unsolvable' equations.

A1 Finds the correct x value of the maximum point $x=\sqrt{5}$.

Ignore the solution $x=-\sqrt{5}$ but withhold this mark if other positive values found.

M1 Substitutes their answer into their $h'(x)=0$ in $h(x)$ to determine the maximum value

A1 Cso-the maximum value of $h(x) = \frac{\sqrt{5}}{5}$. Accept equivalents such as $\frac{2\sqrt{5}}{10}$ **but not** 0.447

A1ft Range of $h(x)$ is $0 \leq h(x) \leq \frac{\sqrt{5}}{5}$. Follow through on their maximum value if the M's have been

scored. Allow $0 \leq y \leq \frac{\sqrt{5}}{5}$, $0 \leq \text{Range} \leq \frac{\sqrt{5}}{5}$, $\left[0, \frac{\sqrt{5}}{5}\right]$ but not $0 \leq x \leq \frac{\sqrt{5}}{5}$, $\left(0, \frac{\sqrt{5}}{5}\right)$

If a candidate attempts to work out $h^{-1}(x)$ in (b) and does all that is required for (b) in (c), then allow.

Do not allow $h^{-1}(x)$ to be used for $h'(x)$ in part (c). For this question (b) and (c) can be scored together.

Alternative to (b) using the product rule

M1 Sets $h(x) = 2x(x^2+5)^{-1}$ and applies the product rule $vu'+uv'$ with terms being $2x$ and $(x^2+5)^{-1}$
 If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out $u=..., u'=..., v=..., v'=...$ followed by their $vu'+uv'$) then only accept answers of the form

$$(x^2+5)^{-1} \times A + 2x \times \pm Bx(x^2+5)^{-2}$$

A1 Correct un simplified answer $(x^2+5)^{-1} \times 2 + 2x \times -2x(x^2+5)^{-2}$

A1 The question asks for $h'(x)$ to be put in its simplest form. Hence in this method the terms need to be combined to form a single correct expression.

For a correct simplified answer accept

$$h'(x) = \frac{10 - 2x^2}{(x^2 + 5)^2} = \frac{2(5 - x^2)}{(x^2 + 5)^2} = \frac{-2(x^2 - 5)}{(x^2 + 5)^2} = (10 - 2x^2)(x^2 + 5)^{-2}$$

Question Number	Scheme	Marks
8.	(a) (£) 19500 (b) $9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$ $17e^{-0.25t} + 2e^{-0.5t} = 9$ $(\times e^{0.5t}) \Rightarrow 17e^{0.25t} + 2 = 9e^{0.5t}$ $0 = 9e^{0.5t} - 17e^{0.25t} - 2$ $0 = (9e^{0.25t} + 1)(e^{0.25t} - 2)$ $e^{0.25t} = 2$ $t = 4\ln(2) \text{ oe}$ (c) $\left(\frac{dV}{dt}\right) = -4250e^{-0.25t} - 1000e^{-0.5t}$ When $t=8$ Decrease = 593 (£/year)	B1 (1) M1 M1 A1 A1 (4) M1A1 M1A1 (4) (9 marks)

- (a) B1 19500. The £ sign is not important for this mark
- (b) M1 Substitute $V=9500$, collect terms and set on 1 side of an equation $=0$. Indices must be correct
Accept $17000e^{-0.25t} + 2000e^{-0.5t} - 9000 = 0$ and $17000x + 2000x^2 - 9000 = 0$ where $x = e^{-0.25t}$
- M1 Factorise the quadratic in $e^{0.25t}$ or $e^{-0.25t}$
For your information the factorised quadratic in $e^{-0.25t}$ is $(2e^{-0.25t} - 1)(e^{-0.25t} + 9) = 0$
Alternatively let ' x ' = $e^{0.25t}$ or otherwise and factorise a quadratic equation in x
- A1 Correct solution of the quadratic. Either $e^{0.25t} = 2$ or $e^{-0.25t} = \frac{1}{2}$ oe.
- A1 Correct exact value of t . Accept variations of $4\ln(2)$, such as $\ln(16)$, $\frac{\ln(\frac{1}{2})}{-0.25}$, $\frac{\ln(2)}{0.25}$, $-4\ln(\frac{1}{2})$
- (c) M1 Differentiates $V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$ by the chain rule.
Accept answers of the form $\left(\frac{dV}{dt}\right) = \pm Ae^{-0.25t} \pm Be^{-0.5t}$ A, B are constants $\neq 0$
- A1 Correct derivative $\left(\frac{dV}{dt}\right) = -4250e^{-0.25t} - 1000e^{-0.5t}$.
There is no need for it to be simplified so accept
 $\left(\frac{dV}{dt}\right) = 17000 \times -0.25e^{-0.25t} + 2000 \times -0.5e^{-0.5t}$ oe
- M1 Substitute $t=8$ into their $\frac{dV}{dt}$.
This is not dependent upon the first M1 but there must have been some attempt to differentiate.
Do not accept $t=8$ in V

- A1 ± 593 . Ignore the sign and the units. If the candidate then divides by 8, withhold this mark. **This would not be isw. Be aware that sub $t=8$ into V first and then differentiating can achieve 593. This is M0A0M0A0.**

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Core Mathematics C3 (6665)

Summer 2005

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Mark Scheme (Results)

June 2005
6665 Core C3
Mark Scheme

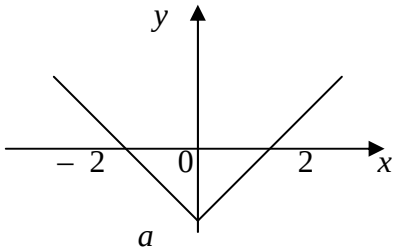
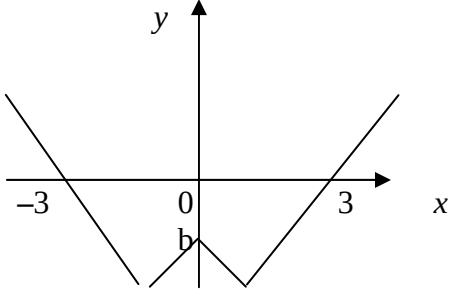
Question Number	Scheme	Marks
1. (a)	Dividing by $\cos^2 \theta$: $\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} \equiv \frac{1}{\cos^2 \theta}$ Completion : $1 + \tan^2 \theta \equiv \sec^2 \theta$ (no errors seen)	M1 A1 (2)
(b)	Use of $1 + \tan^2 \theta = \sec^2 \theta$: $2(\sec^2 \theta - 1) + \sec \theta = 1$ $[2\sec^2 \theta + \sec \theta - 3 = 0]$ Factorising or solving: $(2\sec \theta + 3)(\sec \theta - 1) = 0$ $[\sec \theta = -\frac{3}{2} \text{ or } \sec \theta = 1]$ $\theta = 0$ $\cos \theta = -\frac{2}{3}$; $\theta_1 = 131.8^\circ$ $\theta_2 = 228.2^\circ$ $[A1ft \text{ for } \theta_2 = 360^\circ - \theta_1]$	M1 M1 B1 M1 A1 A1√ (6) [8]

Question Number	Scheme	Marks
2. (a)	(i) $6 \sin x \cos x + 2 \sec 2x \tan 2x$ or $3 \sin 2x + 2 \sec 2x \tan 2x$ [M1 for $6 \sin x$] (ii) $3(x + \ln 2x)^2 \left(1 + \frac{1}{x}\right)$ [B1 for $3(x + \ln 2x)^2$]	M1A1A1 (3) B1M1A1 (3)
(b)	Differentiating numerator to obtain $10x - 10$ Differentiating denominator to obtain $2(x-1)$ Using quotient rule formula correctly: To obtain $\frac{dy}{dx} = \frac{(x-1)^2(10x-10) - (5x^2-10x+9)2(x-1)}{(x-1)^4}$ Simplifying to form $\frac{2(x-1)[5(x-1)^2 - (5x^2-10x+9)]}{(x-1)^4}$ $= -\frac{8}{(x-1)^3}$ * (c.s.o.) Alternatives for (b) Either Using product rule formula correctly: Obtaining $10x - 10$ Obtaining $-2(x-1)^{-3}$ To obtain $\frac{dy}{dx} = (5x^2-10x+9)\{-2(x-1)^{-3}\} + (10x-10)(x-1)^{-2}$ Simplifying to form $\frac{10(x-1)^2 - 2(5x^2-10x+9)}{(x-1)^3}$ $= -\frac{8}{(x-1)^3}$ * (c.s.o.) Or Splitting fraction to give $5 + \frac{4}{(x-1)^2}$ Then differentiating to give answer	B1 B1 M1 A1 M1 A1 (6) [12] M1 B1 B1 A1 cao M1 A1 (6) M1 B1 B1 M1 A1 A1 (6)

Question Number	Scheme	Marks
3(a)	$\frac{5x+1}{(x+2)(x-1)} - \frac{3}{x+2}$ $= \frac{5x+1-3(x-1)}{(x+2)(x-1)}$ M1 for combining fractions even if the denominator is not lowest common $= \frac{2x+4}{(x+2)(x-1)} = \frac{2(x+2)}{(x+2)(x-1)} = \frac{2}{x-1} \quad *$ M1 must have linear numerator	B1 M1 M1 A1 cso (4)
(b)	$y = \frac{2}{x-1} \Rightarrow xy - y = 2 \Rightarrow xy = 2 + y$ $f^{-1}(x) = \frac{2+x}{x} \quad \text{o.e.}$	M1A1 A1 (3)
(c)	$fg(x) = \frac{2}{x^2+4} \quad (\text{attempt}) \quad \left[\frac{2}{g-1} \right]$ Setting $\frac{2}{x^2+4} = \frac{1}{4}$ and finding $x^2 = \dots$; $x = \pm 2$	M1 M1; A1 (3) [10]

Question Number	Scheme	Marks
4	(a) $f'(x) = 3e^x - \frac{1}{2x}$ (b) $3e^x - \frac{1}{2x} = 0$ $\Rightarrow 6\alpha e^\alpha = 1 \quad \Rightarrow \alpha = \frac{1}{6} e^{-\alpha} \quad (*)$ (c) $x_1 = 0.0613..., x_2 = 0.1568..., x_3 = 0.1425..., x_4 = 0.1445...$ [M1 at least x_1 correct, A1 all correct to 4 d.p.] (d) Using $f'(x) = 3e^x - \frac{1}{2x}$ with suitable interval e.g. $f'(0.14425) = -0.0007$ $f'(0.14435) = +0.002(1)$ Accuracy (change of sign and correct values)	M1A1A1 (3) M1 A1 cso (2) M1 A1 (2) M1 A1 (2) [9]

Question Number	Scheme	Marks
5. (a)	$\cos 2A = \cos^2 A - \sin^2 A$ (+ use of $\cos^2 A + \sin^2 A \equiv 1$)	M1
	$= (1 - \sin^2 A); -\sin^2 A = 1 - 2 \sin^2 A$ (*)	A1 (2)
	(b) $2 \sin 2\theta - 3 \cos 2\theta - 3 \sin \theta + 3 \equiv 4 \sin \theta \cos \theta; -3(1 - 2 \sin^2 \theta) - 3 \sin \theta + 3$	B1; M1
	$\equiv 4 \sin \theta \cos \theta + 6 \sin^2 \theta - 3 \sin \theta$	M1
	$\equiv \sin \theta (4 \cos \theta + 6 \sin \theta - 3)$ (*)	A1 (4)
(c)	$4 \cos \theta + 6 \sin \theta \equiv R \sin \theta \cos \alpha + R \cos \theta \sin \alpha$ Complete method for R (may be implied by correct answer) $[R^2 = 4^2 + 6^2, R \sin \alpha = 4, R \cos \alpha = 6]$ $R = \sqrt{52} \text{ or } 7.21$ Complete method for α; $\alpha = 0.588$ (allow 33.7°)	M1 A1 M1 A1 (4)
(d)	$\sin \theta (4 \cos \theta + 6 \sin \theta - 3) = 0$ $\theta = 0$ $\sin(\theta + 0.588) = \frac{3}{\sqrt{52}} = 0.4160.. \text{ (24.6}^\circ\text{)}$ $\theta + 0.588 = (0.4291), 2.7125 \text{ [or } \theta + 33.7^\circ = (24.6^\circ), 155.4^\circ]$ $\theta = 2.12 \text{ rad}$	M1 B1 M1 dM1 A1 (5)
[15]		

Question Number	Scheme	Marks
6. (a)	 Translation $\leftarrow$ by 1 Intercepts correct	M1 A1 (2)
(b)	 $x \geq 0$, correct “shape” provided graph is not original graph Reflection in y-axis Intercepts correct	B1 B1✓ B1 (3)
(c)	$a = -2, \quad b = -1$	B1B1 (2)
(d)	Intersection of $y = 5x$ with $y = -x - 1$ Solving to give $x = -\frac{1}{6}$ [Notes: (i) If both values found for $5x = -x - 1$ and $5x = x - 3$, or solved algebraically, can score 3 out of 4 for $x = -\frac{1}{6}$ and $x = -\frac{3}{4}$; required to eliminate $x = -\frac{3}{4}$ for final mark. (ii) Squaring approach: M1 correct method, $24x^2 + 22x + 3 = 0$ (correct 3 term quadratic, any form) A1 Solving M1, Final correct answer A1.]	M1A1 M1A1 (4) [11]

7. (a)	Setting $p = 300$ at $t = 0 \Rightarrow 300 = \frac{2800a}{1+a}$ $(300 = 2500a); \quad a = 0.12 \text{ (c.s.o) } *$	M1
(b)	$1850 = \frac{2800(0.12)e^{0.2t}}{1 + 0.12e^{0.2t}} ; \quad e^{0.2t} = 16.2...$	dM1A1 (3)
	Correctly taking logs to $0.2 t = \ln k$ $t = 14 \quad (13.9..)$	M1A1
(c)	Correct derivation: (Showing division of num. and den. by $e^{0.2t}$; using a)	M1
		A1 (4)
(d)	Using $t \rightarrow \infty, e^{-0.2t} \rightarrow 0,$ $p \rightarrow \frac{336}{0.12} = 2800$	B1 (1)
		M1
		A1 (2)
		[10]

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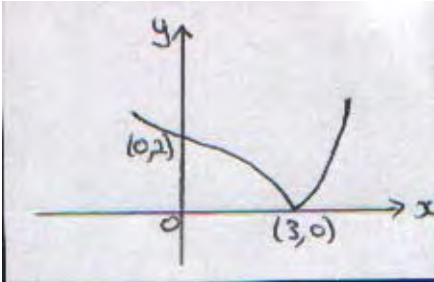
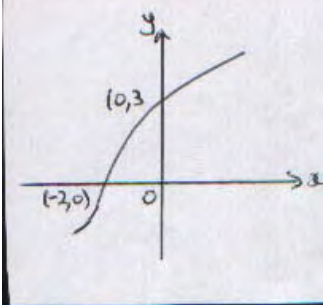
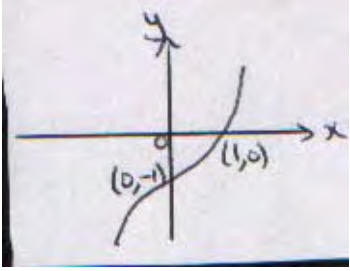
Core Mathematics C3 (6665)

June 2006

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Mark Scheme (Results)

Question Number	Scheme	Marks
1. (a)	$\frac{(3x+2)(x-1)}{(x+1)(x-1)}, = \frac{3x+2}{x+1}$ Notes M1 attempt to factorise numerator, <i>usual rules</i> B1 factorising denominator seen anywhere in (a), A1 given answer If factorisation of denom. not seen, correct answer implies B1	M1B1, A1 (3)
(b)	Expressing over common denominator $\frac{3x+2}{x+1} - \frac{1}{x(x+1)} = \frac{x(3x+2)-1}{x(x+1)}$ [Or “Otherwise” : $\frac{(3x^2 - x - 2)x - (x-1)}{x(x^2 - 1)}$] Multiplying out numerator and attempt to factorise $[3x^2 + 2x - 1 \equiv (3x-1)(x+1)]$ Answer: $\frac{3x-1}{x}$	M1 M1 A1 (3) (6 marks)
2. (a)	$\frac{dy}{dx} = 3e^{3x} + \frac{1}{x}$ Notes B1 $3e^{3x}$ M1 : $\frac{a}{bx}$ A1: $3e^{3x} + \frac{1}{x}$	B1M1A1(3)
(b)	$(5+x^2)^{\frac{1}{2}}$ $\frac{3}{2}(5+x^2)^{\frac{1}{2}} \cdot 2x = 3x(5+x^2)^{\frac{1}{2}}$ M1 for $kx(5+x^2)^m$	B1 M1 A1 (3) (6 marks)

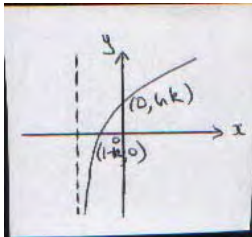
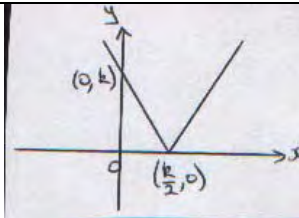
Question Number	Scheme	Marks
3. (a)	 Mod graph, reflect for $y < 0$ (0, 2), (3, 0) or marked on axes Correct shape, including cusp	M1 A1 A1 (3)
(b)	 Attempt at reflection in $y = x$ Curvature correct (-2, 0), (0, 3) or equiv.	M1 A1 B1 (3)
(c)	 Attempt at 'stretches' (0, -1) or equiv. (1, 0)	M1 B1 B1 (3) (9 marks)
4. (a)	425 °C	B1 (1)
(b)	$300 = 400e^{-0.05t} + 25 \Rightarrow 400e^{-0.05t} = 275$ sub. $T = 300$ and attempt to rearrange to $e^{-0.05t} = a$, where $a \in \mathbb{Q}$ $e^{-0.05t} = \frac{275}{400}$ M1 correct application of logs $t = 7.49$	M1 A1 M1 A1 (4)
(c)	$\frac{dT}{dt} = -20 e^{-0.05t} \quad (\text{M1 for } ke^{-0.05t})$ At $t = 50$, rate of decrease = $(\pm) 1.64$ °C/min	M1 A1 A1 (3)
(d)	$T > 25$, (since $e^{-0.05t} \rightarrow 0$ as $t \rightarrow \infty$)	B1 (1)
		(9 marks)

Question Number	Scheme	Marks
5. (a)	Using product rule: $\frac{dy}{dx} = 2 \tan 2x + 2(2x - 1) \sec^2 2x$ Use of “$\tan 2x = \frac{\sin 2x}{\cos 2x}$” and “$\sec 2x = \frac{1}{\cos 2x}$” $[= 2 \frac{\sin 2x}{\cos 2x} + 2(2x - 1) \frac{1}{\cos^2 2x}]$ Setting $\frac{dy}{dx} = 0$ and multiplying through to eliminate fractions $[\Rightarrow 2 \sin 2x \cos 2x + 2(2x - 1) = 0]$ Completion: producing $4k + \sin 4k - 2 = 0$ with no wrong working seen and at least previous line seen. AG	M1 A1 A1 M1 M1 A1* (6)
(b)	$x_1 = 0.2670, x_2 = 0.2809, x_3 = 0.2746, x_4 = 0.2774,$ Note: M1 for first correct application, first A1 for two correct, second A1 for all four correct Max –1 deduction, if ALL correct to > 4 d.p. M1 A0 A1 SC: degree mode: M1 $x_1 = 0.4948$, A1 for $x_2 = 0.4914$, then A0; max 2	M1 A1 A1 (3)
(c)	Choose suitable interval for k: e.g. $[0.2765, 0.2775]$ and evaluate $f(x)$ at these values Show that $4k + \sin 4k - 2$ changes sign and deduction $[f(0.2765) = -0.000087.., f(0.2775) = +0.0057]$ Note: Continued iteration: (no marks in degree mode) Some evidence of further iterations leading to 0.2765 or better M1; Deduction A1	M1 A1 (2) (11 marks)

EDEXCEL CORE MATHEMATICS C3 (6665) – JUNE 2006

MARK SCHEME

Question Number	Scheme	Marks
6.	(a) Dividing $\sin^2 \theta + \cos^2 \theta \equiv 1$ by $\sin^2 \theta$ to give $\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} \equiv \frac{1}{\sin^2 \theta}$ Completion: $1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta \Rightarrow \operatorname{cosec}^2 \theta - \cot^2 \theta \equiv 1$ AG	M1 A1* (2)
	(b) $\operatorname{cosec}^4 \theta - \cot^4 \theta \equiv (\operatorname{cosec}^2 \theta - \cot^2 \theta)(\operatorname{cosec}^2 \theta + \cot^2 \theta)$ $\equiv (\operatorname{cosec}^2 \theta + \cot^2 \theta)$ using (a) AG Notes: (i) Using LHS = $(1 + \cot^2 \theta)^2 - \cot^4 \theta$, using (a) & elim. $\cot^4 \theta$ M1, conclusion {using (a) again} A1* (ii) Conversion to sines and cosines: needs $\frac{(1 - \cos^2 \theta)(1 + \cos^2 \theta)}{\sin^4 \theta}$ for M1	M1 A1* (2)
	(c) Using (b) to form $\operatorname{cosec}^2 \theta + \cot^2 \theta \equiv 2 - \cot \theta$ Forming quadratic in $\cot \theta$ $\Rightarrow 1 + \cot^2 \theta + \cot^2 \theta \equiv 2 - \cot \theta$ {using (a)} $2 \cot^2 \theta + \cot \theta - 1 = 0$ Solving: $(2 \cot \theta - 1)(\cot \theta + 1) = 0$ to $\cot \theta =$ $\left(\cot \theta = \frac{1}{2} \right)$ or $\cot \theta = -1$ $\theta = 135^\circ$ (or correct value(s) for candidate dep. on 3Ms) Note: Ignore solutions outside range Extra “solutions” in range loses A1√, but candidate may possibly have more than one “correct” solution.	M1 M1 A1 M1 A1 A1√ (6) (10 marks)

Question Number	Scheme	Marks
7. (a)		Log graph: Shape B1
		Intersection with -ve x-axis dB1
		$(0, \ln k), (1 - k, 0)$ B1
		Mod graph :V shape, vertex on +ve x-axis B1
		$(0, k) \text{ and } \left(\frac{k}{2}, 0\right)$ B1 (5)
(b)	$f(x) \in \mathbb{R} \quad , \quad -\infty < f(x) < \infty \quad , \quad -\infty < y < \infty$	B1 (1)
(c)	$fg\left(\frac{k}{4}\right) = \ln\left\{k + \left \frac{2k}{4} - k\right \right\} \quad \text{or} \quad f\left(\left -\frac{k}{2}\right \right)$	M1
	$= \ln\left(\frac{3k}{2}\right)$	A1 (2)
(d)	$\frac{dy}{dx} = \frac{1}{x + k}$	B1
	Equating (with $x = 3$) to grad. of line;	$\frac{1}{3 + k} = \frac{2}{9}$ M1; A1
		$k = 1\frac{1}{2}$ A1✓ (4)
		(12 marks)

Question Number	Scheme	Marks
8. (a)	Method for finding $\sin A$ $\sin A = -\frac{\sqrt{7}}{4}$ Note: First A1 for $\frac{\sqrt{7}}{4}$, exact. Second A1 for sign (even if dec. answer given) Use of $\sin 2A \equiv 2 \sin A \cos A$ $\sin 2A = -\frac{3\sqrt{7}}{8} \text{ or equivalent exact}$ Note: $\pm$ f.t. Requires exact value, dependent on 2nd M	M1 A1 A1 M1 A1√ (5)
(b)(i)	$\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x \cos \frac{\pi}{3} - \sin 2x \sin \frac{\pi}{3} + \cos 2x \cos \frac{\pi}{3} + \sin 2x \sin \frac{\pi}{3}$ $\equiv 2 \cos 2x \cos \frac{\pi}{3}$ [This can be just written down (using factor formulae) for M1A1] $\equiv \cos 2x \quad \text{AG}$ Note: M1A1 earned, if $\equiv 2 \cos 2x \cos \frac{\pi}{3}$ just written down, using factor theorem Final A1* requires some working after first result.	M1 A1 A1* (3)
(b)(ii)	$\frac{dy}{dx} = 6 \sin x \cos x - 2 \sin 2x$ or $6 \sin x \cos x - 2 \sin\left(2x + \frac{\pi}{3}\right) - 2 \sin\left(2x - \frac{\pi}{3}\right)$ $= 3 \sin 2x - 2 \sin 2x$ $= \sin 2x \quad \text{AG}$ Note: First B1 for $6 \sin x \cos x$; second B1 for remaining term(s)	B1 B1 M1 A1* (4) (12 marks)

Mark Scheme (Results)

Summer 2007

GCE

GCE Mathematics

Core Mathematics C3 (6665)



June 2007
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1. (a)	$\ln 3x = \ln 6$ or $\ln x = \ln \left(\frac{6}{3}\right)$ [implied by 0.69...] or $\ln \left(\frac{3x}{6}\right) = 0$	M1
	$x = 2$ (only this answer)	A1 (cso) (2)
(b)	$(e^x)^2 - 4e^x + 3 = 0$ (any 3 term form) $(e^x - 3)(e^x - 1) = 0$ $e^x = 3$ or $e^x = 1$ Solving quadratic $x = \ln 3, x = 0$ (or $\ln 1$)	M1 M1 dep M1 A1 (4) (6 marks)

Notes: (a) Answer $x = 2$ with no working or no incorrect working seen: M1A1

Beware $x = 2$ from $\ln x = \frac{\ln 6}{\ln 3} = \ln 2$ M0A0

$\ln x = \ln 6 - \ln 3 \Rightarrow x = e^{(\ln 6 - \ln 3)}$ allow M1, $x = 2$ (no wrong working) A1

(b) 1st M1 for attempting to multiply through by e^x : Allow y, X , even x , for e^x

Be generous for M1 e.g. $e^{2x} + 3 = 4$, $e^{x^2} + 3 = 4e^x$,

$$3y^2 + 1 = 12y \text{ (from } 3e^{-x} = \frac{1}{3e^x}), e^x + 3 = 4e^x$$

2nd M1 is for solving quadratic (may be by formula or completing the square)
as far as getting two values for e^x or y or X etc

3rd M1 is for converting their answer(s) of the form $e^x = k$ to $x = \ln k$ (must be exact)

A1 is for $\ln 3$ **and** $\ln 1$ or 0 (Both required and no further solutions)

2. (a)	$2x^2 + 3x - 2 = (2x - 1)(x + 2)$ $f(x) = \frac{(2x+3)(2x-1) - (9+2x)}{(2x-1)(x+2)}$ (need not be single fraction) Simplifying numerator to quadratic form $\left[= \frac{4x^2 + 4x - 3 - 9 - 2x}{(2x-1)(x+2)} \right]$ Correct numerator $= \frac{4x^2 + 2x - 12}{[(2x-1)(x+2)]}$ Factorising numerator, with a denominator $= \frac{2(2x-3)(x+2)}{(2x-1)(x+2)}$ o.e. $\left[= \frac{2(2x-3)}{2x-1} \right] = \frac{4x-6}{2x-1} \quad (*)$	B1 M1, A1√ M1 A1 M1 A1 cso (7)
Alt.(a)	$2x^2 + 3x - 2 = (2x - 1)(x + 2)$ $f(x) = \frac{(2x+3)(2x^2+3x-2) - (9+2x)(x+2)}{(x+2)(2x^2+3x-2)}$ $= \frac{4x^3 + 10x^2 - 8x - 24}{(x+2)(2x^2+3x-2)}$ $= \frac{2(x+2)(2x^2+x-6)}{(x+2)(2x^2+3x-2)} \text{ or } \frac{2(2x-3)(x^2+4x+4)}{(x+2)(2x^2+3x+2)} \text{ o.e.}$ Any one linear factor $\times$ quadratic factor in numerator $= \frac{2(x+2)(x+2)(2x-3)}{(x+2)(2x^2+3x-2)} \text{ o.e.}$ $= \frac{2(2x-3)}{2x-1} \quad \frac{4x-6}{2x-1} \quad (*)$	B1 M1A1 f.t. M1, A1 M1 A1
(b)	Complete method for $f'(x)$; e.g. $f'(x) = \frac{(2x-1) \times 4 - (4x-6) \times 2}{(2x-1)^2}$ o.e $= \frac{8}{(2x-1)^2} \text{ or } 8(2x-1)^{-2}$ Not treating f^{-1} (for f') as misread	M1 A1 A1 (3) (10 marks)

Notes: (a) 1st M1 in either version is for correct method

1st A1 Allow $\frac{2x+3(2x-1)-(9+2x)}{(2x-1)(x+2)}$ or $\frac{(2x+3)(2x-1)-9+2x}{(2x-1)(x+2)}$ or $\frac{2x+3(2x-1)-9+2x}{(2x-1)(x+2)}$ (fractions)

2nd M1 in (main a) is for forming 3 term quadratic in **numerator**

3rd M1 is for factorising their quadratic (usual rules); factor of 2 need not be extracted

(*) A1 is given answer so is cso

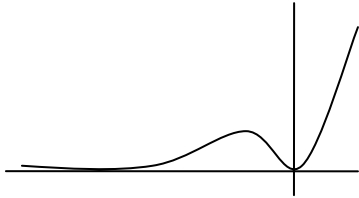
Alt (a) 3rd M1 is for factorising resulting quadratic

Notice that B1 likely to be scored very late but on ePen scored first

(b) SC: For M allow $\pm$ given expression or one error in product rule

Alt: Attempt at $f(x) = 2 - 4(2x-1)^{-1}$ and diff. M1; $k(2x-1)^{-2}$ A1; A1 as above

Accept $8(4x^2 - 4x + 1)^{-2}$. Differentiating original function – mark as scheme.

Question Number	Scheme	Marks
3. (a)	$\frac{dy}{dx} = x^2e^x + 2xe^x$	M1,A1,A1 (3)
(b)	If $\frac{dy}{dx} = 0$, $e^x(x^2 + 2x) = 0$ setting (a) = 0 $[e^x \neq 0]$ $x(x + 2) = 0$ $(x = 0)$ or $x = -2$ $x = 0, y = 0$ and $x = -2, y = 4e^{-2} (= 0.54...)$	M1 A1 A1 ✓ (3)
(c)	$\frac{d^2y}{dx^2} = x^2e^x + 2xe^x + 2xe^x + 2e^x \quad \left[= (x^2 + 4x + 2)e^x \right]$	M1, A1 (2)
(d)	$x = 0, \frac{d^2y}{dx^2} > 0 (=2)$ $x = -2, \frac{d^2y}{dx^2} < 0 \quad [= -2e^{-2} (= -0.270...)]$ M1: Evaluate, or state sign of, candidate's (c) for at least one of candidate's x value(s) from (b) $\therefore$ minimum $\therefore$ maximum	M1 A1 (cso) (2)
Alt.(d)	For M1: Evaluate, or state sign of, $\frac{dy}{dx}$ at two appropriate values – on either side of at least one of their answers from (b) or Evaluate y at two appropriate values – on either side of at least one of their answers from (b) or Sketch curve 	(10 marks)

Notes: (a) Generous M for attempt at $f(x)g'(x) + f'(x)g(x)$

1st A1 for one correct, 2nd A1 for the other correct.

Note that x^2e^x on its own scores no marks

(b) 1st A1 ($x = 0$) may be omitted, but for

2nd A1 both sets of coordinates needed ; f.t only on candidate's $x = -2$

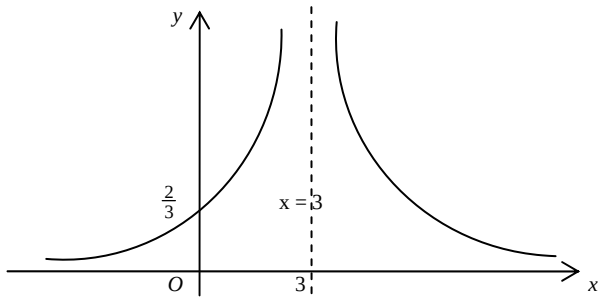
(c) M1 requires complete method for candidate's (a), result may be unsimplified for A1

(d) A1 is cso; $x = 0$, min, and $x = -2$, max and no incorrect working seen.,

or (in alternative) sign of $\frac{dy}{dx}$ either side correct, or values of y appropriate to t.p.

Need only consider the quadratic, as may assume $e^x > 0$.

If all marks gained in (a) and (c), and correct x values, give M1A1 for correct statements with no working

Question Number	Scheme	Marks
4.	(a) $x^2(3-x) - 1 = 0$ o.e. (e.g. $x^2(-x+3) = 1$) $x = \sqrt{\frac{1}{3-x}}$ (*) Note(*), answer is given: need to see appropriate working and A1 is cso [Reverse process: Squaring and non-fractional equation M1, form f(x) A1]	M1 A1 (cso) (2)
	(b) $x_2 = 0.6455$, $x_3 = 0.6517$, $x_4 = 0.6526$ 1 st B1 is for one correct, 2 nd B1 for other two correct If all three are to greater accuracy, award B0 B1	B1; B1 (2)
	(c) Choose values in interval (0.6525, 0.6535) or tighter and evaluate both $f(0.6525) = -0.0005$ (372... $f(0.6535) = 0.002$ (101... At least one correct "up to bracket", i.e. -0.0005 or 0.002 Change of sign , $\therefore x = 0.653$ is a root (correct) to 3 d.p. Requires both correct "up to bracket" and conclusion as above	M1 A1 A1 (3) (7 marks)
	Alt (i) Continued iterations at least as far as x_6 M1 $x_5 = 0.6527$, $x_6 = 0.6527$, $x_7 = \dots$ two correct to at least 4 s.f. A1 Conclusion : Two values correct to 4 d.p., so 0.653 is root to 3 d.p. A1	
	Alt (ii) If use $g(0.6525) = 0.6527.. > 0.6525$ and $g(0.6535) = 0.6528.. < 0.6535$ M1A1 Conclusion : Both results correct, so 0.653 is root to 3 d.p. A1	
5.	(a) Finding $g(4) = k$ and $f(k) = \dots$ or $fg(x) = \ln\left(\frac{4}{x-3} - 1\right)$ $[f(2) = \ln(2 \times 2 - 1) \quad fg(4) = \ln(4 - 1)] = \ln 3$	M1 A1 (2)
	(b) $y = \ln(2x-1) \Rightarrow e^y = 2x-1$ or $e^x = 2y-1$ $f^{-1}(x) = \frac{1}{2}(e^x + 1)$ Allow $y = \frac{1}{2}(e^x + 1)$ Domain $x \in \mathbb{R}$ [Allow $\mathbb{R}$, all reals, $(-\infty, \infty)$] independent	M1, A1 A1 B1 (4)
	(c)  Shape, and x-axis should appear to be asymptote Equation $x = 3$ needed, may see in diagram (ignore others) Intercept $(0, \frac{2}{3})$ no other; accept $y = \frac{2}{3}$ (0.67) or on graph	B1 B1 ind. B1 ind (3)
	(d) $\frac{2}{x-3} = 3 \Rightarrow x = 3\frac{2}{3}$ or exact equiv. $\frac{2}{x-3} = -3, \Rightarrow x = 2\frac{1}{3}$ or exact equiv. Note: $2 = 3(x+3)$ or $2 = 3(-x-3)$ o.e. is M0A0	B1 M1, A1 (3)
	Alt: Squaring to quadratic $(9x^2 - 54x + 77 = 0)$ and solving M1; B1A1	(12 marks)

6.	(a)	Complete method for R: e.g. $R \cos \alpha = 3$, $R \sin \alpha = 2$, $R = \sqrt{3^2 + 2^2}$ $R = \sqrt{13}$ or 3.61 (or more accurate) Complete method for $\tan \alpha = \frac{2}{3}$ [Allow $\tan \alpha = \frac{3}{2}$] $\alpha = 0.588$ (Allow 33.7°)	M1 A1 M1 A1 (4)
	(b)	Greatest value = $(\sqrt{13})^4 = 169$	M1, A1 (2)
	(c)	$\sin(x + 0.588) = \frac{1}{\sqrt{13}}$ (= 0.27735...) $\sin(x + \text{their } \alpha) = \frac{1}{\text{their } R}$ ($x + 0.588$) = 0.281(03... or 16.1°) ($x + 0.588$) = $\pi - 0.28103...$ Must be $\pi - \text{their } 0.281$ or $180^\circ - \text{their } 16.1^\circ$ or ($x + 0.588$) = $2\pi + 0.28103...$ Must be $2\pi + \text{their } 0.281$ or $360^\circ + \text{their } 16.1^\circ$ $x = 2.273$ or $x = 5.976$ (awrt) Both (radians only) If 0.281 or 16.1° not seen, correct answers imply this A mark	M1 A1 M1 M1 A1 (5) (11 marks)

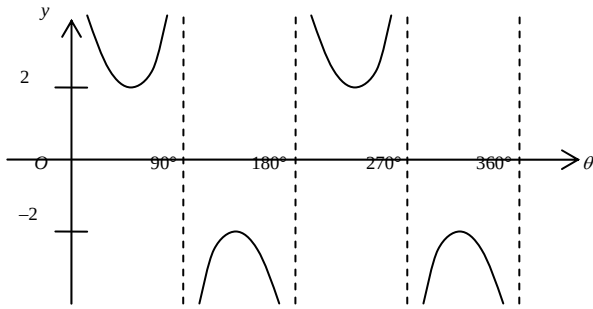
Notes: (a) 1st M1 on Epen for correct method for R, even if found second
2nd M1 for correct method for $\tan \alpha$
No working at all: M1A1 for $\sqrt{13}$, M1A1 for 0.588 or 33.7° .
N.B. $R \cos \alpha = 2$, $R \sin \alpha = 3$ used, can still score M1A1 for R, but loses the A mark for α .
 $\cos \alpha = 3$, $\sin \alpha = 2$: apply the same marking.

(b) M1 for realising $\sin(x + \alpha) = \pm 1$, so finding R^4 .

(c) Working in mixed degrees/rads : first two marks available
Working consistently in degrees: Possible to score first 4 marks
[Degree answers, just for reference, Only are 130.2° and 342.4°]
Third M1 can be gained for candidate's 0.281 – candidate's $0.588 + 2\pi$ or equiv. in degrees
One of the answers correct in radians or degrees implies the corresponding M mark.

Alt: (c) (i) Squaring to form quadratic in $\sin x$ or $\cos x$ M1
[$13\cos^2 x - 4\cos x - 8 = 0$, $13\sin^2 x - 6\sin x - 3 = 0$]
Correct values for $\cos x = 0.953...$, -0.646 ; or $\sin x = 0.767$, 2.27 awrt A1
For any one value of $\cos x$ or $\sin x$, correct method for two values of x M1
 $x = 2.273$ or $x = 5.976$ (awrt) Both seen anywhere A1
Checking other values (0.307, 4.011 or 0.869, 3.449) and discarding M1

(ii) Squaring and forming equation of form $a \cos 2x + b \sin 2x = c$
 $9\sin^2 x + 4\cos^2 x + 12\sin 2x = 1 \Rightarrow 12\sin 2x + 5\cos 2x = 11$
Setting up to solve using R formula e.g. $\sqrt{13} \cos(2x - 1.176) = 11$ M1
 $(2x - 1.176) = \cos^{-1}\left(\frac{11}{\sqrt{13}}\right) = 0.562(0... (\alpha)$ A1
 $(2x - 1.176) = 2\pi - \alpha, 2\pi + \alpha, \dots$ M1
 $x = 2.273$ or $x = 5.976$ (awrt) Both seen anywhere A1
Checking other values and discarding M1

Question Number	Scheme	Marks
7. (a)	$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$ M1 Use of common denominator to obtain single fraction $= \frac{1}{\cos \theta \sin \theta}$ M1 Use of appropriate trig identity (in this case $\sin^2 \theta + \cos^2 \theta = 1$) $= \frac{1}{\frac{1}{2} \sin 2\theta}$ Use of $\sin 2\theta = 2 \sin \theta \cos \theta$ $= 2 \operatorname{cosec} 2\theta \quad (*)$	M1 M1 M1 A1 cso (4)
Alt.(a)	$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \tan \theta + \frac{1}{\tan \theta} = \frac{\tan^2 \theta + 1}{\tan \theta}$ M1 $= \frac{\sec^2 \theta}{\tan \theta}$ M1 $= \frac{1}{\cos \theta \sin \theta} = \frac{1}{\frac{1}{2} \sin 2\theta}$ M1 $= 2 \operatorname{cosec} 2\theta \quad (*) \quad (\text{cso}) \quad \text{A1}$ If show two expressions are equal, need conclusion such as QED, tick, true.	
(b)	 Shape (May be translated but need to see 4 "sections") T.P.s at $y = \pm 2$, asymptotic at correct x-values (dotted lines not required)	B1 B1 dep. (2)
(c)	$2 \operatorname{cosec} 2\theta = 3$ $\sin 2\theta = \frac{2}{3} \quad \text{Allow } \frac{2}{\sin 2\theta} = 3 \quad [\text{M1 for equation in } \sin 2\theta]$ $(2\theta) = [41.810\dots^\circ, 138.189\dots^\circ; 401.810\dots^\circ, 498.189\dots^\circ]$ 1st M1 for $\alpha, 180 - \alpha$; 2nd M1 adding 360° to at least one of values $\theta = 20.9^\circ, 69.1^\circ, 200.9^\circ, 249.1^\circ$ (1 d.p.) awrt	M1, A1 M1; M1
Note	1st A1 for any two correct, 2nd A1 for other two Extra solutions in range lose final A1 only SC: Final 4 marks: $\theta = 20.9^\circ$, after M0M0 is B1; record as M0M0A1A0	A1, A1 (6)
Alt.(c)	$\tan \theta + \frac{1}{\tan \theta} = 3 \quad \text{and form quadratic, } \tan^2 \theta - 3 \tan \theta + 1 = 0 \quad \text{M1, A1}$ (M1 for attempt to multiply through by $\tan \theta$, A1 for correct equation above) Solving quadratic $[\tan \theta = \frac{3 \pm \sqrt{5}}{2} = 2.618\dots \text{ or } 0.3819\dots]$ M1 $\theta = 69.1^\circ, 249.1^\circ \quad \theta = 20.9^\circ, 200.9^\circ$ (1 d.p.) M1, A1, A1 (M1 is for one use of $180^\circ + \alpha^\circ$, A1A1 as for main scheme)	(12 marks)

Question Number	Scheme	Marks
8.	(a) $D = 10, t = 5, \quad x = 10e^{-\frac{1}{8} \times 5}$ $= 5.353$ awrt	M1 A1 (2)
	(b) $D = 10 + 10e^{-\frac{5}{8}}, t = 1, \quad x = 15.3526... \times e^{-\frac{1}{8}}$ $x = 13.549$ (*)	M1 A1 cso (2)
	Alt.(b) $x = 10e^{-\frac{1}{8} \times 6} + 10e^{-\frac{1}{8} \times 1}$ M1 $x = 13.549$ (*) A1 cso	
	(c) $15.3526...e^{-\frac{1}{8}T} = 3$ $e^{-\frac{1}{8}T} = \frac{3}{15.3526...} = 0.1954...$ $-\frac{1}{8}T = \ln 0.1954...$ $T = 13.06... \text{ or } 13.1 \text{ or } 13$	M1 M1 A1 (3) (7 marks)

Notes: (b) (main scheme) M1 is for $(10 + 10e^{-\frac{5}{8}})e^{-\frac{1}{8}}$, or $\{10 + \text{their(a)}\}e^{-(1/8)}$

N.B. The answer is given. There are many correct answers seen which deserve M0A0
or M1A0

(If adding two values, these should be 4.724 and 8.825)

(c) 1st M is for $(10 + 10e^{-\frac{5}{8}})e^{-\frac{T}{8}} = 3$

2nd M is for converting $e^{-\frac{T}{8}} = k$ ($k > 0$) to $-\frac{T}{8} = \ln k$. This is independent of 1st M.

Trial and improvement: M1 as scheme,
M1 correct process for their equation (two equal to 3 s.f.)
A1 as scheme

Mark Scheme (Results)

Summer 2008

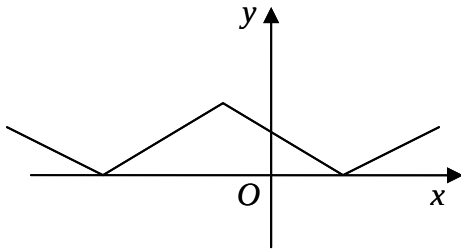
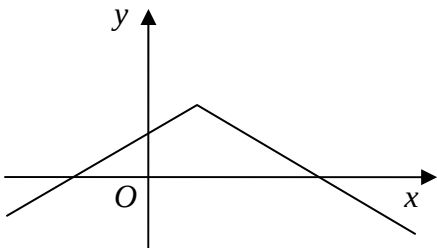
GCE

GCE Mathematics (6665/01)



June 2008
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $e^{2x+1} = 2$ $2x+1 = \ln 2$ $x = \frac{1}{2}(\ln 2 - 1)$ (b) $\frac{dy}{dx} = 8e^{2x+1}$ $x = \frac{1}{2}(\ln 2 - 1) \Rightarrow \frac{dy}{dx} = 16$ $y - 8 = 16 \left(x - \frac{1}{2}(\ln 2 - 1) \right)$ $y = 16x + 16 - 8 \ln 2$	M1 A1 (2) B1 B1 M1 A1 (4) [6]

Question Number	Scheme	Marks
3.	(a)   shape Vertices correctly placed	B1 B1 (2)
	(b)   shape Vertex and intersections with axes correctly placed	B1 B1 (2)
	(c) $P: (-1, 2)$ $Q: (0, 1)$ $R: (1, 0)$	B1 B1 B1 (3)
	(d) $x > -1; \quad 2 - x - 1 = \frac{1}{2}x$ Leading to $x = \frac{2}{3}$ $x < -1; \quad 2 + x + 1 = \frac{1}{2}x$ Leading to $x = -6$	M1 A1 A1 M1 A1 (5) [12]

Question Number	Scheme	Marks
4.	(a) $x^2 - 2x - 3 = (x-3)(x+1)$ $f(x) = \frac{2(x-1) - (x+1)}{(x-3)(x+1)} \left(\text{or } \frac{2(x-1)}{(x-3)(x+1)} - \frac{x+1}{(x-3)(x+1)} \right)$ $= \frac{x-3}{(x-3)(x+1)} = \frac{1}{x+1} *$ cso (b) $\left(0, \frac{1}{4}\right)$ Accept $0 < y < \frac{1}{4}$, $0 < f(x) < \frac{1}{4}$ etc. (c) Let $y = f(x)$ $y = \frac{1}{x+1}$ $x = \frac{1}{y+1}$ $yx + x = 1$ $y = \frac{1-x}{x}$ or $\frac{1}{x} - 1$ $f^{-1}(x) = \frac{1-x}{x}$ Domain of f^{-1} is $\left(0, \frac{1}{4}\right)$ ft their part (b) (d) $fg(x) = \frac{1}{2x^2 - 3 + 1}$ $\frac{1}{2x^2 - 2} = \frac{1}{8}$ $x^2 = 5$ $x = \pm\sqrt{5}$ both	B1 M1 A1 A1 (4) B1 B1 (2) M1 A1 B1 ft (3) M1 A1 A1 (3) [12]

Question Number	Scheme	Marks
5.	(a) $\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$ $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$ * cs0 Alternative for (a) $1 + \cot^2 \theta = 1 + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$ $= \operatorname{cosec}^2 \theta$ * cs0 (b) $2(\operatorname{cosec}^2 \theta - 1) - 9 \operatorname{cosec} \theta = 3$ $2 \operatorname{cosec}^2 \theta - 9 \operatorname{cosec} \theta - 5 = 0$ or $5 \sin^2 \theta + 9 \sin \theta - 2 = 0$ $(2 \operatorname{cosec} \theta + 1)(\operatorname{cosec} \theta - 5) = 0$ or $(5 \sin \theta - 1)(\sin \theta + 2) = 0$ $\operatorname{cosec} \theta = 5$ or $\sin \theta = \frac{1}{5}$ $\theta = 11.5^\circ, 168.5^\circ$	M1 A1 (2) M1 A1 M1 M1 A1 A1 A1 (6) [8]

Question Number	Scheme	Marks
6.	(a)(i) $\frac{d}{dx}(e^{3x}(\sin x + 2 \cos x)) = 3e^{3x}(\sin x + 2 \cos x) + e^{3x}(\cos x - 2 \sin x)$ $(= e^{3x}(\sin x + 7 \cos x))$	M1 A1 A1 (3)
	(ii) $\frac{d}{dx}(x^3 \ln(5x+2)) = 3x^2 \ln(5x+2) + \frac{5x^3}{5x+2}$	M1 A1 A1 (3)
	(b) $\frac{dy}{dx} = \frac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$ $= \frac{(x+1)(6x^2+12x+6-6x^2-12x+14)}{(x+1)^4}$ $= \frac{20}{(x+1)^3} \quad *$	M1 $\frac{A1}{A1}$ M1 CSO A1 (5)
	(c) $\frac{d^2y}{dx^2} = -\frac{60}{(x+1)^4} = -\frac{15}{4}$ $(x+1)^4 = 16$ $x = 1, -3$	M1 M1 both A1 (3)
	<i>Note:</i> The simplification in part (b) can be carried out as follows $\frac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$ $= \frac{(6x^3+18x^2+18x+6) - (6x^3+18x^2-2x-14)}{(x+1)^4}$ $= \frac{20x+20}{(x+1)^4} = \frac{20(x+1)}{(x+1)^4} = \frac{20}{(x+1)^3}$	M1 A1

[14]

Question Number	Scheme	Marks
7.	(a) $f(1.4) = -0.568 \dots < 0$ $f(1.45) = 0.245 \dots > 0$ Change of sign (and continuity) $\Rightarrow \alpha \in (1.4, 1.45)$ (b) $3x^3 = 2x + 6$ $x^3 = \frac{2x}{3} + 2$ $x^2 = \frac{2}{3} + \frac{2}{x}$ $x = \sqrt[3]{\left(\frac{2}{x} + \frac{2}{3}\right)} *$ cs0 (c) $x_1 = 1.4371$ $x_2 = 1.4347$ $x_3 = 1.4355$ (d) Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval. $f(1.4345) = -0.01 \dots$ $f(1.4355) = 0.003 \dots$ Change of sign (and continuity) $\Rightarrow \alpha \in (1.4345, 1.4355)$ $\Rightarrow \alpha = 1.435$, correct to 3 decimal places * cs0 <i>Note: $\alpha = 1.435\,304\,553 \dots$</i>	M1 A1 (2) M1 A1 A1 (3) B1 B1 B1 (3) M1 M1 A1 (3) [11]

Mark Scheme (Results) Summer 2009

GCE

GCE Mathematics (6665/01)



June 2009
6665 Core Mathematics C3
Mark Scheme

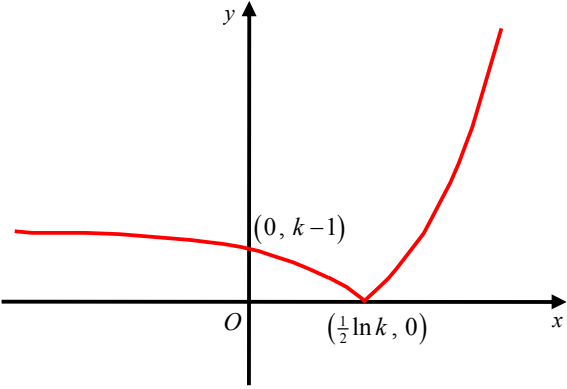
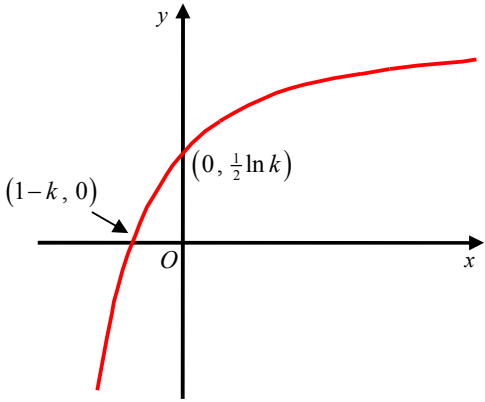
Question Number	Scheme	Marks
Q1	(a) Iterative formula: $x_{n+1} = \frac{2}{(x_n)^2} + 2$, $x_0 = 2.5$ $x_1 = \frac{2}{(2.5)^2} + 2$ $x_1 = 2.32$ $x_2 = 2.371581451\dots$ $x_3 = 2.355593575\dots$ $x_4 = 2.360436923\dots$	An attempt to substitute $x_0 = 2.5$ into the iterative formula. Can be implied by $x_1 = 2.32$ or 2.320 Both $x_1 = 2.32(0)$ and $x_2 = \text{awrt } 2.372$ Both $x_3 = \text{awrt } 2.356$ and $x_4 = \text{awrt } 2.360$ or 2.36 M1 A1 A1 cso (3)
	(b) Let $f(x) = -x^3 + 2x^2 + 2 = 0$ $f(2.3585) = 0.00583577\dots$ $f(2.3595) = -0.00142286\dots$ Sign change (and $f(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)	Choose suitable interval for x, e.g. $[2.3585, 2.3595]$ or tighter any one value awrt 1 sf or truncated 1 sf both values correct, sign change and conclusion M1 dM1 A1 (3) [6]

Question Number	Scheme	Marks
Q2	(a) $\cos^2 \theta + \sin^2 \theta = 1 \quad (\div \cos^2 \theta)$	
	$\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$	Dividing $\cos^2 \theta + \sin^2 \theta = 1$ by $\cos^2 \theta$ to give <u>underlined</u> equation. M1
	$1 + \tan^2 \theta = \sec^2 \theta$	
	$\tan^2 \theta = \sec^2 \theta - 1$ (as required) AG	Complete proof. No errors seen. A1 cso
	(b) $2 \tan^2 \theta + 4 \sec \theta + \sec^2 \theta = 2, \quad (eqn *) \quad 0 \leq \theta < 360^\circ$	(2)
	$2(\sec^2 \theta - 1) + 4 \sec \theta + \sec^2 \theta = 2$	Substituting $\tan^2 \theta = \sec^2 \theta - 1$ into eqn * to get a quadratic in $\sec \theta$ only M1
	$2\sec^2 \theta - 2 + 4 \sec \theta + \sec^2 \theta = 2$	
	$3\sec^2 \theta + 4 \sec \theta - 4 = 0$	Forming a three term "one sided" quadratic expression in $\sec \theta$. M1
	$(\sec \theta + 2)(3 \sec \theta - 2) = 0$	Attempt to factorise or solve a quadratic. M1
	$\sec \theta = -2$ or $\sec \theta = \frac{2}{3}$	
	$\frac{1}{\cos \theta} = -2$ or $\frac{1}{\cos \theta} = \frac{2}{3}$	
	$\cos \theta = -\frac{1}{2};$ or $\cos \theta = \frac{3}{2}$	$\cos \theta = -\frac{1}{2}$ A1;
	$\alpha = 120^\circ$ or $\alpha =$ no solutions	
	$\theta_1 = 120^\circ$	120° A1
	$\theta_2 = 240^\circ$	240° or $\theta_2 = 360^\circ - \theta_1$ when solving using $\cos \theta = \dots$ B1 $\sqrt{}$
	$\theta = \{120^\circ, 240^\circ\}$	Note the final A1 mark has been changed to a B1 mark. (6)
		[8]

Question Number	Scheme	Marks
Q3	$P = 80e^{\frac{t}{5}}$	
(a)	$t = 0 \Rightarrow P = 80e^{\frac{0}{5}} = 80(1) = \underline{80}$	B1 (1)
(b)	$P = 1000 \Rightarrow 1000 = 80e^{\frac{t}{5}} \Rightarrow \frac{1000}{80} = e^{\frac{t}{5}}$ $\therefore t = 5 \ln\left(\frac{1000}{80}\right)$ $t = 12.6286\dots$	Substitutes $P = 1000$ and rearranges equation to make $e^{\frac{t}{5}}$ the subject. M1 awrt 12.6 or 13 years Note $t = 12$ or $t = \text{awrt } 12.6 \Rightarrow t = 12$ will score A0 A1 (2)
(c)	$\frac{dP}{dt} = 16e^{\frac{t}{5}}$	$ke^{\frac{1}{5}t}$ and $k \neq 80$. $16e^{\frac{1}{5}t}$ M1 A1 (2)
(d)	$50 = 16e^{\frac{t}{5}}$ $\therefore t = 5 \ln\left(\frac{50}{16}\right) \quad \{= 5.69717\dots\}$ $P = 80e^{\frac{1}{5}\left(5 \ln\left(\frac{50}{16}\right)\right)} \quad \text{or} \quad P = 80e^{\frac{1}{5}(5.69717\dots)}$ $P = \frac{80(50)}{16} = \underline{250}$	Using $50 = \frac{dP}{dt}$ and an attempt to solve to find the value of t or $\frac{t}{5}$. M1 Substitutes their value of t back into the equation for P . dM1 $\underline{250}$ or awrt 250 A1 (3)
		[8]

Question Number	Scheme	Marks
Q4 (i)(a)	$y = x^2 \cos 3x$ Apply product rule: $\left\{ \begin{array}{l} u = x^2 \\ \frac{du}{dx} = 2x \end{array} \quad \begin{array}{l} v = \cos 3x \\ \frac{dv}{dx} = -3 \sin 3x \end{array} \right\}$ $\frac{dy}{dx} = 2x \cos 3x - 3x^2 \sin 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha x \cos 3x \pm \beta x^2 \sin 3x$ M1 Any one term correct A1 Both terms correct and no further simplification to terms in $\cos \alpha x^2$ or $\sin \beta x^3$. A1 (3)
(b)	$y = \frac{\ln(x^2 + 1)}{x^2 + 1}$ $u = \ln(x^2 + 1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2 + 1}$ Apply quotient rule: $\left\{ \begin{array}{l} u = \ln(x^2 + 1) \\ \frac{du}{dx} = \frac{2x}{x^2 + 1} \end{array} \quad \begin{array}{l} v = x^2 + 1 \\ \frac{dv}{dx} = 2x \end{array} \right\}$ $\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2 + 1}\right)(x^2 + 1) - 2x \ln(x^2 + 1)}{(x^2 + 1)^2}$	$\ln(x^2 + 1) \rightarrow \frac{\text{something}}{x^2 + 1}$ M1 $\ln(x^2 + 1) \rightarrow \frac{2x}{x^2 + 1}$ A1 Applying $\frac{vu' - uv'}{v^2}$ M1 Correct differentiation with correct bracketing but allow recovery. A1 {Ignore subsequent working.} (4)

Question Number	Scheme	Marks
(ii)	$y = \sqrt{4x+1}, \quad x > -\frac{1}{4}$ At P , $y = \sqrt{4(2)+1} = \sqrt{9} = 3$ $\frac{dy}{dx} = \frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)$ $\frac{dy}{dx} = \frac{2}{(4x+1)^{\frac{1}{2}}}$ At P , $\frac{dy}{dx} = \frac{2}{(4(2)+1)^{\frac{1}{2}}}$ Hence $m(T) = \frac{2}{3}$ Either $T: y - 3 = \frac{2}{3}(x - 2);$ or $y = \frac{2}{3}x + c$ and $3 = \frac{2}{3}(2) + c \Rightarrow c = 3 - \frac{4}{3} = \frac{5}{3};$ Either $T: 3y - 9 = 2(x - 2);$ $T: 3y - 9 = 2x - 4$ $T: \underline{2x - 3y + 5 = 0}$ or $T: y = \frac{2}{3}x + \frac{5}{3}$ $T: 3y = 2x + 5$ $T: \underline{2x - 3y + 5 = 0}$	At P , $y = \sqrt{9}$ or 3 $\pm k(4x+1)^{-\frac{1}{2}}$ $2(4x+1)^{-\frac{1}{2}}$ Substituting $x = 2$ into an equation involving $\frac{dy}{dx};$ $y - y_1 = m(x - 2)$ or $y - y_1 = m(x - \text{their stated } x)$ with ‘their TANGENT gradient’ and their y_1 ; or uses $y = mx + c$ with ‘their TANGENT gradient’, their x and their y_1 . $\underline{2x - 3y + 5 = 0}$ Tangent must be stated in the form $ax + by + c = 0$, where a, b and c are integers. (6) [13]

Question Number	Scheme	Marks
Q5 (a)	 Curve retains shape when $x > \frac{1}{2} \ln k$ Curve reflects through the x-axis when $x < \frac{1}{2} \ln k$ $(0, k-1)$ and $(\frac{1}{2} \ln k, 0)$ marked in the correct positions.	B1 B1 B1 (3)
(b)	 Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote) $(1-k, 0)$ and $(0, \frac{1}{2} \ln k)$	B1 B1 (2)
(c)	Range of f: $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k, \infty)}$	Either $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k, \infty)}$ or $\underline{f > -k}$ or $\underline{\text{Range} > -k}$. B1 (1)
(d)	$y = e^{2x} - k \Rightarrow y + k = e^{2x}$ $\Rightarrow \ln(y + k) = 2x$ $\Rightarrow \frac{1}{2} \ln(y + k) = x$ Hence $f^{-1}(x) = \underline{\frac{1}{2} \ln(x + k)}$	Attempt to make x (or swapped y) the subject Makes e^{2x} the subject and takes $\ln$ of both sides $\underline{\frac{1}{2} \ln(x + k)}$ or $\underline{\ln \sqrt{x + k}}$ M1 M1 A1 cao (3)
(e)	$f^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k, \infty)}$	Either $\underline{x > -k}$ or $\underline{(-k, \infty)}$ or Domain $> -k$ or x "ft one sided inequality" their part (c) RANGE answer B1 $\sqrt{}$ (1)
		[10]

Question Number	Scheme	Marks
Q6	(a) $A = B \Rightarrow \cos(A + A) = \cos 2A = \underline{\cos A \cos A - \sin A \sin A}$ Applies $A = B$ to $\cos(A + B)$ to give the <u>underlined</u> equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$ $\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives $\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required) <u>Complete proof, with a link between LHS and RHS. No errors seen.</u>	M1 A1 AG (2)
	(b) $C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$ Eliminating y correctly. Using result in part (a) to substitute for $\sin^2 x$ as $\frac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\frac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles. $3\sin 2x = 4\left(\frac{1 - \cos 2x}{2}\right) - 2\cos 2x$ $3\sin 2x = 2(1 - \cos 2x) - 2\cos 2x$ $3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$ $3\sin 2x + 4\cos 2x = 2$ Rearranges to give correct result	M1 M1 A1 AG (3)
	(c) $3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$ $3\sin 2x + 4\cos 2x = R\cos 2x \cos \alpha + R\sin 2x \sin \alpha$ Equate $\sin 2x$: $3 = R\sin \alpha$ Equate $\cos 2x$: $4 = R\cos \alpha$ $R = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ $R = 5$ $\tan \alpha = \pm \frac{3}{4}$ or $\tan \alpha = \pm \frac{4}{3}$ or $\tan \alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765\dots^\circ$ $\sin \alpha = \pm \frac{3}{\text{their } R}$ or $\cos \alpha = \pm \frac{4}{\text{their } R}$ awrt 36.87 Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$	B1 M1 A1 (3)

9

Question Number	Scheme	Marks
Q7	$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}$ $x \in \mathbb{R}, x \neq -4, x \neq 2.$	
	(a)	
	$f(x) = \frac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$	An attempt to combine to one fraction M1
	$= \frac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$	Correct result of combining all three fractions A1
	$= \frac{x^2 + x - 12}{[(x+4)(x-2)]}$	Simplifies to give the correct numerator. Ignore omission of denominator A1
	$= \frac{(x+4)(x-3)}{[(x+4)(x-2)]}$	An attempt to factorise the numerator. dM1
	$= \frac{(x-3)}{(x-2)}$	Correct result A1 cso AG
		(5)
	(b)	
	$g(x) = \frac{e^x - 3}{e^x - 2} \quad x \in \mathbb{R}, x \neq \ln 2.$	
	Apply quotient rule: $\left\{ \begin{array}{ll} u = e^x - 3 & v = e^x - 2 \\ \frac{du}{dx} = e^x & \frac{dv}{dx} = e^x \end{array} \right\}$	
	$g'(x) = \frac{e^x(e^x - 2) - e^x(e^x - 3)}{(e^x - 2)^2}$	Applying $\frac{vu' - uv'}{v^2}$ M1
	$= \frac{e^{2x} - 2e^x - e^{2x} + 3e^x}{(e^x - 2)^2}$	Correct differentiation A1
	$= \frac{e^x}{(e^x - 2)^2}$	Correct result A1 AG cso
		(3)

Question Number	Scheme	Marks
(c)	$g'(x) = 1 \Rightarrow \frac{e^x}{(e^x - 2)^2} = 1$ $e^x = (e^x - 2)^2$ $e^x = e^{2x} - 2e^x - 2e^x + 4$ $\underline{e^{2x} - 5e^x + 4 = 0}$ $(e^x - 4)(e^x - 1) = 0$ $e^x = 4 \text{ or } e^x = 1$ $x = \ln 4 \text{ or } x = 0$	Puts their differentiated numerator equal to their denominator. M1 $\underline{e^{2x} - 5e^x + 4}$ A1 Attempt to factorise or solve quadratic in e^x M1 both $x = 0, \ln 4$ A1 (4) [12]

Question Number	Scheme	Marks
Q8 (a)	$\sin 2x = \underline{2 \sin x \cos x}$	B1 aef (1)
(b)	$\operatorname{cosec} x - 8 \cos x = 0, \quad 0 < x < \pi$ $\frac{1}{\sin x} - 8 \cos x = 0$ Using $\operatorname{cosec} x = \frac{1}{\sin x}$ $\frac{1}{\sin x} = 8 \cos x$ $1 = 8 \sin x \cos x$ $1 = 4(2 \sin x \cos x)$ $1 = 4 \sin 2x$ $\sin 2x = \frac{1}{4}$ Radians $2x = \{0.25268..., 2.88891...\}$ Degrees $2x = \{14.4775..., 165.5225...\}$ Radians $x = \{0.12634..., 1.44445...\}$ Degrees $x = \{7.23875..., 82.76124...\}$	M1 M1 A1 A1 A1 cao (5)
	$\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$ Either arwt 7.24 or 82.76 or 0.13 or 1.44 or 1.45 or awrt 0.04π or awrt 0.46π. Both <u>0.13</u> and <u>1.44</u> Solutions for the final two A marks must be given in x only. If there are any EXTRA solutions inside the range $0 < x < \pi$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 < x < \pi$.	[6]

Mark Scheme (Results) Summer 2010

GCE

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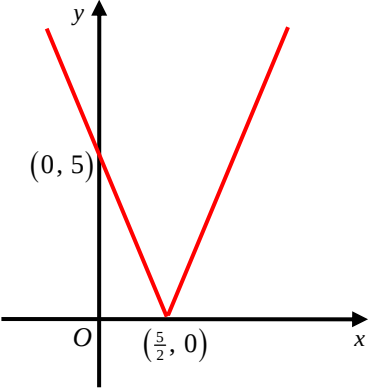
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6665 Core Mathematics C3
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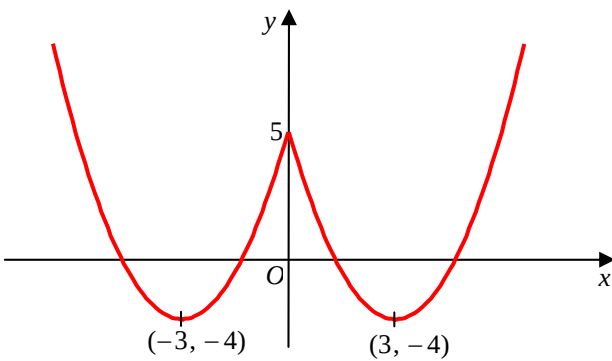
Question Number	Scheme	Marks
1.	(a) $\frac{2 \sin \theta \cos \theta}{1 + 2 \cos^2 \theta - 1}$ $\frac{\cancel{2} \sin \theta \cancel{\cos \theta}}{\cancel{2} \cos \theta \cancel{\cos \theta}} = \tan \theta$ (as required) AG (b) $2 \tan \theta = 1 \Rightarrow \tan \theta = \frac{1}{2}$ $\theta_1 = \text{awrt } 26.6^\circ$ $\theta_2 = \text{awrt } -153.4^\circ$	M1 A1 cso (2) M1 A1 A1 $\sqrt{}$ (3) [5]
	(a) M1: Uses both a correct identity for $\sin 2\theta$ and a correct identity for $\cos 2\theta$. Also allow a candidate writing $1 + \cos 2\theta = 2 \cos^2 \theta$ on the denominator. Also note that angles must be consistent in when candidates apply these identities. A1: Correct proof. No errors seen. (b) 1st M1 for either $2 \tan \theta = 1$ or $\tan \theta = \frac{1}{2}$, seen or implied. A1: awrt 26.6 A1 $\sqrt{}$: awrt -153.4° or $\theta_2 = -180^\circ + \theta_1$ Special Case: For candidate solving, $\tan \theta = k$, where $k \neq \frac{1}{2}$, to give θ_1 and $\theta_2 = -180^\circ + \theta_1$, then award M0A0B1 in part (b). Special Case: Note that those candidates who writes $\tan \theta = 1$, and gives ONLY two answers of 45° and -135° that are inside the range will be awarded SC M0A0B1.	

Question Number	Scheme	Marks
2.	At P, $y = 3$ $\frac{dy}{dx} = \frac{3(-2)(5-3x)^{-3}(-3)}{(5-3(2))^3} \left\{ \text{or } \frac{18}{(5-3x)^3} \right\}$ $\frac{dy}{dx} = \frac{18}{(5-3(2))^3} \{ = -18 \}$ $m(N) = \frac{-1}{-18} \text{ or } \frac{1}{18}$ N: $y - 3 = \frac{1}{18}(x - 2)$ N: $\underline{x - 18y + 52 = 0}$	B1 M1A1 M1 M1 M1 A1 [7]
	1st M1: $\pm k(5-3x)^{-3}$ can be implied. See appendix for application of the quotient rule. 2nd M1: Substituting $x = 2$ into an equation involving their $\frac{dy}{dx}$; 3rd M1: Uses $m(N) = -\frac{1}{\text{their } m(T)}$. 4th M1: $y - y_1 = m(x - 2)$ with 'their NORMAL gradient' or a "changed" tangent gradient and their y_1. Or uses a complete method to express the equation of the tangent in the form $y = mx + c$ with 'their NORMAL ("changed" numerical) gradient', their y_1 and $x = 2$. Note: To gain the final A1 mark all the previous 6 marks in this question need to be earned. Also there must be a completely correct solution given.	

Question Number	Scheme	Marks
3.	(a) $f(1.2) = 0.49166551\dots$, $f(1.3) = -0.048719817\dots$ Sign change (and as $f(x)$ is continuous) therefore a root α is such that $\alpha \in [1.2, 1.3]$ (b) $4\operatorname{cosec} x - 4x + 1 = 0 \Rightarrow 4x = 4\operatorname{cosec} x + 1$ $\Rightarrow x = \operatorname{cosec} x + \frac{1}{4} \Rightarrow x = \frac{1}{\sin x} + \frac{1}{4}$ (c) $x_1 = \frac{1}{\sin(1.25)} + \frac{1}{4}$ $x_1 = 1.303757858\dots$, $x_2 = 1.286745793\dots$ $x_3 = 1.291744613\dots$ (d) $f(1.2905) = 0.00044566695\dots$, $f(1.2915) = -0.00475017278\dots$ Sign change (and as $f(x)$ is continuous) therefore a root α is such that $\alpha \in (1.2905, 1.2915) \Rightarrow \alpha = 1.291$ (3 dp)	M1A1 (2) M1 A1 * (2) M1 A1 A1 (3) M1 A1 (2) [9]
	(a) M1: Attempts to evaluate both $f(1.2)$ and $f(1.3)$ and evaluates at least one of them correctly to awrt (or truncated) 1 sf. A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion. (b) M1: Attempt to make $4x$ or x the subject of the equation. A1: Candidate must then rearrange the equation to give the required result. It must be clear that candidate has made their initial $f(x) = 0$. (c) M1: An attempt to substitute $x_0 = 1.25$ into the iterative formula Eg $= \frac{1}{\sin(1.25)} + \frac{1}{4}$. Can be implied by $x_1 = \text{awrt } 1.3$ or $x_1 = \text{awrt } 46^\circ$. A1: Both $x_1 = \text{awrt } 1.3038$ and $x_2 = \text{awrt } 1.2867$ A1: $x_3 = \text{awrt } 1.2917$ (d) M1: Choose suitable interval for x, e.g. $[1.2905, 1.2915]$ or tighter and at least one attempt to evaluate $f(x)$. A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion.	

Question Number	Scheme	Marks
4.	(a) 	M1A1
(b)	$x = 20$ $2x - 5 = -(15 + x) ; \Rightarrow x = -\frac{10}{3}$	(2) B1 M1;A1 oe.
(c)	$fg(2) = f(-3) = 2(-3) - 5 = -11 = 11$	(3) M1;A1
(d)	$g(x) = x^2 - 4x + 1 = (x - 2)^2 - 4 + 1 = (x - 2)^2 - 3$. Hence $g_{\min} = -3$ Either $g_{\min} = -3$ or $g(x) \geq -3$ or $g(5) = 25 - 20 + 1 = 6$ $-3 \leq g(x) \leq 6$ or $-3 \leq y \leq 6$	(2) M1 B1 A1
		(3) [10]
	(a) M1: V or  or  graph with vertex on the x-axis. A1: $(\frac{5}{2}, \{0\})$ and $(\{0\}, 5)$ seen and the graph appears in both the first and second quadrants. (b) M1: Either $2x - 5 = -(15 + x)$ or $-(2x - 5) = 15 + x$ (c) M1: Full method of inserting $g(2)$ into $f(x) = 2x - 5$ or for inserting $x = 2$ into $2(x^2 - 4x + 1) - 5$. There must be evidence of the modulus being applied. (d) M1: Full method to establish the minimum of g. Eg: $(x \pm \alpha)^2 + \beta$ leading to $g_{\min} = \beta$. Or for candidate to differentiate the quadratic, set the result equal to zero, find x and insert this value of x back into $f(x)$ in order to find the minimum. B1: For either finding the correct minimum value of g (can be implied by $g(x) \geq -3$ or $g(x) > -3$) or for stating that $g(5) = 6$. A1: $-3 \leq g(x) \leq 6$ or $-3 \leq y \leq 6$ or $-3 \leq g \leq 6$. Note that: $-3 \leq x \leq 6$ is A0. Note that: $-3 \leq f(x) \leq 6$ is A0. Note that: $-3 \geq g(x) \geq 6$ is A0. Note that: $g(x) \geq -3$ or $g(x) > -3$ or $x \geq -3$ or $x > -3$ with no working gains M1B1A0. Note that for the final Accuracy Mark: If a candidate writes down $-3 < g(x) < 6$ or $-3 < y < 6$, then award M1B1A0. If, however, a candidate writes down $g(x) \geq -3$, $g(x) \leq 6$, then award A0. If a candidate writes down $g(x) \geq -3$ or $g(x) \leq 6$, then award A0.	

Question Number	Scheme	Marks
5.	(a) Either $y = 2$ or $(0, 2)$	B1 (1)
	(b) When $x = 2$, $y = (8 - 10 + 2)e^{-2} = 0e^{-2} = 0$ $(2x^2 - 5x + 2) = 0 \Rightarrow (x - 2)(2x - 1) = 0$ Either $x = 2$ (for possibly B1 above) or $x = \frac{1}{2}$.	B1 M1 A1 (3)
	(c) $\frac{dy}{dx} = (4x - 5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$	M1A1A1 (3)
	(d) $(4x - 5)e^{-x} - (2x^2 - 5x + 2)e^{-x} = 0$ $2x^2 - 9x + 7 = 0 \Rightarrow (2x - 7)(x - 1) = 0$ $x = \frac{7}{2}, 1$ When $x = \frac{7}{2}$, $y = 9e^{-\frac{7}{2}}$, when $x = 1$, $y = -e^{-1}$	M1 M1 A1 ddM1A1 (5) [12]
	(b) If the candidate believes that $e^{-x} = 0$ solves to $x = 0$ or gives an extra solution of $x = 0$, then withhold the final accuracy mark. (c) M1: (their u') $e^{-x} + (2x^2 - 5x + 2)(\text{their } v')$ A1: Any one term correct. A1: Both terms correct. (d) 1 st M1: For setting their $\frac{dy}{dx}$ found in part (c) equal to 0. 2 nd M1: Factorise or eliminate out e^{-x} correctly and an attempt to factorise a 3-term quadratic or apply the formula to candidate's $ax^2 + bx + c$. See rules for solving a three term quadratic equation on page 1 of this Appendix. 3 rd ddM1: An attempt to use at least one x -coordinate on $y = (2x^2 - 5x + 2)e^{-x}$. Note that this method mark is dependent on the award of the two previous method marks in this part. Some candidates write down corresponding y -coordinates without any working. It may be necessary on some occasions to use your calculator to check that at least one of the two y -coordinates found is correct to awrt 2 sf. Final A1: Both $\{x = 1\}$, $y = -e^{-1}$ and $\{x = \frac{7}{2}\}$, $y = 9e^{-\frac{7}{2}}$. cao Note that both exact values of y are required.	

Question Number	Scheme	Marks
6. (a) (i) (ii) (b) (c) (d)	(3, 4) (6, -8)  $f(x) = (x - 3)^2 - 4$ or $f(x) = x^2 - 6x + 5$ Either: The function f is a many-one {mapping}. Or: The function f is not a one-one {mapping}.	B1 B1 B1 B1 (4) B1 B1 B1 (3) M1A1 (2) B1 (1) [10]
	(b) B1: Correct shape for $x \geq 0$, with the curve meeting the positive y-axis and the turning point is found below the x-axis. (providing candidate does not copy the whole of the original curve and adds nothing else to their sketch.). B1: Curve is symmetrical about the y-axis or correct shape of curve for $x < 0$. Note: The first two B1B1 can only be awarded if the curve has the correct shape, with a cusp on the positive y-axis and with both turning points located in the correct quadrants. Otherwise award B1B0. B1: Correct turning points of $(-3, -4)$ and $(3, -4)$. Also, $(\{0\}, 5)$ is marked where the graph cuts through the y-axis. Allow $(5, 0)$ rather than $(0, 5)$ if marked in the "correct" place on the y-axis. (c) M1: Either states $f(x)$ in the form $(x \pm \alpha)^2 \pm \beta$; $\alpha, \beta \neq 0$ Or uses a complete method on $f(x) = x^2 + ax + b$, with $f(0) = 5$ and $f(3) = -4$ to find both a and b. A1: Either $(x - 3)^2 - 4$ or $x^2 - 6x + 5$ (d) B1: Or: The inverse is a one-many {mapping and not a function}. Or: Because $f(0) = 5$ and also $f(6) = 5$. Or: One y-coordinate has 2 corresponding x-coordinates {and therefore cannot have an inverse}.	

Question Number	Scheme	Marks
7.	(a) $R = \sqrt{6.25}$ or 2.5 $\tan \alpha = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \alpha = \text{awrt } 0.6435$ (b) (i) Max Value = 2.5 (ii) $\sin(\theta - 0.6435) = 1$ or $\theta - \text{their } \alpha = \frac{\pi}{2}; \Rightarrow \theta = \text{awrt } 2.21$ (c) $H_{\text{Max}} = 8.5$ (m) $\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1$ or $\frac{4\pi t}{25} = \text{their (b) answer}; \Rightarrow t = \text{awrt } 4.41$ (d) $\Rightarrow 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7; \Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4$ $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4)$ or awrt 0.41 Either $t = \text{awrt } 2.1$ or awrt 6.7 So, $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \{\pi - 0.411517... \text{ or } 2.730076...^c\}$ Times = $\{14:06, 18:43\}$	B1 M1A1 (3) B1 $\sqrt{\quad}$ M1;A1 $\sqrt{\quad}$ (3) B1 $\sqrt{\quad}$ M1;A1 (3) M1;M1 A1 A1 ddM1 A1 (6) [15]
	(a) B1: $R = 2.5$ or $R = \sqrt{6.25}$. For $R = \pm 2.5$, award B0. M1: $\tan \alpha = \pm \frac{1.5}{2}$ or $\tan \alpha = \pm \frac{2}{1.5}$ A1: $\alpha = \text{awrt } 0.6435$ (b) B1 $\sqrt{\quad}$: 2.5 or follow through the value of R in part (a). M1: For $\sin(\theta - \text{their } \alpha) = 1$ A1 $\sqrt{\quad}$: awrt 2.21 or $\frac{\pi}{2} + \text{their } \alpha$ rounding correctly to 3 sf. (c) B1 $\sqrt{\quad}$: 8.5 or 6 + their R found in part (a) as long as the answer is greater than 6. M1: $\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 1$ or $\frac{4\pi t}{25} = \text{their (b) answer}$ A1: For $\sin^{-1}(0.4)$ This can be implied by awrt 4.41 or awrt 4.40. (d) M1: $6 + (\text{their } R)\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 7$, M1: $\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = \frac{1}{\text{their } R}$ A1: For $\sin^{-1}(0.4)$. This can be implied by awrt 0.41 or awrt 2.73 or other values for different α's. Note this mark can be implied by seeing 1.055. A1: Either $t = \text{awrt } 2.1$ or $t = \text{awrt } 6.7$ ddM1: either $\pi - \text{their PV}^c$. Note that this mark is dependent upon the two M marks. This mark will usually be awarded for seeing either 2.730... or 3.373... A1: Both $t = 14:06$ and $t = 18:43$ or both 126 (min) and 403 (min) or both 2 hr 6 min and 6 hr 43 min.	

Question Number	Scheme	Marks
8.	(a) $\frac{(x+5)(2x-1)}{(x+5)(x-3)} = \frac{(2x-1)}{(x-3)}$ (b) $\ln\left(\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}\right) = 1$ $\frac{2x^2 + 9x - 5}{x^2 + 2x - 15} = e$ $\frac{2x - 1}{x - 3} = e \Rightarrow 3e - 1 = x(e - 2)$ $\Rightarrow x = \frac{3e - 1}{e - 2}$	M1 B1 A1 aef (3) M1 dM1 M1 A1 aef cso (4) [7]
	(a) M1: An attempt to factorise the numerator. B1: Correct factorisation of denominator to give $(x+5)(x-3)$. Can be seen anywhere. (b) M1: Uses a correct law of logarithms to combine at least two terms. This usually is achieved by the subtraction law of logarithms to give $\ln\left(\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}\right) = 1$. The product law of logarithms can be used to achieve $\ln(2x^2 + 9x - 5) = \ln(e(x^2 + 2x - 15))$. The product and quotient law could also be used to achieve $\ln\left(\frac{2x^2 + 9x - 5}{e(x^2 + 2x - 15)}\right) = 0$. dM1: Removing $\ln$'s correctly by the realisation that the anti-$\ln$ of 1 is e. Note that this mark is dependent on the previous method mark being awarded. M1: Collect x terms together and factorise. Note that this is not a dependent method mark. A1: $\frac{3e - 1}{e - 2}$ or $\frac{3e^1 - 1}{e^1 - 2}$ or $\frac{1 - 3e}{2 - e}$. aef Note that the answer needs to be in terms of e. The decimal answer is 9.9610559... Note that the solution must be correct in order for you to award this final accuracy mark. Note: See Appendix for an alternative method of long division.	

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Mark Scheme (Results)

June 2011

GCE Core Mathematics C3 (6665) Paper 1

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June 2011

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

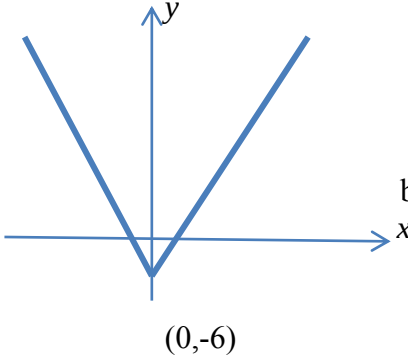
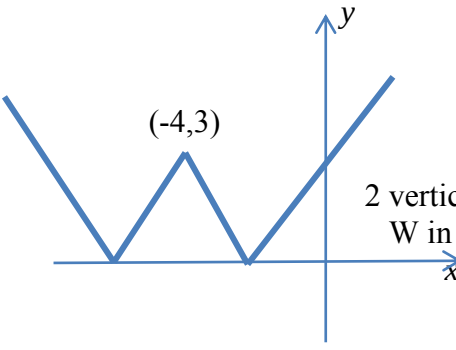
1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
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 - B marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\frac{\Delta}{\Delta}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
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- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- $\square$ The second mark is dependent on gaining the first mark

Question Number	Scheme	Marks
1 (a)	$\frac{1}{(x^2+3x+5)} \times \dots = \frac{2x+3}{(x^2+3x+5)}$	M1,A1 (2)
(b)	Applying $\frac{vu'-uv'}{v^2}$ $\frac{x^2 \times -\sin x - \cos x \times 2x}{(x^2)^2} = \frac{-x^2 \sin x - 2x \cos x}{x^4} = \frac{-x \sin x - 2 \cos x}{x^3} \text{ oe}$	M1, A2,1,0 (3) 5 Marks
2 (a)	$f(0.75) = -0.18\dots$ $f(0.85) = 0.17\dots$ Change of sign, hence root between $x=0.75$ and $x=0.85$	M1 A1 (2)
(b)	Sub $x_0=0.8$ into $x_{n+1} = [\arcsin(1 - 0.5x_n)]^{\frac{1}{2}}$ to obtain x_1 Awrt $x_1=0.80219$ and $x_2=0.80133$ Awrt $x_3 = 0.80167$	M1 A1 A1 (3)
(c)	$f(0.801565) = -2.7\dots \times 10^{-5}$ $f(0.801575) = +8.6\dots \times 10^{-6}$ Change of sign and conclusion See Notes for continued iteration method	M1A1 A1 (3) 8 Marks

Question Number	Scheme	Marks
3 (a)	 V shape vertex on y axis & both branches of graph cross x axis ∴ y-co-ordinate of R is -6 (0, -6)	B1 B1 B1 (3)
(b)	 W shape 2 vertices on the negative x axis. W in both quad 1 & quad 2. $R = (-4, 3)$	B1 B1dep B1 (3) 6 Marks
4 (a)	$y = 4 - \ln(x + 2)$ $\ln(x + 2) = 4 - y$ $x + 2 = e^{4-y}$ $x = e^{4-y} - 2$ $f^{-1}(x) = e^{4-x} - 2$ oe	M1 M1A1 (3)
(b)	$x \leq 4$	B1 (1)
(c)	$fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$ $fg(x) = 4 - x^2$	M1 dM1A1 (3)
(d)	$fg(x) \leq 4$	B1ft (1) 8 Marks

Question Number	Scheme	Marks
5 (a)	$p=7.5$	B1
(b)	$2.5 = 7.5e^{-4k}$ $e^{-4k} = \frac{1}{3}$ $-4k = \ln\left(\frac{1}{3}\right)$ $-4k = -\ln(3)$ $k = \frac{1}{4}\ln(3)$	M1 M1 dM1 A1*
	See notes for additional correct solutions and the last A1	(4)
(c)	$\frac{dm}{dt} = -kpe^{-kt}$ ft on their p and k $-\frac{1}{4}\ln 3 \times 7.5e^{-\frac{1}{4}(\ln 3)t} = -0.6\ln 3$ $e^{-\frac{1}{4}(\ln 3)t} = \frac{2.4}{7.5} = (0.32)$ $-\frac{1}{4}(\ln 3)t = \ln(0.32)$ $t=4.1486\dots$ 4.15 or awrt 4.1	M1A1ft M1A1 dM1 A1
		(6)
		11Marks

Question Number	Scheme	Marks
6 (a)	$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \frac{1 - \cos 2\theta}{\sin 2\theta}$ $= \frac{2\sin^2 \theta}{2\sin \theta \cos \theta}$ $= \frac{\sin \theta}{\cos \theta} = \tan \theta$	M1 M1A1 A1* (4)
(b)(i)	$\tan 15^\circ = \frac{1}{\sin 30^\circ} - \frac{\cos 30^\circ}{\sin 30^\circ}$ $\tan 15^\circ = \frac{1}{\frac{1}{2}} - \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2 - \sqrt{3}$	M1 CSO dM1 A1* (3)
(b)(ii)	$\tan 2x = 1$ $2x = 45^\circ$ $2x = 45^\circ + 180^\circ$ $x = 22.5^\circ, 112.5^\circ, 202.5^\circ, 292.5^\circ$	M1 A1 M1 A1(any two) A1 (5)
	Alt for (b)(i) $\tan 15^\circ = \tan(60^\circ - 45^\circ) \text{ or } \tan(45^\circ - 30^\circ)$ $\tan 15^\circ = \frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45} \text{ or } \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$ $\tan 15^\circ = \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \text{ or } \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$ Rationalises to produce $\tan 15^\circ = 2 - \sqrt{3}$	12 Marks M1 M1 A1*

Question Number	Scheme	Marks
7 (a)	$x^2 - 9 = (x + 3)(x - 3)$ $\frac{4x - 5}{(2x + 1)(x - 3)} - \frac{2x}{(x + 3)(x - 3)}$ $= \frac{(4x - 5)(x + 3)}{(2x + 1)(x - 3)(x + 3)} - \frac{2x(2x + 1)}{(2x + 1)(x + 3)(x - 3)}$ $= \frac{5x - 15}{(2x + 1)(x - 3)(x + 3)}$ $= \frac{5\cancel{(x - 3)}}{(2x + 1)\cancel{(x - 3)}(x + 3)} = \frac{5}{(2x + 1)(x + 3)}$	B1 M1 M1A1 A1* (5)
(b)	$f(x) = \frac{5}{2x^2 + 7x + 3}$ $f'(x) = \frac{-5(4x + 7)}{(2x^2 + 7x + 3)^2}$ $f'(-1) = -\frac{15}{4}$ Uses $m_1 m_2 = -1$ to give gradient of normal $= \frac{4}{15}$ $\frac{y - (-\frac{5}{2})}{(x - -1)} = \text{their } \frac{4}{15}$ $y + \frac{5}{2} = \frac{4}{15}(x + 1) \text{ or any equivalent form}$	 M1M1A1 M1A1 M1 M1 A1 (8) 13 Marks

Question Number	Scheme	Marks
8	(a) $R^2 = 2^2 + 3^2$ $R = \sqrt{13} \text{ or } 3.61 \dots$ $\tan \alpha = \frac{3}{2}$ $\alpha = 0.983 \dots$	M1 A1 M1 A1 (4)
	(b) $f'(x) = 2e^{2x} \cos 3x - 3e^{2x} \sin 3x$ $= e^{2x} (2 \cos 3x - 3 \sin 3x)$ $= e^{2x} (R \cos(3x + \alpha))$ $= R e^{2x} \cos(3x + \alpha)$	M1A1A1 M1 A1* cso (5)
	(c) $f'(x) = 0 \quad \cos(3x + \alpha) = 0$ $3x + \alpha = \frac{\pi}{2}$ $x = 0.196 \dots \quad \text{awrt } 0.20$	M1 M1 A1 (3)
	Alternative to part (c) $f'(x) = 0 \quad 2 \cos 3x - 3 \sin 3x = 0$ $\tan 3x = \frac{2}{3}$ $x = 0.196 \dots \quad \text{awrt } 0.20$	M1 M1 A1 (3)
		12 Marks

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Mark Scheme (Results)

Summer 2012

GCE Core Mathematics C3
(6665) Paper 1

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Summer 2012

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Summer 2012
6665 Core Mathematics
C3 Mark Scheme

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

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 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - $\square$ The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c), leading to $x = \dots$

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1.	$9x^2 - 4 = (3x - 2)(3x + 2)$ At any stage Eliminating the common factor of $(3x+2)$ at any stage $\frac{2(3x+2)}{(3x-2)(3x+2)} = \frac{2}{3x-2}$	B1
	Use of a common denominator	B1
	$\frac{2(3x+2)(3x+1)}{(9x^2-4)(3x+1)} - \frac{2(9x^2-4)}{(9x^2-4)(3x+1)} \text{ or } \frac{2(3x+1)}{(3x-2)(3x+1)} - \frac{2(3x-2)}{(3x+1)(3x-2)}$	M1
	$\frac{6}{(3x-2)(3x+1)} \text{ or } \frac{6}{9x^2-3x-2}$	A1
		(4 marks)

Notes

- B1 For factorising $9x^2 - 4 = (3x - 2)(3x + 2)$ using difference of two squares. It can be awarded at any stage of the answer but it must be scored on E pen as the first mark
- B1 For eliminating/cancelling out a factor of $(3x+2)$ at any stage of the answer.
- M1 For combining two fractions to form a single fraction with a common denominator. Allow slips on the numerator but at least one must have been adapted. Condone invisible brackets. Accept two separate fractions with the same denominator as shown in the mark scheme. Amongst possible (incorrect) options scoring method marks are

$$\frac{2(3x+2)}{(9x^2-4)(3x+1)} - \frac{2(9x^2-4)}{(9x^2-4)(3x+1)} \quad \text{Only one numerator adapted, separate fractions}$$

$$\frac{2 \times 3x + 1 - 2 \times 3x - 2}{(3x-2)(3x+1)} \quad \text{Invisible brackets, single fraction}$$

$$\text{A1 } \frac{6}{(3x-2)(3x+1)}$$

This is not a given answer so you can allow recovery from 'invisible' brackets.

Alternative method

$$\frac{2(3x+2)}{(9x^2-4)} - \frac{2}{(3x+1)} = \frac{2(3x+2)(3x+1) - 2(9x^2-4)}{(9x^2-4)(3x+1)} = \frac{18x+12}{(9x^2-4)(3x+1)} \quad \text{has scored 0,0,1,0 so far}$$

$$= \frac{6(3x+2)}{(3x+2)(3x-2)(3x+1)} \quad \text{is now 1,1,1,0}$$

$$= \frac{6}{(3x-2)(3x+1)} \quad \text{and now 1,1,1,1}$$

Question Number	Scheme	Marks
2.	(a) $x^3 + 3x^2 + 4x - 12 = 0 \Rightarrow x^3 + 3x^2 = 12 - 4x$ $\Rightarrow x^2(x+3) = 12 - 4x$ $\Rightarrow x^2 = \frac{12-4x}{(x+3)} \Rightarrow x = \sqrt{\frac{4(3-x)}{(x+3)}}$ (b) $x_1 = 1.41$, awrt $x_2 = 1.20$ $x_3 = 1.31$ (c) Choosing (1.2715, 1.2725) or tighter containing root 1.271998323 $f(1.2725) = (+)0.00827...$ $f(1.2715) = -0.00821....$ Change of sign $\Rightarrow \alpha = 1.272$	M1 dM1A1* (3) M1A1, A1 (3) M1 M1 A1 (3) (9 marks)

Notes

- (a) M1 Moves from $f(x)=0$, which may be implied by subsequent working, to $x^2(x \pm 3) = \pm 12 \pm 4x$ by separating terms and factorising in either order. No need to factorise rhs for this mark.
- dM1 Divides by ' $(x+3)$ ' term to make x^2 the subject, then takes square root. No need for rhs to be factorised at this stage
- A1* CSO. This is a given solution. Do not allow sloppy algebra or notation with root on just numerator for instance. The $12-4x$ needs to have been factorised.
- (b) **Note that this appears B1, B1, B1 on EPEN**
- M1 An attempt to substitute $x_0 = 1$ into the iterative formula to calculate x_1 .
- This can be awarded for the sight of $\sqrt{\frac{4(3-1)}{(3+1)}}$, $\sqrt{\frac{8}{4}}$, $\sqrt{2}$ and even 1.4
- A1 $x_1 = 1.41$. The subscript is not important. Mark as the first value found, $\sqrt{2}$ is A0
- A1 $x_2 = \text{awrt } 1.20$ $x_3 = \text{awrt } 1.31$. Mark as the second and third values found. Condone 1.2 for x_2
- (c) **Note that this appears M1A1A1 on EPEN**
- M1 Choosing the interval (1.2715, 1.2725) or tighter containing the root 1.271998323. Continued iteration is not allowed for this question and is M0
- M1 Calculates $f(1.2715)$ and $f(1.2725)$, or **the** tighter interval with at least 1 correct to 1 sig fig rounded or truncated.
- Accept $f(1.2715) = -0.008$ 1sf rounded or truncated. Also accept $f(1.2715) = -0.01$ 2dp
- Accept $f(1.2725) = (+)0.008$ 1sf rounded or truncated. Also accept $f(1.2725) = (+)0.01$ 2dp
- A1 Both values correct (see above),
- A valid reason; Accept change of sign, or $>0 <0$, or $f(1.2715) \times f(1.2725) < 0$
- And a (minimal) conclusion; Accept hence root or $\alpha = 1.272$ or QED or □

Alternative to (a) working backwards

2(a)

	$x = \sqrt{\frac{4(3-x)}{(x+3)}} \Rightarrow x^2 = \frac{4(3-x)}{(x+3)} \Rightarrow x^2(x+3) = 4(3-x)$ $x^3 + 3x^2 = 12 - 4x \Rightarrow x^3 + 3x^2 + 4x - 12 = 0$ States that this is $f(x)=0$	M1 dM1 A1*
		(3)

Alternative starting with the given result and working backwards

M1 Square (both sides) and multiply by $(x+3)$

dM1 Expand brackets and collect terms on one side of the equation =0

A1 A statement to the effect that this is $f(x)=0$ **An acceptable answer to (c) with an example of a tighter interval**

M1 Choosing the interval (1.2715, 1.2720). This contains the root 1.2719(98323)

M1 Calculates $f(1.2715)$ and $f(1.2720)$, with at least 1 correct to 1 sig fig rounded or truncated.Accept $f(1.2715) = -0.008$ 1sf rounded or truncated $f(1.2715) = -0.01$ 2dpAccept $f(1.2720) = (+)0.00003$ 1sf rounded or $f(1.2720) = (+)0.00002$ truncated 1sf

A1 Both values correct (see above),

A valid reason; Accept change of sign, or $>0 <0$, or $f(1.2715) \times f(1.2720) < 0$ And a (minimal) conclusion; Accept hence root or $\alpha=1.272$ or QED or $\square$

x	f(x)
1.2715	-0.00821362
1.2716	-0.00656564
1.2717	-0.00491752
1.2718	-0.00326927
1.2719	-0.00162088
1.2720	+0.00002765
1.2721	+0.00167631
1.2722	+0.00332511
1.2723	+0.00497405
1.2724	+0.00662312
1.2725	+0.00827233

An acceptable answer to (c) using $g(x)$ where $g(x) = \sqrt{\frac{4(3-x)}{(x+3)}} - x$ 2nd M1 Calculates $g(1.2715)$ and $g(1.2725)$, or **the** tighter interval with at least 1 correct to 1 sig fig rounded or truncated. $g(1.2715) = 0.0007559$. Accept $g(1.2715) = \text{awrt } (+)0.0008$ 1sf rounded or awrt 0.0007 truncated. $g(1.2725) = -0.00076105$. Accept $g(1.2725) = \text{awrt } -0.0008$ 1sf rounded or awrt -0.0007 truncated.

Question Number	Scheme	Marks
3.	(a) $\frac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}} \sin 3x + 3e^{x\sqrt{3}} \cos 3x$	M1A1
	$\frac{dy}{dx} = 0 \quad e^{x\sqrt{3}}(\sqrt{3} \sin 3x + 3 \cos 3x) = 0$	M1
	$\tan 3x = -\sqrt{3}$	A1
	$3x = \frac{2\pi}{3} \Rightarrow x = \frac{2\pi}{9}$	M1A1
	(b) At $x=0$ $\frac{dy}{dx} = 3$ Equation of normal is $-\frac{1}{3} = \frac{y-0}{x-0}$ or any equivalent $y = -\frac{1}{3}x$	B1 M1A1 (3) (9 marks)

- (a) M1 Applies the product rule $vu' + uv'$ to $e^{x\sqrt{3}} \sin 3x$. If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, ie. terms are written out $u = \dots, u' = \dots, v = \dots, v' = \dots$ followed by their

$vu' + uv'$) only accept answers of the form $\frac{dy}{dx} = Ae^{x\sqrt{3}} \sin 3x + e^{x\sqrt{3}} \times \pm B \cos 3x$

A1 Correct expression for $\frac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}} \sin 3x + 3e^{x\sqrt{3}} \cos 3x$

M1 Sets **their** $\frac{dy}{dx} = 0$, factorises out or divides by $e^{x\sqrt{3}}$ producing an equation in $\sin 3x$ and $\cos 3x$

A1 Achieves either $\tan 3x = -\sqrt{3}$ or $\tan 3x = -\frac{3}{\sqrt{3}}$

M1 Correct order of arctan, followed by $\div 3$.

Accept $3x = \frac{5\pi}{3} \Rightarrow x = \frac{5\pi}{9}$ or $3x = \frac{-\pi}{3} \Rightarrow x = \frac{-\pi}{9}$ but not $x = \arctan(\frac{-\sqrt{3}}{3})$

A1 CS0 $x = \frac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

- (b) B1 Sight of **3** for the gradient

M1 A full method for finding an equation of the normal.

Their tangent gradient m must be modified to $-\frac{1}{m}$ and used together with $(0, 0)$.

Eg $-\frac{1}{\text{their 'm'}} = \frac{y-0}{x-0}$ or equivalent is acceptable

A1 $y = -\frac{1}{3}x$ or any correct equivalent including $-\frac{1}{3} = \frac{y-0}{x-0}$.

Alternative in part (a) using the form $R\sin(3x + \alpha)$ JUST LAST 3 MARKS

Question Number	Scheme	Marks
3.	(a) $\frac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}} \sin 3x + 3e^{x\sqrt{3}} \cos 3x$ $\frac{dy}{dx} = 0 \quad e^{x\sqrt{3}}(\sqrt{3} \sin 3x + 3 \cos 3x) = 0$ $(\sqrt{12}) \sin(3x + \frac{\pi}{3}) = 0$ $3x = \frac{2\pi}{3} \Rightarrow x = \frac{2\pi}{9}$	M1A1 M1 A1 M1A1 (6)

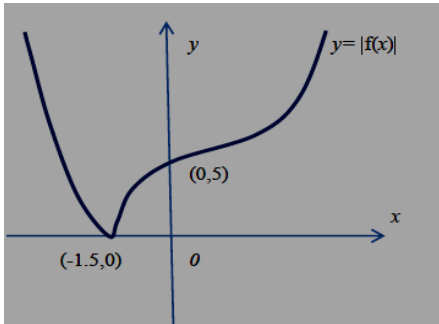
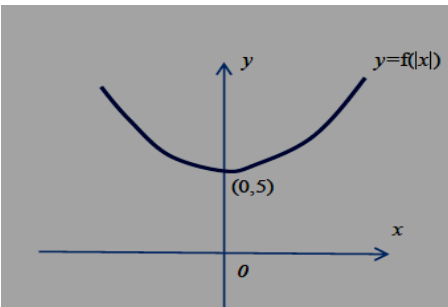
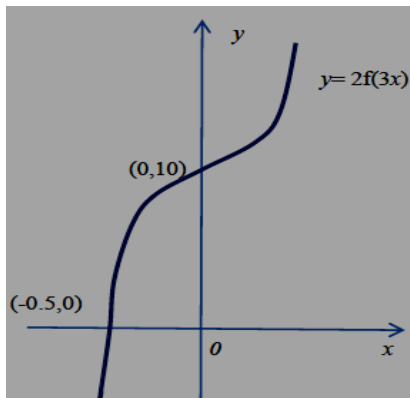
A1 Achieves either $(\sqrt{12}) \sin(3x + \frac{\pi}{3}) = 0$ or $(\sqrt{12}) \cos(3x - \frac{\pi}{6}) = 0$

M1 Correct order of arcsin or arcos, etc to produce a value of x
 Eg accept $3x + \frac{\pi}{3} = 0$ or π or $2\pi \Rightarrow x = \dots$

A1 Cao $x = \frac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

Alternative to part (a) squaring both sides JUST LAST 3 MARKS

Question Number	Scheme	Marks
3.	(a) $\frac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}} \sin 3x + 3e^{x\sqrt{3}} \cos 3x$ $\frac{dy}{dx} = 0 \quad e^{x\sqrt{3}}(\sqrt{3} \sin 3x + 3 \cos 3x) = 0$ $\sqrt{3} \sin 3x = -3 \cos 3x \Rightarrow \cos^2(3x) = \frac{1}{4}$ or $\sin^2(3x) = \frac{3}{4}$ $x = \frac{1}{3} \arccos(\pm \sqrt{\frac{1}{4}})$ oe $x = \frac{2\pi}{9}$	M1A1 M1 A1 M1 A1

Question Number	Scheme	Marks
4.(a)	 Shape including cusp (-1.5, 0) and (0, 5)	B1 B1 (2)
(b)	 Shape (0,5)	B1 B1 (2)
(c)	 Shape (0,10) (-0.5, 0)	B1 B1 B1 (3) (7 marks)

(a) **Note that this appears as M1A1 on EPEN**

B1 Shape (inc cusp) with graph in just quadrants 1 and 2. Do not be overly concerned about relative gradients, but the left hand section of the curve should not bend back beyond the cusp

B1 This is independent, and for the curve touching the x-axis at (-1.5, 0) **and** crossing the y-axis at (0,5)

(b) **Note that this appears as M1A1 on EPEN**

B1 For a U shaped curve symmetrical about the y- axis

B1 (0,5) lies on the curve

(c) **Note that this appears as M1B1B1 on EPEN**

B1 Correct shape- do not be overly concerned about relative gradients. Look for a similar shape to $f(x)$

B1 Curve **crosses** the y axis at (0, 10). The curve must appear in both quadrants 1 and 2

B1 Curve **crosses** the x axis at (-0.5, 0). The curve must appear in quadrants 3 and 2.

In all parts accept the following for any co-ordinate. Using (0,3) as an example, accept both (3,0) or 3 written on the y axis (as long as the curve passes through the point)

Special case with (a) and (b) completely correct but the wrong way around mark - SC(a) 0,1 SC(b) 0,1 Otherwise follow scheme

Question Number	Scheme	Marks
5.	(a) $4\cos\text{ec}^2 2\theta - \cos\text{ec}^2 \theta = \frac{4}{\sin^2 2\theta} - \frac{1}{\sin^2 \theta}$ $= \frac{4}{(2\sin\theta\cos\theta)^2} - \frac{1}{\sin^2 \theta}$ (b) $\frac{4}{(2\sin\theta\cos\theta)^2} - \frac{1}{\sin^2 \theta} = \frac{4}{4\sin^2 \theta \cos^2 \theta} - \frac{1}{\sin^2 \theta}$ $= \frac{1}{\sin^2 \theta \cos^2 \theta} - \frac{\cos^2 \theta}{\sin^2 \theta \cos^2 \theta}$ Using $1 - \cos^2 \theta = \sin^2 \theta$ $= \frac{\sin^2 \theta}{\sin^2 \theta \cos^2 \theta}$ $= \frac{1}{\cos^2 \theta} = \sec^2 \theta$ (c) $\sec^2 \theta = 4 \Rightarrow \sec \theta = \pm 2 \Rightarrow \cos \theta = \pm \frac{1}{2}$ $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$	B1 B1 (2) M1 M1 M1A1* (4) M1 A1,A1 (3) (9 marks)

Note (a) and (b) can be scored together

(a) B1 One term correct. Eg. writes $4\cos\text{ec}^2 2\theta$ as $\frac{4}{(2\sin\theta\cos\theta)^2}$ **or** $\cos\text{ec}^2 \theta$ as $\frac{1}{\sin^2 \theta}$. Accept terms like

$\cos\text{ec}^2 \theta = 1 + \cot^2 \theta = 1 + \frac{\cos^2 \theta}{\sin^2 \theta}$. The question merely asks for an expression in $\sin \theta$ and $\cos \theta$

B1 A fully correct expression in $\sin \theta$ and $\cos \theta$. Eg. $\frac{4}{(2\sin\theta\cos\theta)^2} - \frac{1}{\sin^2 \theta}$ **Accept equivalents**

Allow a different variable say x 's instead of θ 's but do not allow mixed units.

b) M1 Attempts to combine their expression in $\sin\theta$ and $\cos\theta$ using a common denominator. The terms can be separate but the denominator must be correct and one of the numerators must have been adapted

M1 Attempts to form a 'single' term on the numerator by using the identity $1 - \cos^2 \theta = \sin^2 \theta$

M1 Cancels correctly by $\sin^2 \theta$ terms and replaces $\frac{1}{\cos^2 \theta}$ with $\sec^2 \theta$

A1* Cso. This is a given answer. All aspects must be correct

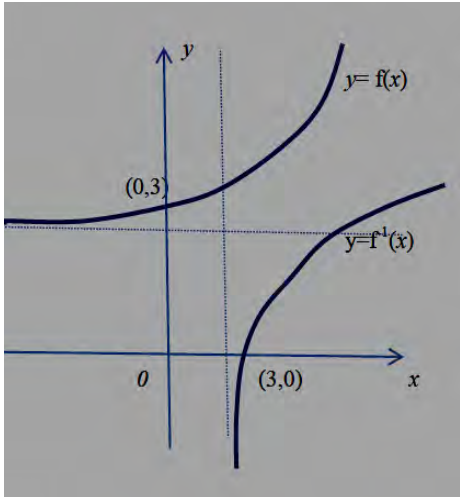
IF IN ANY DOUBT SEND TO REVIEW OR CONSULT YOUR TEAM LEADER

c) M1 For $\sec^2 \theta = 4$ leading to a solution of $\cos \theta$ by taking the root and inverting in either order.

Similarly accept $\tan^2 \theta = 3$, $\sin^2 \theta = \frac{3}{4}$ leading to solutions of $\tan \theta$, $\sin \theta$. Also accept $\cos 2\theta = -\frac{1}{2}$

A1 Obtains one correct answer usually $\theta = \frac{\pi}{3}$ Do not accept decimal answers or degrees

A1 Obtains both correct answers. $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$ Do not award if there are extra solutions inside the range.
Ignore solutions outside the range.

Question Number	Scheme	Marks
6.	(a) $f(x) > 2$	B1 (1)
	(b) $fg(x) = e^{\ln x} + 2, = x + 2$	M1,A1 (2)
	(c) $e^{2x+3} + 2 = 6 \Rightarrow e^{2x+3} = 4$ $\Rightarrow 2x + 3 = \ln 4$ $\Rightarrow x = \frac{\ln 4 - 3}{2} \text{ or } \ln 2 - \frac{3}{2}$	M1A1 M1A1 (4)
	(d) Let $y = e^x + 2 \Rightarrow y - 2 = e^x \Rightarrow \ln(y - 2) = x$ $f^{-1}(x) = \ln(x - 2), \quad x > 2.$	M1 A1 , B1ft (3)
	(e) 	Shape for $f(x)$ (0, 3) Shape for $f^{-1}(x)$ (3, 0) (4)
		(14 marks)

- (a) B1 Range of $f(x) > 2$. Accept $y > 2$, $(2, \infty)$, $f > 2$, as well as 'range is the set of numbers bigger than 2' but **don't accept** $x > 2$
- (b) M1 For applying the correct order of operations. Look for $e^{\ln x} + 2$. Note that $\ln e^x + 2$ is M0
A1 Simplifies $e^{\ln x} + 2$ to $x + 2$. Just the answer is acceptable for both marks
- (c) M1 Starts with $e^{2x+3} + 2 = 6$ and proceeds to $e^{2x+3} = \dots$
A1 $e^{2x+3} = 4$
M1 Takes $\ln$'s both sides, $2x + 3 = \ln \dots$ and proceeds to $x = \dots$
A1 $x = \frac{\ln 4 - 3}{2}$ oe. eg $\ln 2 - \frac{3}{2}$ Remember to isw any incorrect working after a correct answer

(d) Note that this is marked M1A1A1 on EPEN

M1 Starts with $y = e^x + 2$ or $x = e^y + 2$ and attempts to change the subject.

All $\ln$ work must be correct. The 2 must be dealt with first.

Eg. $y = e^x + 2 \Rightarrow \ln y = x + \ln 2 \Rightarrow x = \ln y - \ln 2$ is M0

A1 $f^{-1}(x) = \ln(x-2)$ or $y = \ln(x-2)$ or $y = \ln|x-2|$ There must be some form of bracket

B1ft Either $x > 2$, or follow through on their answer to part (a), provided that it wasn't $y \in \mathbb{R}$

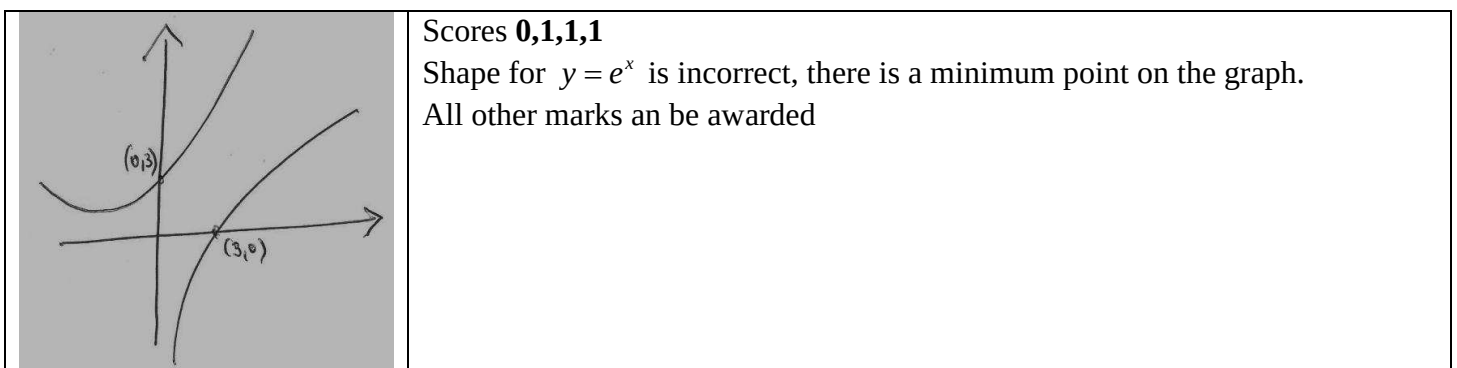
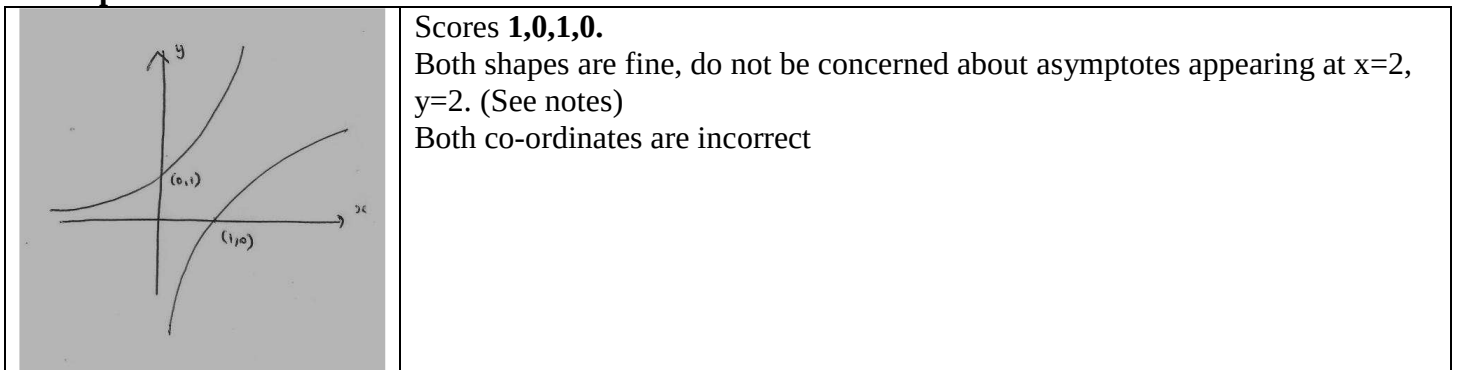
Do not accept $y > 2$ or $f^{-1}(x) > 2$.

(e) B1 Shape for $y = e^x$. The graph should only lie in quadrants 1 and 2. It should start out with a gradient that is approx. 0 above the x axis in quadrant 2 and increase in gradient as it moves into quadrant 1. You should not see a minimum point on the graph.

B1 (0, 3) lies on the curve. Accept 3 written on the y axis as long as the point lies on the curve

B1 Shape for $y = \ln x$. The graph should only lie in quadrants 4 and 1. It should start out with gradient that is approx. infinite to the right of the y axis in quadrant 4 and decrease in gradient as it moves into quadrant 1. You should not see a maximum point. Also withhold this mark if it intersects $y = e^x$

B1 (3, 0) lies on the curve. Accept 3 written on the x axis as long as the point lies on the curve

Condone lack of labels in this part**Examples**

Question Number	Scheme	Marks
7.	(a)(i) $\frac{d}{dx}(\ln(3x)) = \frac{3}{3x}$ $\frac{d}{dx}(x^{\frac{1}{2}} \ln(3x)) = \ln(3x) \times \frac{1}{2} x^{-\frac{1}{2}} + x^{\frac{1}{2}} \times \frac{3}{3x}$ (ii) $\frac{dy}{dx} = \frac{(2x-1)^5 \times -10 - (1-10x) \times 5(2x-1)^4 \times 2}{(2x-1)^{10}}$ $\frac{dy}{dx} = \frac{80x}{(2x-1)^6}$ (b) $x = 3 \tan 2y \Rightarrow \frac{dx}{dy} = 6 \sec^2 2y$ $\Rightarrow \frac{dy}{dx} = \frac{1}{6 \sec^2 2y}$ Uses $\sec^2 2y = 1 + \tan^2 2y$ and uses $\tan 2y = \frac{x}{3}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{6(1+(\frac{x}{3})^2)} = (\frac{3}{18+2x^2})$	M1 M1A1 (3) M1A1 A1 (3) M1A1 M1 M1A1 (5) (11 marks)

Note that this is marked B1M1A1 on EPEN

(a)(i) M1 Attempts to differentiate $\ln(3x)$ to $\frac{B}{x}$. Note that $\frac{1}{3x}$ is fine.

M1 Attempts the product rule for $x^{\frac{1}{2}} \ln(3x)$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the rule is not quoted nor implied from their stating of u , u' , v , v' and their subsequent expression, only accept answers of the form

$$\ln(3x) \times Ax^{-\frac{1}{2}} + x^{\frac{1}{2}} \times \frac{B}{x}, \quad A, B > 0$$

A1 Any correct (un simplified) form of the answer. Remember to isw any incorrect further work

$$\frac{d}{dx}(x^{\frac{1}{2}} \ln(3x)) = \ln(3x) \times \frac{1}{2} x^{-\frac{1}{2}} + x^{\frac{1}{2}} \times \frac{3}{3x} = \left(\frac{\ln(3x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}} \right) = x^{-\frac{1}{2}} \left(\frac{1}{2} \ln 3x + 1 \right)$$

Note that this part does not require the answer to be in its simplest form

(ii) M1 Applies the quotient rule, a version of which appears in the formula booklet. If the formula is quoted it must be correct. There must have been an attempt to differentiate both terms. If the formula is not quoted nor implied from their stating of u , u' , v , v' and their subsequent expression, only accept answers of the form

$$\frac{(2x-1)^5 \times \pm 10 - (1-10x) \times C(2x-1)^4}{(2x-1)^{10 \text{ or } 7 \text{ or } 25}}$$

A1 Any un simplified form of the answer. Eg $\frac{dy}{dx} = \frac{(2x-1)^5 \times -10 - (1-10x) \times 5(2x-1)^4 \times 2}{((2x-1)^5)^2}$

A1 Cao. It must be simplified as required in the question $\frac{dy}{dx} = \frac{80x}{(2x-1)^6}$

(b) M1 Knows that $3 \tan 2y$ differentiates to $C \sec^2 2y$. The lhs can be ignored for this mark. If they write $3 \tan 2y$ as $\frac{3 \sin 2y}{\cos 2y}$ this mark is awarded for a correct attempt of the quotient rule.

A1 Writes down $\frac{dx}{dy} = 6 \sec^2 2y$ or implicitly to get $1 = 6 \sec^2 2y \frac{dy}{dx}$

Accept from the quotient rule $\frac{6}{\cos^2 2y}$ or even $\frac{\cos 2y \times 6 \cos 2y - 3 \sin 2y \times -2 \sin 2y}{\cos^2 2y}$

M1 An attempt to invert 'their' $\frac{dx}{dy}$ to reach $\frac{dy}{dx} = f(y)$, or changes the subject of their implicit differential to achieve a similar result $\frac{dy}{dx} = f(y)$

M1 Replaces an expression for $\sec^2 2y$ in their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ with x by attempting to use

$\sec^2 2y = 1 + \tan^2 2y$. Alternatively, replaces an expression for y in $\frac{dx}{dy}$ or $\frac{dy}{dx}$ with $\frac{1}{2} \arctan\left(\frac{x}{3}\right)$

A1 Any correct form of $\frac{dy}{dx}$ in terms of x . $\frac{dy}{dx} = \frac{1}{6(1+(\frac{x}{3})^2)} \frac{dy}{dx} = \frac{3}{18+2x^2}$ or $\frac{1}{6 \sec^2(\arctan(\frac{x}{3}))}$

Question Number	Scheme	Marks
7.	(a)(ii) Alt using the product rule Writes $\frac{1-10x}{(2x-1)^5}$ as $(1-10x)(2x-1)^{-5}$ and applies $vu'+uv'$. See (a)(i) for rules on how to apply $(2x-1)^{-5} \times -10 + (1-10x) \times -5(2x-1)^{-6} \times 2$ Simplifies as main scheme to $80x(2x-1)^{-6}$ or equivalent (b) Alternative using arctan. They must attempt to differentiate to score any marks. Technically this is M1A1M1A2 Rearrange $x = 3 \tan 2y$ to $y = \frac{1}{2} \arctan\left(\frac{x}{3}\right)$ and attempt to differentiate Differentiates to a form $\frac{A}{1+(\frac{x}{3})^2}$, $= \frac{1}{2} \times \frac{1}{(1+(\frac{x}{3})^2)} \times \frac{1}{3}$ or $\frac{1}{6(1+(\frac{x}{3})^2)}$ oe	M1A1 A1 (3) M1A1 M1, A2 (5)

Question Number	Scheme	Marks
8.	(a) $R=25$ $\tan \alpha = \frac{24}{7} \Rightarrow \alpha = (\text{awrt})73.7^\circ$ (b) $\cos(2x + \text{their } \alpha) = \frac{12.5}{\text{their } R}$ $2x + \text{their } \alpha' = 60^\circ$ $2x + \text{their } \alpha' = \text{their } 300^\circ \text{ or their } 420^\circ \Rightarrow x = ..$ $x = \text{awrt } 113.1^\circ, 173.1^\circ$ (c) Attempts to use $\cos 2x = 2\cos^2 x - 1$ AND $\sin 2x = 2\sin x \cos x$ in the expression $14\cos^2 x - 48\sin x \cos x = 7(\cos 2x + 1) - 24\sin 2x$ $= 7\cos 2x - 24\sin 2x + 7$ (d) $14\cos^2 x - 48\sin x \cos x = R\cos(2x + \alpha) + 7$ Maximum value = 'R' + 'c' $= 32 \text{ cao}$	B1 M1A1 (3) M1 A1 M1 A1A1 (5) M1 A1 (2) M1 A1 (2) (12 marks)

(a) B1 Accept 25, awrt 25.0, $\sqrt{625}$. Condone ± 25

M1 For $\tan \alpha = \pm \frac{24}{7}$ $\tan \alpha = \pm \frac{7}{24}$ $\sin \alpha = \pm \frac{24}{\text{their } R}$, $\cos \alpha = \pm \frac{7}{\text{their } R}$

A1 $\alpha = (\text{awrt})73.7^\circ$. The answer 1.287 (radians) is A0

(b) M1 For using part (a) and dividing by their R to reach $\cos(2x + \text{their } \alpha) = \frac{12.5}{\text{their } R}$

A1 Achieving $2x + \text{their } \alpha = 60^{(0)}$. This can be implied by $113.1^{(0)}/113.2^{(0)}$ or $173.1^{(0)}/173.2^{(0)}$ or $-6.8^{(0)}/-6.85^{(0)}/-6.9^{(0)}$

M1 Finding a secondary value of x from their principal value. A correct answer will imply this mark
Look for $\frac{360 \pm \text{'their' principal value} \pm \text{'their' } \alpha}{2}$

A1 $x = \text{awrt } 113.1^\circ / 113.2^\circ$ OR $173.1^\circ / 173.2^\circ$.

A1 $x = \text{awrt } 113.1^\circ$ AND 173.1° . Ignore solutions outside of range. Penalise this mark for extra solutions inside the range

- (c) M1 Attempts to use $\cos 2x = 2\cos^2 x - 1$ **and** $\sin 2x = 2\sin x \cos x$ in expression.
 Allow slips in sign on the $\cos 2x$ term. So accept $2\cos^2 x = \pm \cos 2x \pm 1$
 A1 $\text{Cao} = 7\cos 2x - 24\sin 2x + 7$. The order of terms is not important. Also accept $a=7$, $b=-24$, $c=7$
- (d) M1 This mark is scored for adding their R to their c
 A1 $\text{cao } 32$

Radian solutions- they will lose the first time it occurs (usually in a with 1.287 radians) Part b will then be marked as follows

- (b) M1 For using part (a) and dividing by their R to reach $\cos(2x + \text{their } \alpha) = \frac{12.5}{\text{their } R}$
- A1 The correct principal value of $\frac{\pi}{3}$ or awrt 1.05 radians. Accept $60^{(0)}$
 This can be implied by awrt -0.12 radians or awrt 1.97 radians or awrt 3.02 radians
- M1 Finding a secondary value of x from their principal value. A correct answer will imply this mark
 Look for $\frac{2\pi \pm \text{'their' principal value} \pm \text{'their' } \alpha}{2}$ Do not allow mixed units.
- A1 $x = \text{awrt } 1.97 \text{ OR } 3.02$.
- A1 $x = \text{awrt } 1.97 \text{ AND } 3.02$. Ignore solutions outside of range. Penalise this mark for extra solutions inside the range

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Mark Scheme (Results)

Summer 2013

GCE Core Mathematics 3 (6665/01R)

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Summer 2013

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\surd$ will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - $\square$ The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
 7. Ignore wrong working or incorrect statements following a correct answer.
 8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x =$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x =$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1.	(a) $x^2 + x - 12 = (x + 4)(x - 3)$ Attempt as a single fraction $\frac{(3x+5)(x-3) - 2(x^2 + x - 12)}{(x^2 + x - 12)(x-3)}$ or $\frac{3x+5 - 2(x+4)}{(x+4)(x-3)}$ $= \frac{x-3}{(x+4)(x-3)}, = \frac{1}{(x+4)}$ cao	B1 M1 A1, A1 (4 marks)

Notes for Question 1

B1 For correctly factorising $x^2 + x - 12 = (x + 4)(x - 3)$. It could appear anywhere in their solution

M1 For an attempt to combine two fractions. The denominator must be correct for 'their' fractions. The terms could be separate but one term must have been modified. Condone invisible brackets.

Examples of work scoring this mark are;

$$\frac{(3x+5)(x-3)}{(x^2+x-12)(x-3)} - \frac{2(x^2+x-12)}{(x^2+x-12)(x-3)} \quad \text{Two separate terms}$$

$$\frac{3x+5-2x+4}{(x+4)(x-3)} \quad \text{Single term, invisible bracket}$$

$$\frac{(3x+5)}{(x^2+x-12)(x-3)} - \frac{2(x^2+x-12)}{(x^2+x-12)(x-3)} \quad \text{Separate terms, only one numerator modified}$$

A1 Correct un simplified answer $\frac{x-3}{(x+4)(x-3)}$

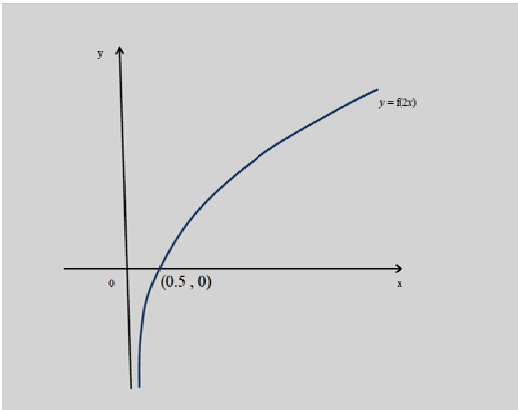
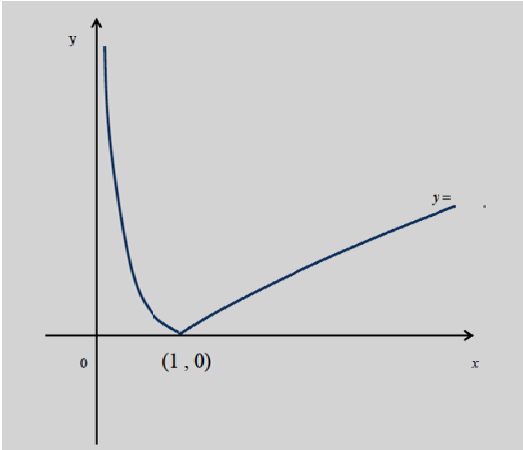
If $\frac{x^2-6x-9}{(x^2+x-12)(x-3)}$ scored M1 the fraction must be subsequently be reduced to a correct $\frac{x-3}{x^2+x-12}$ or $\frac{(x-3)(x-3)}{(x+4)(x-3)(x-3)}$ to score this mark.

A1 cao $\frac{1}{(x+4)}$

Do Not isw in this question.

The method of partial fractions is perfectly acceptable and can score full marks

$$\underbrace{\frac{3x+5}{(x+4)(x-3)}}_{B1} - \frac{2}{x-3} = \underbrace{\frac{1}{x+4} + \frac{2}{x-3}}_{M1A1} - \frac{2}{x-3} = \underbrace{\frac{1}{x+4}}_{A1}$$

Question Number	Scheme	Marks
2.(a)		Shape B1 (0.5, 0) B1 (2)
(b)		 Shape B1 (1,0) B1 Cusp at (1,0) B1 (3) (5 marks)

Notes for Question 2

(a)

B1 Award for the correct shape. Look for an increasing function with decreasing gradient. Condone linear looking functions in the first quadrant. It needs to look asymptotic at the y axis and have no obvious maximum point. It must be wholly contained in quadrants 1 and 4
See practice and qualification items for clarification.

B1 Crosses x axis at $\left(\frac{1}{2}, 0\right)$. Accept $\frac{1}{2}$, 0.5 or even $\left(0, \frac{1}{2}\right)$ marked on the correct axis.
There must be a graph for this mark to be scored.

(b)

B1 Correct shape wholly contained in quadrant 1.
The shape to the rhs of the cusp must not have an obvious maximum.
Accept linear, or positive with decreasing gradient. The gradient of the curve to the lhs of the cusp/minimum should always be negative. The curve in this section should not 'bend' back past (1, 0) forming a 'C' shape or have incorrect curvature.
See practice and qualification for clarification.

B1 The curve touches or crosses the x axis at (1, 0). Allow for the curve passing through a point marked '1' on the x axis. Condone the point marked on the correct axis as (0, 1)

B1 Award for a cusp, not a minimum at (1,0)

Note that $f(|x|)$ scores B0 B1 B0 under the scheme.

If the graphs are not labelled (a) and (b), then they are to be marked in the order they are presented

Question Number	Scheme	Marks
3.(a)	$7 \cos x + \sin x = R \cos(x - \alpha)$ $R = \sqrt{7^2 + 1^2} = \sqrt{50} = (5\sqrt{2})$ $\alpha = \arctan\left(\frac{1}{7}\right) = 8.13... = \text{awrt } 8.1^\circ$	B1 M1A1 (3)
(b)	$\sqrt{50} \cos(x - 8.1) = 5 \Rightarrow \cos(x - 8.1) = \frac{5}{\sqrt{50}}$ $x - 8.1 = 45 \Rightarrow x = 53.1^\circ$ $\text{AND } x - 8.1 = 315 \Rightarrow x = 323.1^\circ$	M1 M1,A1 M1A1 (5)
(c)	One solution if $\frac{k}{\sqrt{50}} = \pm 1, \Rightarrow k = \pm\sqrt{50}$	ft on R M1A1ft (2) (10 marks)

Notes for Question 3

- (a)
- B1 $R = \sqrt{50}$. Accept $5\sqrt{2}$ Accept $R = \pm\sqrt{50}$
 Do not accept $R = \sqrt{(7^2 + 1^2)}$ or the decimal equivalent 7.07... unless you see $\sqrt{50}$ or $5\sqrt{2}$ as well
- M1 For $\tan \alpha = \pm \frac{1}{7}$ or $\tan \alpha = \pm \frac{7}{1}$. Condone if this comes from $\cos \alpha = 7$, $\sin \alpha = 1$
 If R is used then only accept $\sin \alpha = \pm \frac{1}{R}$ or $\cos \alpha = \pm \frac{7}{R}$
- A1 $\alpha = \text{awrt } 8.1$.
 Be aware that $\tan \alpha = 7 \Rightarrow \alpha = 81.9$ can easily be mistaken for the correct answer
 Note that the radian answer awrt 0.1418... is A0
- (b)
- M1 For using their answers to part (a) and moving from $R \cos(x \pm \alpha) = 5 \Rightarrow \cos(x \pm \alpha) = \frac{5}{R}$ using their numerical values of R and α
 This may be implied for sight of 53.1 if R and α were correct
- M1 For achieving $x \pm \alpha = \text{awrt } 45^\circ$ or 315, leading to one value of x **in the range**
 Note that for this to be scored R has to be correct (to 2sf) as awrt 45, 315 must be achieved
 This may be implied for achieving an answer of either $45 + \text{their } \alpha$ or $315 + \text{their } \alpha$
- A1 One correct answer, either awrt 53.1° or 323.1°
- M1 For an attempt at finding a secondary value of x **in the range**.
 Usually this is an attempt at solving $x - \text{their } 8.1^\circ = 360^\circ - \text{their } 45^\circ \Rightarrow x = ..$
- A1 Both values correct awrt 53.1° and 323.1° .
 Withhold this mark if there are extra values in the range.
 Ignore extra values outside the range
- (c)
- M1 For stating that $\frac{k}{\text{their } R} = 1$ OR $\frac{k}{\text{their } R} = -1$
 This may be implied by seeing $k = (\pm)\text{their } R$
- A1ft Both values $k = \pm\sqrt{50}$ oe. Follow through on their numerical R

Answers all in radians. Lose the first time that it appears but demand an accuracy of 2dp.

Part (a) $R = \sqrt{50}$ $\alpha = \text{awrt } 0.14$

Part (b) $x = \text{awrt } 0.927, 5.64$. Accuracy must be to 3 sf.

With correct working this would score (a) B1M1A0 (b) M1A1A1M1A1

Mixed degrees and radians refer to the main scheme

Question Number	Scheme	Marks
4.(a)	$f(x) \geq 3$	M1A1 (2)
(b)	An attempt to find $2 3-4x +3$ when $x=1$ Correct answer $fg(1)=5$	M1 A1 (2)
(c)	$y=3-4x \Rightarrow 4x=3-y \Rightarrow x=\frac{3-y}{4}$ $g^{-1}(x)=\frac{3-x}{4}$	M1 A1 (2)
(d)	$[g(x)]^2 = (3-4x)^2$ $gg(x) = 3-4(3-4x)$ $gg(x) + [g(x)]^2 = 0 \Rightarrow -9+16x+9-24x+16x^2 = 0$ $16x^2 - 8x = 0$ $8x(2x-1) = 0 \Rightarrow x = 0, 0.5$ oe	B1 M1 A1 M1A1 (5) (11 marks)

Notes for Question 4

- (a)
- M1 Attempt at calculating f at $x=0$. Sight of 3 is sufficient. Accept $f(x) > 3$ and $x > 3$ for M1,
- A1 $f(x) \geq 3$. Accept $y \geq 3$, range ≥ 3 , $[3, \infty)$
Do not accept $f(x) > 3$, $x \geq 3$
The correct answer is sufficient for both marks.
- (b)
- M1 A full method of finding $fg(1)$. The order of substituting into the expressions must be correct and $2|x| + 3$ must be used as opposed to $2x + 3$
Accept an attempt to calculate $2|x| + 3$ when $x = -1$.
Accept an attempt to put $x=1$ into $3 - 4x$ and then substituting their answer to $3 - 4x|_{x=1}$ into $2|x| + 3$
Do not accept the substitution of $x=1$ into $2|x| + 3$, followed by their result into ' $3-4x$ '
This is evidence of incorrect order.
- A1 $fg(1)=5$.
Watch for $1 \xrightarrow{3-4x} 1 \xrightarrow{2|x|+3} 5$ which is M1A0
- (c)
- M1 Award for an attempt to make x or a swapped y the subject of the formula. It must be a full method and cannot finish $4x = ..$
You can condone at most one 'arithmetic' error for this method mark.
 $y = 3 - 4x \Rightarrow 4x = 3 + y \Rightarrow x = \frac{3+y}{4}$ is fine for the M1 as there is only one error
 $y = 3 - 4x \Rightarrow 4x = 3 - y \Rightarrow x = \frac{3}{4} - y$ is fine for the M1 as there is only one error
 $y = 3 - 4x \Rightarrow 4x = 3 + y \Rightarrow x = \frac{3}{4} + y$ is M0 as there are two arithmetic errors
- A1 Obtaining a correct expression $g^{-1}(x) = \frac{3-x}{4}$ or such as $g^{-1}(x) = \frac{x-3}{-4}$, $g^{-1}(x) = \frac{3}{4} - \frac{x}{4}$
It must be in terms of x , but could be expressed ' y ' or $g^{-1}(x) \rightarrow$
- (d)
- B1 Sight of $[g(x)]^2 = (3 - 4x)^2$. If only the expanded version appears it must be correct
- M1 A full attempt to find $gg(x) = 3 - 4(3 - 4x)$
Condone invisible brackets. Note that it may appear in an equation
- A1 $16x^2 - 8x = 0$ Accept other alternatives such as $2x^2 = x$
- M1 For factorising their quadratic or cancelling their $Ax^2 = Bx$ by x to get ≥ 1 value of x
If they have a 3TQ then usual methods are applicable.
- A1 Both values correct $x = 0, 0.5$ or

Question Number	Scheme	Marks
5.(a)	$\frac{d}{dx}(\cos 2x) = -2 \sin 2x$ $\text{Applies } \frac{vu' - uv'}{v^2} \text{ to } \frac{\cos 2x}{\sqrt{x}} = \frac{\sqrt{x} \times -2 \sin 2x - \cos 2x \times \frac{1}{2} x^{-\frac{1}{2}}}{(\sqrt{x})^2}$ $= \frac{-2\sqrt{x} \sin 2x - \frac{1}{2} x^{-\frac{1}{2}} \cos 2x}{x}$	B1 M1A1 (3)
(b)	$\frac{d}{dx}(\sec^2 3x) = 2 \sec 3x \times 3 \sec 3x \tan 3x (= 6 \sec^2 3x \tan 3x)$ $= 6(1 + \tan^2 3x) \tan 3x$ $= 6(\tan 3x + \tan^3 3x)$	M1 dM1 A1 (3)
(c)	$x = 2 \sin\left(\frac{y}{3}\right) \Rightarrow \frac{dx}{dy} = \frac{2}{3} \cos\left(\frac{y}{3}\right)$ $\frac{dy}{dx} = \frac{1}{\frac{2}{3} \cos\left(\frac{y}{3}\right)} = \frac{1}{\frac{2}{3} \sqrt{1 - \sin^2\left(\frac{y}{3}\right)}} = \frac{1}{\frac{2}{3} \sqrt{1 - \left(\frac{x}{2}\right)^2}}$ $\frac{dy}{dx} = \frac{3}{\sqrt{4 - x^2}} \quad \text{cao}$	M1A1 dM1 A1 (4) (10 marks)
Alt 5(c)	$y = 3 \arcsin\left(\frac{x}{2}\right) \Rightarrow \frac{dy}{dx} = \frac{3}{\sqrt{1 - \left(\frac{x}{2}\right)^2}} \times \frac{1}{2}$ $\frac{dy}{dx} = \frac{3}{\sqrt{4 - x^2}}$ M1 Rearranging to $y = A \arcsin Bx$ and differentiating to $\frac{dy}{dx} = \frac{A}{\sqrt{1 - Bx^2}}$ dM1 As above, but form of the rhs must be correct $\frac{dy}{dx} = \frac{C}{\sqrt{1 - \left(\frac{x}{2}\right)^2}}$ A1 Correct but un simplified answer	M1dM1A1 A1 (4)

Notes for Question 5

(a)

B1 Award for the sight of $\frac{d}{dx}(\cos 2x) = -2 \sin 2x$. This could be seen in their differential.

M1 Applies $\frac{vu' - uv'}{v^2}$ to $\frac{\cos 2x}{\sqrt{x}}$

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out $u=...$, $u'=...$, $v=...$, $v'=...$ followed by their $\frac{vu' - uv'}{v^2}$) then only accept answers of the form

$$\frac{\sqrt{x} \times \pm A \sin 2x - \cos 2x \times Bx^{-\frac{1}{2}}}{(\sqrt{x})^2 \text{ or } x^{\frac{1}{4}}}$$

A1 Award for a correct answer. This does not need to be simplified.

Alt (a) using the product rule

B1 Award for the sight of $\frac{d}{dx}(\cos 2x) = -2 \sin 2x$. This could be seen in their differential.

M1 Applies $vu' + uv'$ to $x^{-\frac{1}{2}} \cos 2x$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out $u=...$, $u'=...$, $v=...$, $v'=...$ followed by their $vu' + uv'$) then only accept answers of the form

$$\pm Ax^{-\frac{1}{2}} \sin 2x - Bx^{-\frac{3}{2}} \cos 2x$$

A1 Award for a correct answer. This does not need to be simplified.

$$-2x^{-\frac{1}{2}} \sin 2x - \frac{1}{2}x^{-\frac{3}{2}} \cos 2x$$

(b)

M1 Award for a correct application of the chain rule on $\sec^2 3x$
Sight of $C \sec 3x \sec 3x \tan 3x$ is sufficient

dM1 Replacing $\sec^2 3x = 1 + \tan^2 3x$ in their derivative to create an expression in just $\tan 3x$. It is dependent upon the first M being scored.

A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

Alt (b) using the product rule

M1 Writes $\sec^2 3x$ as $\sec 3x \times \sec 3x$ and uses the product rule with $u' = A \sec 3x \tan 3x$ and $v' = B \sec 3x \tan 3x$ to produce a derivative of the form $A \sec 3x \tan 3x \sec 3x + B \sec 3x \tan 3x \sec 3x$

dM1 Replaces $\sec^2 3x$ with $1 + \tan^2 3x$ to produce an expression in just $\tan 3x$. It is dependent upon the first M being scored.

Notes for Question 5 Continued

A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

Alt (b) using $\sec 3x = \frac{1}{\cos 3x}$ **and proceeding by the chain or quotient rule**

M1 Writes $\sec^2 3x$ as $(\cos 3x)^{-2}$ and differentiates to $A(\cos 3x)^{-3} \sin 3x$

Alternatively writes $\sec^2 3x$ as $\frac{1}{(\cos 3x)^2}$ and achieves $\frac{(\cos 3x)^2 \times 0 - 1 \times A \cos 3x \sin 3x}{(\cos^2 3x)^2}$

dM1 Uses $\frac{\sin 3x}{\cos 3x} = \tan 3x$ and $\frac{1}{\cos^2 3x} = \sec^2 3x$ and $\sec^2 3x = 1 + \tan^2 3x$ in their derivative to create an expression in just $\tan 3x$. It is dependent upon the first M being scored.

A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

Alt (b) using $\sec^2 3x = 1 + \tan^2 3x$

M1 Writes $\sec^2 3x$ as $1 + \tan^2 3x$ and

uses chain rule to produce a derivative of the form $A \tan 3x \sec^2 3x$

or the product rule to produce a derivative of the form $C \tan 3x \sec^2 3x + D \tan 3x \sec^2 3x$

dM1 Replaces $\sec^2 3x = 1 + \tan^2 3x$ to produce an expression in just $\tan 3x$. It is dependent upon the first M being scored.

A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

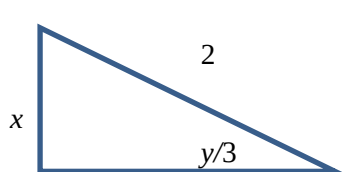
(c)

M1 Award for knowing the method that $\sin\left(\frac{y}{3}\right)$ differentiates to $\cos\left(\frac{y}{3}\right)$ The lhs does not need to be

correct/present. Award for $2 \sin\left(\frac{y}{3}\right) \rightarrow A \cos\left(\frac{y}{3}\right)$

A1 $x = 2 \sin\left(\frac{y}{3}\right) \Rightarrow \frac{dx}{dy} = \frac{2}{3} \cos\left(\frac{y}{3}\right)$. Both sides must be correct

dM1 Award for inverting their $\frac{dx}{dy}$ and using $\sin^2 \frac{y}{3} + \cos^2 \frac{y}{3} = 1$ to produce an expression for $\frac{dy}{dx}$ in terms of x only. It is dependent upon the first M 1 being scored.
An alternative to Pythagoras is a triangle.



$$\sin\left(\frac{y}{3}\right) = \frac{x}{2} \Rightarrow \cos\left(\frac{y}{3}\right) = \frac{\sqrt{4-x^2}}{2}$$

Notes for Question 5 Continued

Candidates who write $\frac{dy}{dx} = \frac{3}{2 \cos \left(\arcsin \left(\frac{x}{2} \right) \right)}$ do not score the mark.

BUT $\frac{dy}{dx} = \frac{3}{2 \sqrt{1 - \sin^2 \left(\arcsin \left(\frac{x}{2} \right) \right)}}$ does score M1 as they clearly use a correct Pythagorean identity as required by the notes.

A1 $\frac{dy}{dx} = \frac{3}{\sqrt{4 - x^2}}$. Expression must be in its simplest form.

Do not accept $\frac{dy}{dx} = \frac{3}{2 \sqrt{1 - \frac{1}{4} x^2}}$ or $\frac{dy}{dx} = \frac{1}{\frac{1}{3} \sqrt{4 - x^2}}$ for the final A1

Question Number	Scheme	Marks
6.(i)	$\operatorname{cosec} 2x = \frac{1}{\sin 2x}$ $= \frac{1}{2 \sin x \cos x}$ $= \frac{1}{2} \operatorname{cosec} x \sec x \Rightarrow \lambda = \frac{1}{2}$	M1 M1 A1 (3)
(ii)	$3 \sec^2 \theta + 3 \sec \theta = 2 \tan^2 \theta \Rightarrow 3 \sec^2 \theta + 3 \sec \theta = 2(\sec^2 \theta - 1)$ $\sec^2 \theta + 3 \sec \theta + 2 = 0$ $(\sec \theta + 2)(\sec \theta + 1) = 0$ $\sec \theta = -2, -1$ $\cos \theta = -0.5, -1$ $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \pi$	M1 M1 A1 M1 A1A1 (6) (9 marks)
ALT (ii)	$3 \sec^2 \theta + 3 \sec \theta = 2 \tan^2 \theta \Rightarrow 3 \times \frac{1}{\cos^2 \theta} + 3 \times \frac{1}{\cos \theta} = 2 \times \frac{\sin^2 \theta}{\cos^2 \theta}$ $3 + 3 \cos \theta = 2 \sin^2 \theta$ $3 + 3 \cos \theta = 2(1 - \cos^2 \theta)$ $2 \cos^2 \theta + 3 \cos \theta + 1 = 0$ $(2 \cos \theta + 1)(\cos \theta + 1) = 0 \Rightarrow \cos \theta = -0.5, -1$ $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \pi$	M1 M1A1 M1, A1, A1 (6) (9 marks)

Notes for Question 6

(i)

M1 Uses the identity $\operatorname{cosec} 2x = \frac{1}{\sin 2x}$

M1 Uses the correct identity for $\sin 2x = 2 \sin x \cos x$ in their expression.
Accept $\sin 2x = \sin x \cos x + \cos x \sin x$

A1 $\lambda = \frac{1}{2}$ following correct working

(ii)

M1 Replaces $\tan^2 \theta$ by $\pm \sec^2 \theta \pm 1$ to produce an equation in just $\sec \theta$

M1 Award for forming a $3\text{TQ}=0$ in $\sec \theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\sec \theta$

If they replace $\sec \theta = \frac{1}{\cos \theta}$ it is for forming a 3TQ in $\cos \theta$ and applying a correct method for finding two answers to $\cos \theta$

A1 Correct answers to $\sec \theta = -2, -1$ or $\cos \theta = -\frac{1}{2}, -1$

M1 Award for using the identity $\sec \theta = \frac{1}{\cos \theta}$ and proceeding to find at least one value for θ .

If the 3TQ was in cosine then it is for finding at least one value of θ .

A1 Two correct values of θ . All method marks must have been scored.

Accept two of $120^\circ, 180^\circ, 240^\circ$ or two of $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19

A1 All three answers correct. They must be given in terms of π as stated in the question.

Accept $0.6\pi, 1.3\pi, \pi$

Withhold this mark if further values in the range are given. All method marks must have been scored.
Ignore any answers outside the range.

Alt (ii)

M1 Award for replacing $\sec^2 \theta$ with $\frac{1}{\cos^2 \theta}$, $\sec \theta$ with $\frac{1}{\cos \theta}$, $\tan^2 \theta$ with $\frac{\sin^2 \theta}{\cos^2 \theta}$ multiplying through by $\cos^2 \theta$ (seen in at least 2 terms) and replacing $\sin^2 \theta$ with $\pm 1 \pm \cos^2 \theta$ to produce an equation in just $\cos \theta$

M1 Award for forming a $3\text{TQ}=0$ in $\cos \theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\cos \theta$

A1 $\cos \theta = -\frac{1}{2}, -1$

M1 Proceeding to finding at least one value of θ from an equation in $\cos \theta$.

A1 Two correct values of θ . All method marks must have been scored

Accept two of $120^\circ, 180^\circ, 240^\circ$ or two of $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19

A1 All three answers correct. They must be given in terms of π as stated in the question.

Notes for Question 6 Continued

Accept $0.6\pi, 1.3\pi, \pi$

All method marks must have been scored. Withhold this mark if further values in the range are given.

Ignore any answers outside the range

Question Number	Scheme	Marks
7.(a)	$f(x) = 0 \Rightarrow x^2 + 3x + 1 = 0$ $\Rightarrow x = \frac{-3 \pm \sqrt{5}}{2} = \text{awrt } -0.382, -2.618$	M1A1 (2)
(b)	Uses $vu' + uv'$ $f'(x) = e^{x^2}(2x+3) + (x^2+3x+1)e^{x^2} \times 2x$	M1A1A1 (3)
(c)	$e^{x^2}(2x+3) + (x^2+3x+1)e^{x^2} \times 2x = 0$ $\Rightarrow e^{x^2} \{2x^3 + 6x^2 + 4x + 3\} = 0$ $\Rightarrow x(2x^2 + 4) = -3(2x^2 + 1)$ $\Rightarrow x = -\frac{3(2x^2 + 1)}{2(x^2 + 2)}$	M1 M1 A1* (3)
(d)	Sub $x_0 = -2.4$ into $x_{n+1} = -\frac{3(2x_n^2 + 1)}{2(x_n^2 + 2)}$ $x_1 = \text{awrt } -2.420, x_2 = \text{awrt } -2.427, x_3 = \text{awrt } -2.430$	M1A1,A1 (3)
(e)	Sub $x = -2.425$ and -2.435 into $f'(x)$ and start to compare signs $f'(-2.425) = +22.4, f'(-2.435) = -15.02$ Change in sign, hence $f'(x) = 0$ in between. Therefore $\alpha = -2.43$ (2dp)	M1 A1 (2)
(13 marks)		
Alt 7.(c)	$x = -\frac{3(2x^2 + 1)}{2(x^2 + 2)} \Rightarrow 2x(x^2 + 2) = -3(2x^2 + 1) \Rightarrow 2x^3 + 6x^2 + 4x + 3 = 0$ $f'(x) = e^{x^2} \{2x^3 + 6x^2 + 4x + 3\} = 0 \text{ when } 2x^3 + 6x^2 + 4x + 3 = 0$ Hence the minimum point occurs when $x = -\frac{3(2x^2 + 1)}{(2x^2 + 4)}$	M1 M1 A1

Question Number	Scheme	Marks
Alt 1 7(e)	Sub $x = -2.425$ and -2.435 into cubic part of $f'(x) = 2x^3 + 6x^2 + 4x + 3$ and start to compare signs Adapted $f'(-2.425) = +0.06$, $f'(-2.435) = -0.04$ Change in sign, hence $f'(x) = 0$ in between. Therefore $\alpha = -2.43$ (2dp)	M1 A1 (2)
Alt 2 7 (e)	Sub $x = -2.425$, -2.43 and -2.435 into $f(x) = (x^2 + 3x + 1)e^{x^2}$ and start to compare sizes $f(-2.425) = -141.2$, $f(-2.435) = -141.2$, $f(-2.43) = -141.3$ $f(-2.43) < f(-2.425)$, $f(-2.43) < f(-2.435)$. Therefore $\alpha = -2.43$ (2dp)	M1 A1 (2)
Notes for Question 7		
(a)	M1 Solves $x^2 + 3x + 1 = 0$ by completing the square or the formula, producing two 'non integer answers. Do not accept factorisation here. Accept awrt -0.4 and -2.6 for this mark A1 Answers correct. Accept awrt -0.382, -2.618. Accept just the answers for both marks. Don't withhold the marks for incorrect labelling.	
(b)	M1 Applies the product rule $vu' + uv'$ to $(x^2 + 3x + 1)e^{x^2}$. If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, ie. terms are written out $u = \dots, u' = \dots, v = \dots, v' = \dots$ followed by their $vu' + uv'$) only accept answers of the form $\left(\frac{dy}{dx}\right) = f'(x) = e^{x^2}(Ax + B) + (x^2 + 3x + 1)Cxe^{x^2}$ A1 One term of $f'(x) = e^{x^2}(2x + 3) + (x^2 + 3x + 1)e^{x^2} \times 2x$ correct. There is no need to simplify A1 A fully correct (un simplified) answer $f'(x) = e^{x^2}(2x + 3) + (x^2 + 3x + 1)e^{x^2} \times 2x$	
(c)	M1 Sets their $f'(x) = 0$ and either factorises out, or cancels by e^{x^2} to produce a polynomial equation in x M1 Rearranges the cubic polynomial to $Ax^3 + Bx = Cx^2 + D$ and factorises to reach $x(Ax^2 + B) = Cx^2 + D$ or equivalent A1* Correctly proceeds to $x = -\frac{3(2x^2 + 1)}{2(x^2 + 2)}$. This is a given answer	

Notes on Question 7 Continued

(c) Alternative to (c) working backwards

M1 Moves correctly from $x = -\frac{3(2x^2+1)}{2(x^2+2)}$ to $2x^3 + 6x^2 + 4x + 3 = 0$

M1 States or implies that $f'(x) = 0$

A1 Makes a conclusion to tie up the argument

For example, hence the minimum point occurs when $x = -\frac{3(2x^2+1)}{(2x^2+4)}$

(d)

M1 Sub $x_0 = -2.4$ into $x_{n+1} = -\frac{3(2x_n^2+1)}{2(x_n^2+2)}$

This may be implied by awrt -2.42, or $x_{n+1} = -\frac{3(2 \times -2.4^2 + 1)}{2(-2.4^2 + 2)}$

A1 Awrt. $x_1 = -2.420$.

The subscript is not important. Mark as the first value given

A1 awrt $x_2 = -2.427$ awrt $x_3 = -2.430$

The subscripts are not important. Mark as the second and third values given

(e)

Note that continued iteration is not allowed

M1 Sub $x = -2.425$ and -2.435 into $f'(x)$, starts to compare signs and gets at least one correct to 1 sf rounded or truncated.

A1 Both values correct (1sf rounded or truncated), a reason and a minimal conclusion
Acceptable reasons are change in sign, positive and negative and $f'(a) \times f'(b) < 0$

Minimal conclusions are hence $\alpha = -2.43$, hence shown, hence root

Alt 1 using adapted $f'(x)$

(e)

M1 Sub $x = -2.425$ and -2.435 into cubic part of $f'(x)$, starts to compare signs and gets at least one correct to 1 sf rounded or truncated.

A1 Both values correct of adapted $f'(x)$ correct (1sf rounded or truncated), a reason and a minimal conclusion

Acceptable reasons are change in sign, positive and negative and $f'(a) \times f'(b) < 0$

Minimal conclusions are hence $\alpha = -2.43$, hence shown, hence root

Alt 2 using $f(x)$

(e)

M1 Sub $x = -2.425$, -2.43 and -2.435 into $f(x)$, starts to compare sizes and gets at least one correct to 4sf rounded

A1 All three values correct of $f(x)$ correct (4sf rounded), a reason and a minimal conclusion

Acceptable reasons are $f(-2.43) < f(-2.425)$, $f(-2.43) < f(-2.435)$, a sketch

Minimal conclusions are hence $\alpha = -2.43$, hence shown, hence root

Question Number	Scheme	Marks
8.(a)	$t = 0 \Rightarrow P = \frac{8000}{1+7} = 1000$	cao M1A1 (2)
(b)	$t \rightarrow \infty \quad P \rightarrow \frac{8000}{1} = 8000$	B1 (1)
(c)	$t = 3, P = 2500 \Rightarrow 2500 = \frac{8000}{1+7e^{-3k}}$ $e^{-3k} = \frac{2.2}{7} = (0.31..) \quad \text{oe}$ $k = -\frac{1}{3} \ln\left(\frac{2.2}{7}\right) = \text{awrt } 0.386$	B1 M1,A1 M1A1 (5)
(d)	Sub t=10 into $P = \frac{8000}{1+7e^{-0.386t}} \Rightarrow P = 6970$	cao M1A1 (2)
(e)	$\frac{dP}{dt} = -\frac{8000}{(1+7e^{-kt})^2} \times -7ke^{-kt}$ Sub t=10 $\frac{dP}{dt} \Big _{t=10} = 346$	M1,A1 A1 (3)
		(13 marks)

Notes for Question 8

- (a)
- M1 Sets $t=0$, giving $e^{-k \times 0} = 1$. Award if candidate attempts $\frac{8000}{1+7 \times 1}, \frac{8000}{8}$
- A1 Correct answer only 1000. Accept 1000 for both marks as long as no incorrect working is seen.
- (b)
- B1 8000. Accept $P < 8000$. Condone $P \leq 8000$ but not $P > 8000$
- (c)
- B1 Sets both $t = 3$, and $P = 2500 \Rightarrow 2500 = \frac{8000}{1+7e^{-3k}}$
This may be implied by a subsequent correct line.
- M1 Rearranges the equation to make $e^{\pm 3k}$ the subject. They need to multiply by the $1+7e^{-3k}$ term, and proceed to $e^{\pm 3k} = A, A > 0$
- A1 The correct intermediate answer of $e^{-3k} = \frac{2.2}{7}, \frac{11}{35}$ or equivalent. Accept awrt 0.31..
Alternatively accept $e^{3k} = \frac{35}{11}, 3.18..$ or equivalent.
- M1 Proceeds from $e^{\pm 3k} = A, A > 0$ by correctly taking $\ln$'s and then making k the subject of the formula.
Award for $e^{-3k} = A \Rightarrow -3k = \ln(A) \Rightarrow k = \frac{\ln(A)}{-3}$
If e^{3k} was found accept $e^{3k} = C \Rightarrow 3k = \ln C \Rightarrow k = \frac{\ln C}{3}$ As with method 1, $C > 0$
- A1 Awrt $k = 0.386$ 3dp
- (d)
- M1 Substitutes $t=10$ into $P = \frac{8000}{1+7e^{-kt}}$ with their numerical value of k to find P
- A1 ($P =$) 6970 or other exact equivalents like 6.97×10^3
- (e)
- M1 Differentiates using the chain rule to a form $\frac{dP}{dt} = \frac{C}{(1+7e^{-kt})^2} \times e^{-kt}$
Accept an application of the quotient rule to achieve $\frac{(1+7e^{-kt}) \times 0 - C \times -e^{-kt}}{(1+7e^{-kt})^2}$
- A1 A correct un simplified $\frac{dP}{dt} = -\frac{8000}{(1+7e^{-kt})^2} \times -7ke^{-kt}$.
The derivative can be given in terms of k . If a numerical value is used you may follow through on incorrect values.
- A1 Awrt 346. Note that M1 must have been achieved. Just the answer scores 0

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Mark Scheme (Results)

Summer 2013

GCE Core Mathematics 3 (6665/01)

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Summer 2013

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - $\square$ The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
 7. Ignore wrong working or incorrect statements following a correct answer.
 8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x =$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x =$$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

$$\text{Solving } x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0, \text{ leading to } x = \dots$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

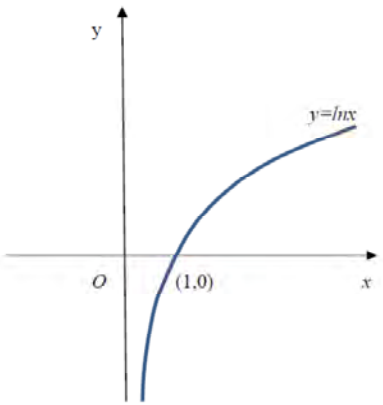
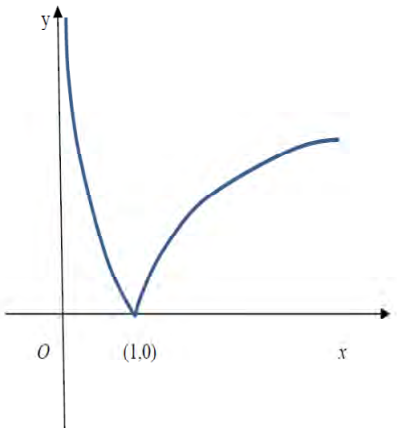
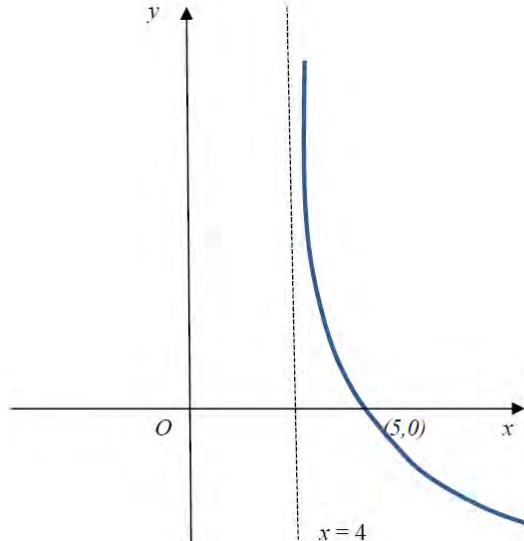
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1	$ \begin{array}{r} 3x^2 - 2x + 7 \\ x^2(+0x) - 4 \overline{) 3x^4 - 2x^3 - 5x^2 + (0x) - 4} \\ \underline{3x^4 + 0x^3 - 12x^2} \\ -2x^3 + 7x^2 + 0x \\ \underline{-2x^3 + 0x^2 + 8x} \\ 7x^2 - 8x - 4 \\ \underline{7x^2 + 0x - 28} \\ -8x + 24 \end{array} $	
By Division	$ \begin{array}{r} 3x^2 - 2x \dots\dots \\ x^2(+0x) - 4 \overline{) 3x^4 - 2x^3 - 5x^2 + (0x) - 4} \\ \underline{3x^4 + 0x^3 - 12x^2} \\ -2x^3 + \dots\dots\dots \\ \underline{-2x^3 + \dots\dots\dots} \end{array} $ Long division as far as	$a = 3$ B1 M1 Two of $b = -2$ $c = 7$ $d = -8$ $e = 24$ A1 All four of $b = -2$ $c = 7$ $d = -8$ $e = 24$ A1 (4 marks)

Notes for Question 1

- B1 Stating $a = 3$. This can also be scored by the coefficient of x^2 in $3x^2 - 2x + 7$
- M1 Using long division by $x^2 - 4$ and getting as far as the 'x' term. The coefficients need not be correct. Award if you see the whole number part as $\dots x^2 + \dots x$ following some working. You may also see this in a table/ grid.
Long division by $(x + 2)$ will not score anything until $(x - 2)$ has been divided into the new quotient. It is very unlikely to score full marks and the mark scheme can be applied.
- A1 Achieving two of $b = -2$ $c = 7$ $d = -8$ $e = 24$.
The answers may be embedded within the division sum and can be implied.
- A1 Achieving all of $b = -2$ $c = 7$ $d = -8$ and $e = 24$
- Accept a correct long division for 3 out of the 4 marks scoring B1M1A1A0
- Need to see $a = \dots$, $b = \dots$, or the values embedded in the rhs for all 4 marks

Question Number	Scheme	Marks
Alt 1 By Multiplication	$* 3x^4 - 2x^3 - 5x^2 - 4 \equiv (ax^2 + bx + c)(x^2 - 4) + dx + e$ Compares the x^4 terms $a = 3$ Compares coefficients to obtain a numerical value of one further constant $-2 = b, \quad -5 = -4a + c \Rightarrow c = ..,$ Two of $b = -2 \quad c = 7 \quad d = -8 \quad e = 24$ All four of $b = -2 \quad c = 7 \quad d = -8 \quad e = 24$	B1 M1 A1 A1 (4 marks)
Notes for Question 2		
B1	Stating $a = 3$. This can also be scored for writing $3x^4 = ax^4$	
M1	Multiply out expression given to get *. Condone slips only on signs of either expression. Then compare the coefficient of any term (other than x^4) to obtain a numerical value of one further constant. In reality this means a valid attempt at either b or c The method may be implied by a correct additional constant to a .	
A1	Achieving two of $b = -2 \quad c = 7 \quad d = -8 \quad e = 24$	
A1	Achieving all of $b = -2 \quad c = 7 \quad d = -8$ and $e = 24$	

Question Number	Scheme	Marks
2(i)	 $y = \ln x$ ln graph crossing x axis at (1,0) and asymptote at $x=0$	B1
2(ii)	 Shape including cusp Touches or crosses the x axis at (1,0) Asymptote given as $x=0$	B1ft B1ft B1
2(iii)	 Shape Crosses at (5, 0) Asymptote given as $x=4$	B1 B1ft B1 (7 marks)

Notes for Question 2

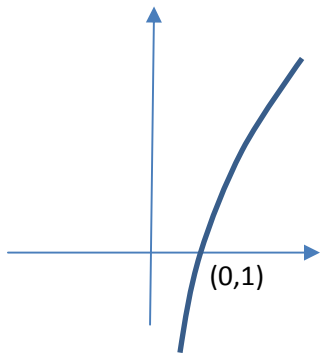
- (i) B1 Correct shape, correct position and passing through (1, 0).
Graph must 'start' to the rhs of the y - axis in quadrant 4 with a gradient that is large. The gradient then decreases as it moves through (1, 0) into quadrant 1. There must not be an obvious maximum point but condone 'slips'. Condone the point marked (0,1) on the correct axis. See practice and qualification for clarification. **Do not withhold this mark if $x=0$ the asymptote is incorrect or not given.**
- (ii) B1ft Correct shape **including the cusp** wholly contained in quadrant 1.
The shape to the rhs of the cusp should have a decreasing gradient and must not have an obvious maximum.. The shape to the lhs of the cusp should not bend backwards past (1,0)
Tolerate a 'linear' looking section here but not one with incorrect curvature (See examples sheet (ii) number 3. For further clarification see practice and qualification items.
Follow through on an incorrect sketch in part (i) as long as it was above and below the x axis.
- B1ft The curve touches or crosses the x axis at (1, 0). Allow for the curve passing through a point marked '1' on the x axis. Condone the point marked on the correct axis as (0, 1)
Follow through on an incorrect intersection in part (i).
- B1 Award for the asymptote to the curve given/ marked as $x = 0$. Do not allow for it given/ marked as 'the y axis'. There must be a graph for this mark to be awarded, and there must be an asymptote on the graph at $x = 0$. Accept if $x=0$ is drawn separately to the y axis.
- (iii)
- B1 Correct shape.
The gradient should always be negative and becoming less steep. It must be approximately infinite at the lh end and not have an obvious minimum. The lh end must not bend 'forwards' to make a C shape. The position is not important for this mark. See practice and qualification for clarification.
- B1ft The graph crosses (or touches) the x axis at (5, 0). Allow for the curve passing through a point marked '5' on the x axis. Condone the point marked on the correct axis as (0, 5)
Follow through on an incorrect intersection in part (i). Allow for $((i) + 4, 0)$
- B1 The asymptote is given/ marked as $x = 4$. There must be a graph for this to be awarded and there must be an asymptote on the graph (in the correct place to the rhs of the y axis).

If the graphs are not labelled as (i), (ii) and (iii) mark them in the order that they are given.

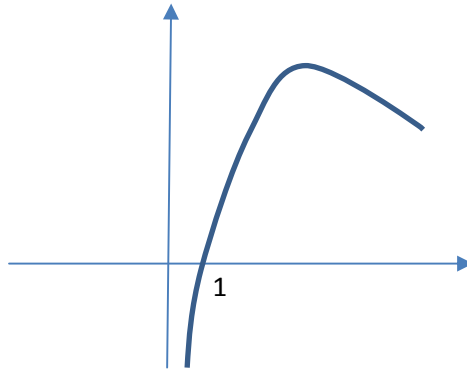
Examples of graphs in number 2

Part (i)

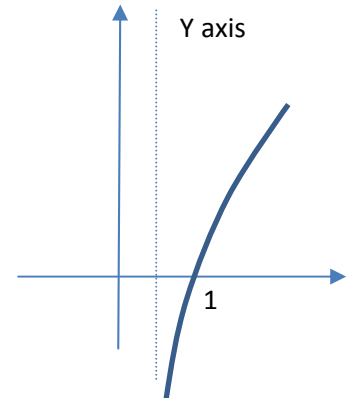
Condoned



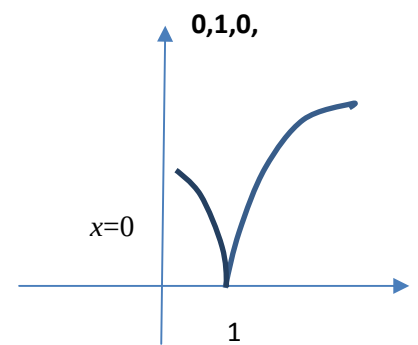
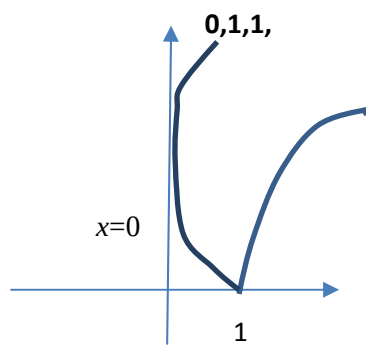
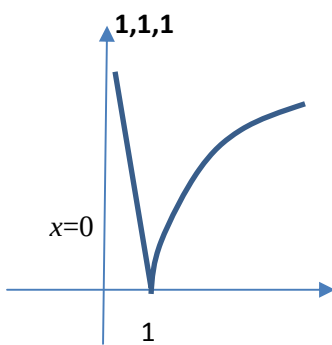
Not condoned



Condone

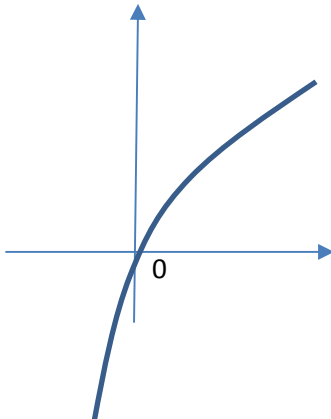


Part (ii)

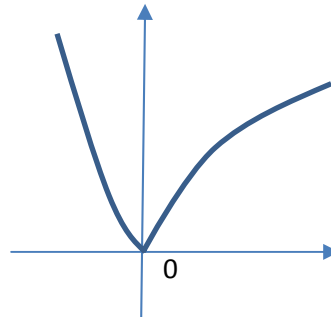


Example of follow through in part (ii) and (iii)

(i) B0

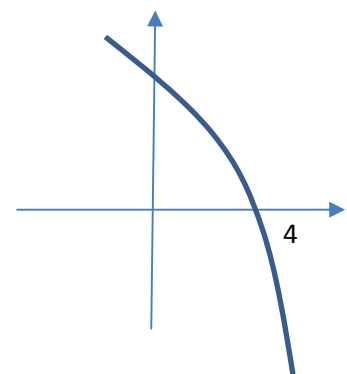


(ii) B1ftB1ftB0



(iii)

B0B1ftB0



Question Number	Scheme	Marks
3(a)	$2 \cos x \cos 50 - 2 \sin x \sin 50 = \sin x \cos 40 + \cos x \sin 40$ $\sin x(\cos 40 + 2 \sin 50) = \cos x(2 \cos 50 - \sin 40)$ $\div \cos x \Rightarrow \tan x(\cos 40 + 2 \sin 50) = 2 \cos 50 - \sin 40$ $\tan x = \frac{2 \cos 50 - \sin 40}{\cos 40 + 2 \sin 50}, \quad (\text{or numerical answer awrt } 0.28)$ States or uses $\cos 50 = \sin 40$ and $\cos 40 = \sin 50$ and so $\tan x^\circ = \frac{1}{3} \tan 40^\circ$ * cao	M1 M1 A1 A1 * (4)
(b)	Deduces $\tan 2\theta = \frac{1}{3} \tan 40$ $2\theta = 15.6 \quad \text{so} \quad \theta = \text{awrt } 7.8(1) \text{ One answer}$ Also $2\theta = 195.6, 375.6, 555.6 \Rightarrow \theta = ..$ $\theta = \text{awrt } 7.8, 97.8, 187.8, 277.8 \quad \text{All 4 answers}$	M1 A1 M1 A1 (4) [8 marks]
Alt 1 3(a)	$2 \cos x \cos 50 - 2 \sin x \sin 50 = \sin x \cos 40 + \cos x \sin 40$ $2 \cos x \sin 40 - 2 \sin x \cos 40 = \sin x \cos 40 + \cos x \sin 40$ $\div \cos x \Rightarrow 2 \sin 40 - 2 \tan x \cos 40 = \tan x \cos 40 + \sin 40$ $\tan x = \frac{\sin 40}{3 \cos 40} \quad (\text{or numerical answer awrt } 0.28), \Rightarrow \tan x = \frac{1}{3} \tan 40$	M1 M1 A1,A1
Alt 2 3(a)	$2 \cos(x + 50) = \sin(x + 40) \Rightarrow 2 \sin(40 - x) = \sin(x + 40)$ $2 \cos x \sin 40 - 2 \sin x \cos 40 = \sin x \cos 40 + \cos x \sin 40$ $\div \cos x \Rightarrow 2 \sin 40 - 2 \tan x \cos 40 = \tan x \cos 40 + \sin 40$ $\tan x = \frac{\sin 40}{3 \cos 40} \quad (\text{or numerical answer awrt } 0.28), \Rightarrow \tan x = \frac{1}{3} \tan 40$	 M1 M1 A1,A1

Notes for Question 3

(a)

M1 Expand both expressions using $\cos(x + 50) = \cos x \cos 50 - \sin x \sin 50$ and $\sin(x + 40) = \sin x \cos 40 + \cos x \sin 40$. Condone a missing bracket on the lhs.
The terms of the expansions must be correct as these are given identities. You may condone a sign error on one of the expressions.
Allow if written separately and not in a connected equation.

M1 Divide by $\cos x$ to reach an equation in $\tan x$.
Below is an example of M1M1 with incorrect sign on left hand side

$$2 \cos x \cos 50 + 2 \sin x \sin 50 = \sin x \cos 40 + \cos x \sin 40$$

$$\Rightarrow 2 \cos 50 + 2 \tan x \sin 50 = \tan x \cos 40 + \sin 40$$
This is independent of the first mark.

A1 $\tan x = \frac{2 \cos 50 - \sin 40}{\cos 40 + 2 \sin 50}$
Accept for this mark $\tan x = \text{awrt } 0.28\dots$ as long as M1M1 has been achieved.

A1* States or uses $\cos 50 = \sin 40$ and $\cos 40 = \sin 50$ leading to showing

$$\tan x = \frac{2 \cos 50 - \sin 40}{\cos 40 + 2 \sin 50} = \frac{\sin 40}{3 \cos 40} = \frac{1}{3} \tan 40$$

This is a given answer and all steps above must be shown. The line above is acceptable.
Do not allow from $\tan x = \text{awrt } 0.28\dots$

(b)

M1 For linking part (a) with (b). Award for writing $\tan 2\theta = \frac{1}{3} \tan 40$

A1 Solves to find one solution of θ which is usually (awrt) 7.8

M1 Uses the correct method to find at least another value of θ . It must be a full method but can be implied by any correct answer.

Accept $\theta = \frac{180 + \text{their } \alpha}{2}, (\text{or}) \frac{360 + \text{their } \alpha}{2}, (\text{or}) \frac{540 + \text{their } \alpha}{2}$

A1 Obtains all four answers awrt 1dp. $\theta = 7.8, 97.8, 187.8, 277.8$.
Ignore any extra solutions outside the range.
Withhold this mark for extras inside the range.
Condone a different variable. Accept $x = 7.8, 97.8, 187.8, 277.8$

Answers fully given in radians, loses the first A mark.

Acceptable answers in rads are awrt 0.136, 1.71, 3.28, 4.85

Mixed units can only score the first M 1

Question Number	Scheme	Marks
4(a)	$f'(x) = 50x^2e^{2x} + 50xe^{2x}$ oe. Puts $f'(x) = 0$ to give $x = -1$ and $x = 0$ or one coordinate Obtains $(0, -16)$ and $(-1, 25e^{-2} - 16)$	M1A1 dM1A1 A1 (5)
(b)	Puts $25x^2e^{2x} - 16 = 0 \Rightarrow x^2 = \frac{16}{25}e^{-2x} \Rightarrow x = \pm \frac{4}{5}e^{-x}$	B1* (1)
(c)	Subs $x_0 = 0.5$ into $x = \frac{4}{5}e^{-x} \Rightarrow x_1 = \text{awrt } 0.485$ $\Rightarrow x_2 = \text{awrt } 0.492, x_3 = \text{awrt } 0.489$	M1A1 A1 (3)
(d)	$\alpha = 0.49$ $f(0.485) = -0.487, f(0.495) = (+)0.485$, sign change and deduction	B1 B1 (2)
(11 marks)		

Notes for Question 4

No marks can be scored in part (a) unless you see differentiation as required by the question.

(a)

M1

Uses $vu' + uv'$. If the rule is quoted it must be correct.

It can be implied by their $u = \dots, v = \dots, u' = \dots, v' = \dots$ followed by their $vu' + uv'$

If the rule is not quoted nor implied only accept answers of the form $Ax^2e^{2x} + Bxe^{2x}$

A1

$f'(x) = 50x^2e^{2x} + 50xe^{2x}$.

Allow unsimplified forms such as $f'(x) = 25x^2 \times 2e^{2x} + 50x \times e^{2x}$

dM1

Sets $f'(x) = 0$, factorises out/ or cancels the e^{2x} leading to at least one solution of x

This is dependent upon the first M1 being scored.

A1

Both $x = -1$ and $x = 0$ or one complete coordinate. Accept $(0, -16)$ and $(-1, 25e^{-2} - 16)$ or $(-1, \text{awrt } -12.6)$

A1

CSO. Obtains both solutions from differentiation. Coordinates can be given in any way.

$x = -1, 0 \quad y = \frac{25}{e^2} - 16, -16$ or linked together by coordinate pairs $(0, -16)$ and $(-1, 25e^{-2} - 16)$ but the 'pairs' must be correct and exact.

Notes for Question 4 Continued

(b)

B1

This is a show that question and all elements must be seen

Candidates must 1) State that $f(x)=0$ or writes $25x^2e^{2x} - 16 = 0$ or $25x^2e^{2x} = 16$

2) Show at least one intermediate (correct) line with either

$$x^2 \text{ or } x \text{ the subject. Eg } x^2 = \frac{16}{25}e^{-2x}, \quad x = \sqrt{\frac{16}{25}e^{-2x}} \text{ oe}$$

$$\text{or square rooting } 25x^2e^{2x} = 16 \Rightarrow 5xe^x = \pm 4$$

$$\text{or factorising by DOTS to give } (5xe^x + 4)(5xe^x - 4) = 0$$

$$3) \text{ Show the given answer } x = \pm \frac{4}{5}e^{-x}.$$

Condone the minus sign just appearing on the final line.

A 'reverse' proof is acceptable as long as there is a statement that $f(x)=0$

(c)

M1

$$\text{Substitutes } x_0 = 0.5 \text{ into } x = \frac{4}{5}e^{-x} \Rightarrow x_1 = \dots$$

$$\text{This can be implied by } x_1 = \frac{4}{5}e^{-0.5}, \text{ or awrt } 0.49$$

A1

 $x_1 = \text{awrt } 0.485 \text{ 3dp. Mark as the first value given. Don't be concerned by the subscript.}$

A1

 $x_2 = \text{awrt } 0.492, x_3 = \text{awrt } 0.489 \text{ 3dp. Mark as the second and third values given.}$

(d)

B1

States $\alpha = 0.49$

B1

Justifies **by****either** calculating correctly $f(0.485)$ and $f(0.495)$ to awrt 1sf or 1dp,

$$f(0.485) = -0.5, f(0.495) = (+)0.5 \text{ rounded}$$

$$f(0.485) = -0.4, f(0.495) = (+)0.4 \text{ truncated}$$

giving a reason – accept change of sign, $>0 <0$ or $f(0.485) \times f(0.495) < 0$ and giving a minimal conclusion. Eg. Accept hence root or $\alpha = 0.49$ A smaller interval containing the root may be used, eg $f(0.49)$ and $f(0.495)$. Root = 0.49007**or by** stating that the iteration is oscillating**or by** calculating by continued iteration to at least the value of $x_4 = \text{awrt } 0.491$ and stating (or seeing each value round to) 0.49

Question Number	Scheme	Marks
5(a)	$\frac{dx}{dy} = 2 \times 3 \sec 3y \sec 3y \tan 3y = (6 \sec^2 3y \tan 3y)$ (oe $\frac{6 \sin 3y}{\cos^3 3y}$)	M1A1 (2)
(b)	Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to obtain $\frac{dy}{dx} = \frac{1}{6 \sec^2 3y \tan 3y}$ $\tan^2 3y = \sec^2 3y - 1 = x - 1$ Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in just x. $\Rightarrow \frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$ CSO	M1 B1 M1 A1* (4)
(c)	$\frac{d^2 y}{dx^2} = \frac{0 - [6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$ $\frac{d^2 y}{dx^2} = \frac{6-9x}{36x^2(x-1)^{\frac{3}{2}}} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$	M1A1 dM1A1 (4)
Alt 1 to 5(a)	$x = (\cos 3y)^{-2} \Rightarrow \frac{dx}{dy} = -2(\cos 3y)^{-3} \times -3 \sin 3y$	M1A1
Alt 2 to 5 (a)	$x = \sec 3y \times \sec 3y \Rightarrow \frac{dx}{dy} = \sec 3y \times 3 \sec 3y \tan 3y + \sec 3y \times 3 \sec 3y \tan 3y$	M1A1
Alt 1 To 5 (c)	$\frac{d^2 y}{dx^2} = \frac{1}{6} [x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$ $= \frac{1}{6} x^{-2}(x-1)^{-\frac{3}{2}} [x(-\frac{1}{2}) + (-1)(x-1)]$ $= \frac{1}{12} x^{-2}(x-1)^{-\frac{3}{2}} [2-3x]$ oe	M1A1 dM1 A1 (4)

(10 marks)

Notes for Question 5

(a)

M1

Uses the chain rule to get $A \sec 3y \sec 3y \tan 3y = (A \sec^2 3y \tan 3y)$.There is no need to get the lhs of the expression. Alternatively could use the chain rule on $(\cos 3y)^{-2} \Rightarrow A(\cos 3y)^{-3} \sin 3y$ or the quotient rule on $\frac{1}{(\cos 3y)^2} \Rightarrow \frac{\pm A \cos 3y \sin 3y}{(\cos 3y)^4}$

A1

 $\frac{dx}{dy} = 2 \times 3 \sec 3y \sec 3y \tan 3y$ or equivalent. There is no need to simplify the rhs but

both sides must be correct.

(b)

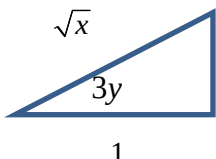
M1

Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to get an expression for $\frac{dy}{dx}$. Follow through on their $\frac{dx}{dy}$

Allow slips on the coefficient but not trig expression.

B1

Writes $\tan^2 3y = \sec^2 3y - 1$ or an equivalent such as $\tan 3y = \sqrt{\sec^2 3y - 1}$ and uses $x = \sec^2 3y$ to obtain either $\tan^2 3y = x - 1$ or $\tan 3y = (x - 1)^{\frac{1}{2}}$ All elements **must be present**.

Accept  $\sqrt{x-1}$ $\cos 3y = \frac{1}{\sqrt{x}} \Rightarrow \tan 3y = \sqrt{x-1}$

If the differential was in terms of $\sin 3y, \cos 3y$ it is awarded for $\sin 3y = \frac{\sqrt{x-1}}{\sqrt{x}}$

M1

Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ or equivalent to get $\frac{dy}{dx}$ in just x . Allow slips on the signs in $\tan^2 3y = \sec^2 3y - 1$.

It may be implied- see below

A1*

CSO. This is a given solution and you must be convinced that all steps are shown.

Note that the two method marks may occur the other way around

$$\text{Eg. } \frac{dx}{dy} = 6 \sec^2 3y \tan 3y = 6x(x-1)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

Scores the 2nd methodScores the 1st method

The above solution will score M1, B0, M1, A0

Notes for Question 5 Continued

Example 1- Scores 0 marks in part (b)

$$\frac{dx}{dy} = 6 \sec^2 3y \tan 3y \Rightarrow \frac{dy}{dx} = \frac{1}{6 \sec^2 3x \tan 3x} = \frac{1}{6 \sec^2 3x \sqrt{\sec^2 3x - 1}} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

Example 2- Scores M1B1M1A0

$$\frac{dx}{dy} = 2 \sec^2 3y \tan 3y \Rightarrow \frac{dy}{dx} = \frac{1}{2 \sec^2 3y \tan 3y} = \frac{1}{2 \sec^2 3y \sqrt{\sec^2 3y - 1}} = \frac{1}{2x(x-1)^{\frac{1}{2}}}$$

(c) Using Quotient and Product Rules

M1 Uses the quotient rule $\frac{vu' - uv'}{v^2}$ with $u = 1$ and $v = 6x(x-1)^{\frac{1}{2}}$ **and** achieving

$$u' = 0 \text{ and } v' = A(x-1)^{\frac{1}{2}} + Bx(x-1)^{-\frac{1}{2}}.$$

If the formulae are quoted, **both** must be correct. If they are not quoted nor implied by their working allow expressions of the form

$$\left(\frac{d^2 y}{dx^2} \right) = \frac{0 - [A(x-1)^{\frac{1}{2}} + Bx(x-1)^{-\frac{1}{2}}]}{\left(6x(x-1)^{\frac{1}{2}} \right)^2} \quad \text{or} \quad \left(\frac{d^2 y}{dx^2} \right) = \frac{0 - A(x-1)^{\frac{1}{2}} \pm Bx(x-1)^{-\frac{1}{2}}}{Cx^2(x-1)}$$

A1 Correct un simplified expression $\frac{d^2 y}{dx^2} = \frac{0 - [6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$ oe

dM1 Multiply numerator and denominator by $(x-1)^{\frac{1}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{1}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{d^2 y}{dx^2} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe

Notes for Question 5 Continued

(c) Using Product and Chain Rules

M1 Writes $\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}} = Ax^{-1}(x-1)^{-\frac{1}{2}}$ and uses the product rule with u or $v = Ax^{-1}$ and

v or $u = (x-1)^{-\frac{1}{2}}$. If any rule is quoted it must be correct.

If the rules are not quoted nor implied then award if you see an expression of the form

$$(x-1)^{-\frac{3}{2}} \times Bx^{-1} \pm C(x-1)^{-\frac{1}{2}} \times x^{-2}$$

A1 ~~$\frac{d^2y}{dx^2} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$~~

dM1 Factorises out / uses a common denominator of $x^{-2}(x-1)^{-\frac{3}{2}}$ producing a linear factor/numerator which must be simplified by collecting like terms. Need a single fraction.

A1 Correct simplified expression $\frac{d^2y}{dx^2} = \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ oe

(c) Using Quotient and Chain rules Rules

M1 Uses the quotient rule $\frac{vu' - uv'}{v^2}$ with $u = (x-1)^{-\frac{1}{2}}$ and $v = 6x$ and achieving

$$u' = A(x-1)^{-\frac{3}{2}} \text{ and } v' = B.$$

If the formulae is quoted, it must be correct. If it is not quoted nor implied by their working allow an expression of the form

~~$$\left(\frac{d^2y}{dx^2}\right) = \frac{Cx(x-1)^{-\frac{3}{2}} - D(x-1)^{-\frac{1}{2}}}{Ex^2}$$~~

A1 Correct un simplified expression ~~$\frac{d^2y}{dx^2} = \frac{6x \times -\frac{1}{2}(x-1)^{-\frac{3}{2}} - (x-1)^{-\frac{1}{2}} \times 6}{(6x)^2}$~~

dM1 Multiply numerator and denominator by $(x-1)^{\frac{3}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{3}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{d^2y}{dx^2} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe $\frac{d^2y}{dx^2} = \frac{(2-3x)x^{-2}(x-1)^{-\frac{3}{2}}}{12}$

Notes for Question 5 Continued

(c) **Using just the chain rule**

M1 Writes $\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}} = \frac{1}{(36x^3 - 36x^2)^{\frac{1}{2}}} = (36x^3 - 36x^2)^{-\frac{1}{2}}$ and proceeds by the chain rule to

$$A(36x^3 - 36x^2)^{-\frac{3}{2}}(Bx^2 - Cx).$$

M1 Would automatically follow under this method if the first M has been scored

Question Number	Scheme	Marks
6(a)	$\ln(4-2x)(9-3x) = \ln(x+1)^2$ $\text{So } 36-30x+6x^2 = x^2+2x+1 \text{ and } 5x^2-32x+35=0$ $\text{Solve } 5x^2-32x+35=0 \text{ to give } x=\frac{7}{5} \text{ oe (Ignore the solution } x=5)$	M1, M1 A1 M1A1 (5)
(b)	$\text{Take log}_e \text{'s to give } \ln 2^x + \ln e^{3x+1} = \ln 10$ $x \ln 2 + (3x+1) \ln e = \ln 10$ $x(\ln 2 + 3 \ln e) = \ln 10 - \ln e \Rightarrow x = ..$ $\text{and uses } \ln e = 1$ $x = \frac{-1 + \ln 10}{3 + \ln 2}$	M1 M1 dM1 M1 A1 (5)
Note that the 4 th M mark may occur on line 2		(10 marks)
Notes for Question 6		
(a)		
M1	Uses addition law on lhs of equation. Accept slips on the signs. If one of the terms is taken over to the rhs it would be for the subtraction law.	
M1	Uses power rule for logs write the $2 \ln(x+1)$ term as $\ln(x+1)^2$. Condone invisible brackets	
A1	Undoes the logs to obtain the 3TQ $=0$. $5x^2-32x+35=0$. Accept equivalences. The equals zero may be implied by a subsequent solution of the equation.	
M1	Solves a quadratic by any allowable method. The quadratic cannot be a version of $(4-2x)(9-3x)=0$ however.	
A1	Deduces $x=1.4$ or equivalent. Accept both $x=1.4$ and $x=5$. Candidates do not have to eliminate $x=5$. You may ignore any other solution as long as it is not in the range $-1 < x < 2$. Extra solutions in the range scores A0.	

Notes for Question 6 Continued

(b)

M1 Takes logs of both sides **and** splits LHS using addition law. If one of the terms is taken to the other side it can be awarded for taking logs of both sides **and** using the subtraction law.

M1 Taking both powers down using power rule. It is not wholly dependent upon the first M1 but logs of both sides must have been taken. Below is an example of M0M1

$$\ln 2^x \times \ln e^{3x+1} = \ln 10 \Rightarrow x \ln 2 + (3x+1) \ln e = \ln 10$$

dM1 This is dependent upon both previous two M's being scored. It can be awarded for a full method to solve their linear equation in x . The terms in x must be collected on one side of the equation and factorised. You may condone slips in signs for this mark but the process must be correct and leading to $x = \dots$

M1 Uses $\ln e = 1$. This could appear in line 2, but it must be part of their equation and not just a statement.

Another example where it could be awarded is $e^{3x+1} = \frac{10}{2^x} \Rightarrow 3x+1 = \dots$

A1 Obtains answer $x = \frac{-1 + \ln 10}{3 + \ln 2} = \left(\frac{\ln 10 - 1}{3 + \ln 2} \right) = \left(\frac{\log_e 10 - 1}{3 + \log_e 2} \right) oe$. **DO NOT ISW HERE**

Note 1: If the candidate takes $\log_{10}$'s of both sides can score M1M1dM1M0A0 for 3 out of 5.

$$\text{Answer} = x = \frac{-\log e + \log 10}{3 \log e + \log 2} = \left(\frac{-\log e + 1}{3 \log e + \log 2} \right)$$

Note 2: If the candidate writes $x = \frac{-1 + \log 10}{3 + \log 2}$ without reference to natural logs then award M4 but with hold the last A1 mark, scoring 4 out of 5.

Question Number	Scheme	Marks
Alt 1 to 6(b)	Writes lhs in e's $2^x e^{3x+1} = 10 \Rightarrow e^{x \ln 2} e^{3x+1} = 10$ $\Rightarrow e^{x \ln 2 + 3x + 1} = 10, \quad x \ln 2 + 3x + 1 = \ln 10$ $x(\ln 2 + 3) = \ln 10 - 1 \Rightarrow x = ..$ $x = \frac{-1 + \ln 10}{3 + \ln 2}$	1st M1 2nd M1, 4th M1 dM1 A1 (5)
Notes for Question 6 Alt 1		
M1	Writes the lhs of the expression in e's. Seeing $2^x = e^{x \ln 2}$ in their equation is sufficient	
M1	Uses the addition law on the lhs to produce a single exponential	
dM1	Takes ln's of both sides to produce and attempt to solve a linear equation in x You may condone slips in signs for this mark but the process must be correct leading to $x = ..$	
M1	Uses $\ln e = 1$. This could appear in line 2	

Question Number	Scheme	Marks
7(a)	$0 \leq f(x) \leq 10$	B1 (1)
(b)	$ff(0) = f(5), = 3$	B1,B1 (2)
(c)	$y = \frac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$ $\Rightarrow 5y - 4 = xy + 3x$ $\Rightarrow 5y - 4 = x(y+3) \Rightarrow x = \frac{5y-4}{y+3}$ $g^{-1}(x) = \frac{5x-4}{3+x}$	M1 dM1 A1 (3)
(d)	$gf(x) = 16 \Rightarrow f(x) = g^{-1}(16) = 4 \quad \text{oe}$ $f(x) = 4 \Rightarrow x = 6$ $f(x) = 4 \Rightarrow 5 - 2.5x = 4 \Rightarrow x = 0.4 \quad \text{oe}$	M1A1 B1 M1A1 (5) (11 marks)
Alt 1 to 7(d)	$gf(x) = 16 \Rightarrow \frac{4+3(ax+b)}{5-(ax+b)} = 16$ $ax+b = x-2 \quad \text{or} \quad 5-2.5x$ $\Rightarrow x = 6$ $\frac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = \dots$ $\Rightarrow x = 0.4 \quad \text{oe}$	M1 A1 B1 M1 A1 (5)

Notes for Question 7

(a)

B1 Correct range. Allow $0 \leq f(x) \leq 10$, $0 \leq f \leq 10$, $0 \leq y \leq 10$, $0 \leq \text{range} \leq 10$, $[0, 10]$ Allow $f(x) \geq 0$ and $f(x) \leq 10$ but not $f(x) \geq 0$ or $f(x) \leq 10$ Do Not Allow $0 \leq x \leq 10$. The inequality must include BOTH ends

(b)

B1 For correct one application of the function at $x=0$ Possible ways to score this mark are $f(0)=5$, $f(5)$ $0 \rightarrow 5 \rightarrow \dots$

B1: 3 ('3' can score both marks as long as no incorrect working is seen.)

(c)

M1 For an attempt to make x or a replaced y the subject of the formula. This can be scored for putting $y = g(x)$, multiplying across, expanding and collecting x terms on one side of the equation. Condone slips on the signsdM1 Take out a common factor of x (or a replaced y) and divide, to make x subject of formula. Only allow **one sign error** for this markA1 Correct answer. No need to state the domain. Allow $g^{-1}(x) = \frac{5x-4}{3+x}$ $y = \frac{5x-4}{3+x}$ Accept alternatives such as $y = \frac{4-5x}{-3-x}$ and $y = \frac{5-\frac{4}{x}}{1+\frac{3}{x}}$

(d)

M1 Stating or implying that $f(x) = g^{-1}(16)$. For example accept $\frac{4+3f(x)}{5-f(x)} = 16 \Rightarrow f(x) = ..$ A1 Stating $f(x) = 4$ or implying that solutions are where $f(x) = 4$ B1 $x = 6$ and may be given if there is no workingM1 Full method to obtain other value from line $y = 5 - 2.5x$ $5 - 2.5x = 4 \Rightarrow x = \dots$ Alternatively this could be done by similar triangles. Look for $\frac{2}{5} = \frac{2-x}{4} (oe) \Rightarrow x = ..$ A1 0.4 or $\frac{2}{5}$ **Alt 1 to (d)**M1 Writes $gf(x) = 16$ with a linear $f(x)$. The order of $gf(x)$ must be correct

Condone invisible brackets. Even accept if there is a modulus sign.

A1 Uses $f(x) = x - 2$ or $f(x) = 5 - 2.5x$ in the equation $gf(x) = 16$ B1 $x = 6$ and may be given if there is no workingM1 Attempt at solving $\frac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = \dots$. The bracketing must be correct and there must be

no more than one error in their calculation

A1 $x = 0.4, \frac{2}{5}$ or equivalent

Question Number	Scheme	Marks
8(a)	$R = \sqrt{(7^2 + 24^2)} = 25$ $\tan \alpha = \frac{24}{7}, \Rightarrow \alpha = \text{awrt } 73.74^\circ$	B1 M1A1 (3)
(b)	maximum value of $24\sin x + 7\cos x = 25$ so $V_{\min} = \frac{21}{25} = (0.84)$	M1A1 (2)
(c)	$\text{Distance } AB = \frac{7}{\sin \theta}, \text{ with } \theta = \alpha$ $\text{So distance} = 7.29\text{m} = \frac{175}{24}\text{m}$	M1, B1 A1 (3)
(d)	$R \cos(\theta - \alpha) = \frac{21}{1.68} \Rightarrow \cos(\theta - \alpha) = 0.5$ $\theta - \alpha = 60 \Rightarrow \theta = .., \theta - \alpha = -60 \Rightarrow \theta = ..$ $\theta = \text{awrt } 133.7, 13.7$	M1, A1 dM1, dM1 A1, A1 (6) (14 marks)

Notes for Question 8

(a)

B1 25. Accept 25.0 but not $\sqrt{625}$ or answers that are not exactly 25. Eg 25.0001M1 For $\tan \alpha = \pm \frac{24}{7}$, $\tan \alpha = \pm \frac{7}{24}$.If the value of R is used only accept $\sin \alpha = \pm \frac{24}{R}$, $\cos \alpha = \pm \frac{7}{R}$

A1 Accept answers which round to 73.74 – must be in degrees for this mark

(b)

M1 Calculates $V = \frac{21}{\text{their 'R'}}$ NOT - RA1 Obtains correct answer. $V = \frac{21}{25}$ Accept 0.84Do not accept if you see incorrect working- ie from $\cos(\theta - \alpha) = -1$ or the minus just disappearing from a previous line.Questions involving differentiation are acceptable. To score M1 the candidate would have to differentiate V by the quotient rule (or similar), set $V'=0$ to find θ and then sub this back into V to find its value.

Notes for Question 8 Continued

(c)

M1 Uses the trig equation $\sin \theta = \frac{7}{AB}$ with a numerical θ to find $AB = \dots$

B1 Uses $\theta =$ their value of α in a trig calculation involving sin. ($\sin \alpha = \frac{AB}{7}$ is condoned)

A1 Obtains answer $\frac{175}{24}$ or awrt 7.29

(d)

M1 Substitutes $V = 1.68$ and their answer to part (a) in $V = \frac{21}{24 \sin \theta + 7 \cos \theta}$ to get an equation of the form $R \cos(\theta \pm \alpha) = \frac{21}{1.68}$ or $1.68R \cos(\theta \pm \alpha) = 21$ or $\cos(\theta \pm \alpha) = \frac{21}{1.68R}$.

Follow through on their R and α

A1 Obtains $\cos(\theta \pm \alpha) = 0.5$ oe. Follow through on their α . It may be implied by later working.

dM1 Obtains one value of θ **in the range** $0 < \theta < 150$ from inverse cos +their α
It is dependent upon the first M being scored.

dM1 Obtains second angle of θ **in the range** $0 < \theta < 150$ from inverse cos +their α
It is dependent upon the first M being scored.

A1 one correct answer awrt $\theta = 133.7$ or 13.7 1dp

A1 both correct answers awrt $\theta = 133.7$ and 13.7 1dp.

Extra solutions in the range loses the last A1.

Answers in radians, lose the first time it occurs. Answers must be to 3dp

For your info $\alpha = 1.287$, $\theta_1 = 2.334$, $\theta_2 = 0.240$

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