RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4737:** Decision Mathematics 2

Mark Scheme for January 2011

Oxford Cambridge and RSA Examinations

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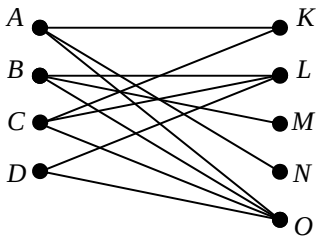
1	**(i)**		B1	Bipartite graph correct	**[1]**
	(ii)	$N = A - K = C - O = D$ Amir sponsors the nightjar Bex sponsors the lark Ceris sponsors the kite Duncan sponsors the owl	B1 B1	This alternating path written down, not just read off from labels on graph This matching written down in words or letters	**[2]**
	(iii)	Amir sponsors the nightjar Bex sponsors the moorhen Ceris sponsors the kite Duncan sponsors the lark	B1	This matching written down in words or symbols	**[1]**
				Total =	**4**

2

	A	B	C	D
Amir	25	15	21	19
Bex	20	25	16	14
Cerys	25	12	25	16
Duncan	24	10	18	25

Reduce rows

A	10	0	6	4
B	6	11	2	0
C	13	0	13	4
D	14	0	8	15

M1 Reduce rows

A1 Correct row reduced matrix (cao) **[2]**

Reduce columns

A	4	0	4	4
B	0	11	0	0
C	7	0	11	4
D	8	0	6	15

M1 Reduce columns

A1 Their correct column reduced matrix (ft) **[2]**

Incomplete matching, cross through zeros

A	4	0	4	4
B	0	11	0	0
C	7	0	11	4
D	8	0	6	15

M1 Cross through zeros using minimum number of lines (may be implied) and augment efficiently

Augment by 4

A	0	0	0	0
B	0	15	0	0
C	3	0	7	0
D	4	0	2	11

A1 Correct augmented matrix (cao) **[2]**

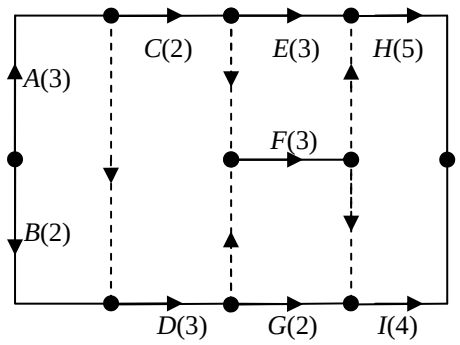
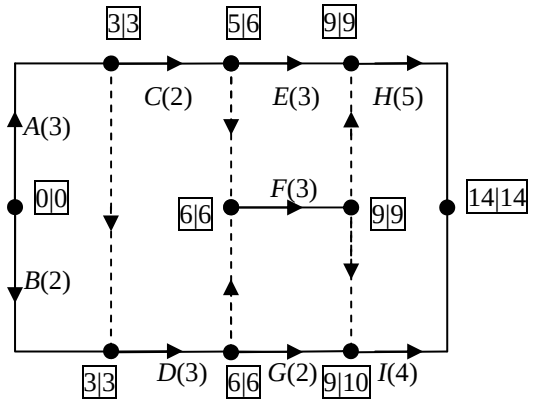
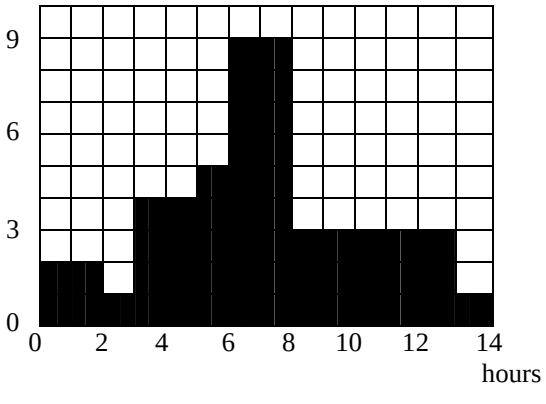
Cannot match A to A
Complete matching

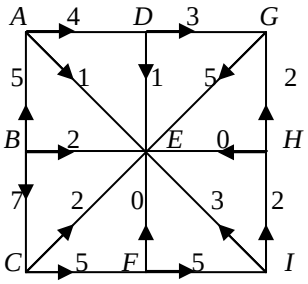
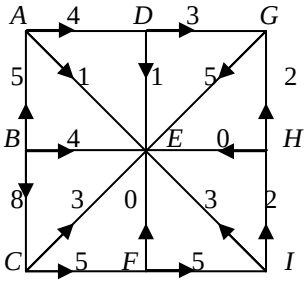
	A	B	C	D
Amir	0	0	0	0
Bex	0	15	0	0
Cerys	3	0	7	0
Duncan	4	0	2	11

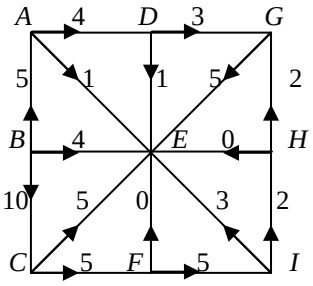
Amir chose Cerys
Bex chose Amir
Cerys chose Duncan
Duncan chose Bex

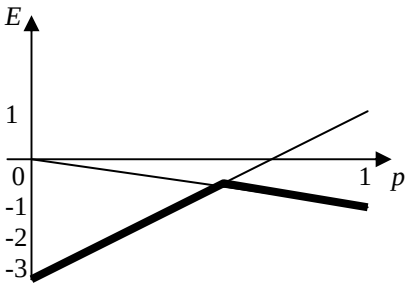
B1 This matching (cao) **[1]**

Total = 7

3	**(i)**		M1 M1d A1	Durations not necessary Correct structure, even without directions shown Activities must be labelled Exactly five directed dummies used correctly Completely correct, with exactly five dummies used and all arcs directed	**[3]**
	(ii)	 Minimum project completion time = 14 hours Critical activities *A*, *D*, *F*, *H*	Follow through their activity network if possible M1 M1 A1ft B1 B1	Substantially correct attempt at forward pass (up to 2 independent errors) Substantially correct attempt at backward pass (up to 2 independent errors) Both passes wholly correct 14 cao *ADFH* cao	**[3]** **[2]**
	(iii)	No. of workers 	Need not be on graph paper M1 A1	Axes scaled appropriately (or implied from lines) and a plausible histogram with no holes or overhangs Axes also labelled and histogram completely correct, cao	**[2]**
	(iv)	Delay *G* by 2 hours, so that it starts after *E* has finished, and delay *I* by 1 hour.	M1 A1	Delay *G* (6 to 8 → 8 to 10) Delay *I* by 1 hour (9 to 13 → 10 to 14) May be shown as a diagram, with activities marked so that shift of *G* and *I* can be seen	**[2]**
					Total = 12

4	**(i)**	*B* is the source (since all flows are out at *B*) *E* is the sink (since all flows are in at *E*)	B1	Both *B* and *E* (assume first answer is source) (reasons not needed)	**[1]**
	(ii)	4+4+4+5+5 = 22 litres per second	M1 A1	Substantially correct, using upper capacities 22	**[2]**
	(iii)	Does not partition source from sink	B1	Source and sink are both in the same set	**[1]**
	(iv)a	3	B1	3	**[1]**
	b	At least 3+1 = 4 must flow out of *D* and 4 is the most that can flow in, so the flow must be 4 At least 1 must flow along $AE \Rightarrow BA = 5$	B1 B1	4 must flow out of vertex *D* *DG* = 3 and *DE* = 1 (at minimum) 5 cao	**[2]**
	c	At least 3+2 = 5 must flow out of *I* so 5 must flow along *FI* and hence at least 5 must flow along *CF* and so at least 2+5 = 7 must flow along *BC* Alternatively may use a cuts argument, eg by considering the min through arcs *CE*, *IE*, *IH*	M1 A1	Substantially correct, starting at *I* and tracing back along *IFCB* 5 must flow along *FI* Wholly correct reasoning *CF* = 5 and *CE* = 2, hence 7 (given)	**[2]**
	(v)	Minimum flow  Maximum flow  	M1 A1 M1 A1	Answered on insert *BA* = 5, *BC* = 7 and *BE* = 2 This flow Assume blank means zero *BA* = 5, *BC* = 8 and *BE* = 4 This flow	**[2]** **[2]**

	(vi)a		B1	Answered on insert Flow out of B = 19 = flow into E Flow in = flow out at A, C, D, F, G, H and I Lower capacity $\leq$ flow $\leq$ upper capacity for every arc	**[1]**
	b	Saturated arcs: AD, BA, BE, CE, CF, DG, FI Cut $\{B, C\}$, $\{A, D, E, F, G, H, I\}$	B1 B1	These arcs, written down (and no others) This cut, represented in any way May be shown on diagram	**[2]**
	(vii)	We have a flow of 19 so max flow $\geq$ 19 We have a cut of 19 so min cut $\leq$ 19 Max flow = min cut Hence 19 is the max flow and the min cut Or, the cut arcs are saturated so no more can flow across the cut	B1 B1	Using or referring to the flow of 19 and cut of 19 that have been found Stating or using 'max flow = min cut' (eg a false cut with a flow of 19 and correct logic given)	**[2]**
				Total =	**18**

5	**(i)**	The number of tokens that the first player gains equals the number that the second player loses. The total number of tokens is unchanged. Collaboration cannot benefit both players No reason to cooperate	B1 B1	Explaining why game is zero-sum Describing a single instance not what happens in the long run Describing what zero-sum means for the way in which the players play the game Not just 'one player can only gaim by making the other lose'	**[2]**
	(ii)	<table><tr><th></th><th>Square</th><th>Triangle</th><th>Circle</th><th>Row min</th></tr><tr><td>Red</td><td>2</td><td>-1</td><td>1</td><td>-1 *</td></tr><tr><td>Yellow</td><td>-2</td><td>0</td><td>-3</td><td>-3</td></tr><tr><td>Blue</td><td>-5</td><td>1</td><td>3</td><td>-5</td></tr><tr><td>Col max</td><td>2</td><td>1 *</td><td>3</td><td></td></tr></table> Col minimax is 1 Row maximin is -1 Play-safe strategy for first player is red Play-safe strategy for second player is triangle Game is unstable since $1 \neq -1$ row maximin ≠ col minimax In a stable game, playing safe is the best strategy for each player in the long run In an unstable game, playing safe cannot be the best strategy for both players	M1 M1 A1 B1 B1 B1	Finding row minima and maximin correctly (numerical values must be seen) Finding col maxima and minimax correctly (or negatives of these), (numerical values must be seen) Finding Red (R) and Triangle (T or Δ), following both method marks gained Unstable and a correct reason (may be explained in words, eg if second chooses triangle then first would do better by choosing blue) Explaining what play-safe strategies mean for a stable game Explaining what play-safe strategies mean for the playing of an unstable game	**[3]** **[3]**
	(iii)	Red: $-2 < -1$ Yellow: $2 < 3$ In each row the entry for square is bigger than the entry for circle, so the second player loses more by choosing square than by choosing circle. The second player should not choose square	B1 B1	Or $1 < 2$ and $-3 < -2$ Showing both comparisons (or equivalent) or in words Circ le dominates square (given) as the pay-off is better (for the second player) in each row Do not choose square	**[2]**
	(iv)	Triangle: $-1(p) + 0(1-p) = -p$ Circle: $1(p) - 3(1-p) = 4p - 3$  $-p = 4p - 3 \Rightarrow p = 0.6$	B1 B1 B1	Both expressions correct (in any form) (may also have square: $2p - 2(1-p) = 4p-2$) Either a correct sketch graph (condone missing scales and/or labels), no ft, except may have $4p-2$ as well or correct reasoning (considering $p=0$, $p=1$ and intersection or using gradients) Calculating intersection on its own is not enough 0.6 cao If circle column was removed in (iii), instead of square then ft for (iv) to $p = 0.4$	**[3]**

	(v)	The new table is |	Square	Triangle	Circle | | Red | 2 | -1 | 1 | | Yellow | -2 | 0 | -3 | | Blue | 5 | -1 | -3 | We add 3 throughout to make all entries non-negative |	Square	Triangle	Circle | | Red | 5 | 2 | 4 | | Yellow | 1 | 3 | 0 | | Blue | 8 | 2 | 0 | When the second player chooses square, the first expects to win $5x + y + 8z$ in this augmented table	M1 A1	Need not draw whole table, could just explain effect on first column. (Values for Blue being multiplied by -1 was given in question) -5 becomes 5, then add 3 to values This table is sufficient for the M mark Square, or first column, explicitly identified as giving the constraint	**[2]**
	(vi)	$5x + y + 8z = 3.4$ $2x + 3y + 2z = 2.4$ $4x = 2.4$ $m \leq 3.4, 2.4, 2.4 \Rightarrow m \leq 2.4$ $M = m - 3 \Rightarrow M \leq -0.6$ Need maximum value of $M \Rightarrow M = -0.6$	M1 A1 B1	At least one of the values 3.4, 2.4, 2.4 correct All three values -0.6 cao	**[3]**
					Total = 18

Answered on insert

6 (i) 10+3+2+3+17 = 35 — B1: 35 — **[1]**

(ii) Visits the kite twice
Does not visit the nightjar at all — B1: Does not visit every bird (in context) — **[1]**

(iii) 18 is the suboptimal min from stage 3, state 4(13)
6 is the time taken to travel from bird 1 to bird 4 (kite to nightjar) — B1: Identifying the 18 with coming from state 4(13)
B1: Identifying the 6 with kite – nightjar in table, or with 1 to 4 or 1(3) to 4(13)
Note: 18 and 6 are given in question — **[2]**

(iv)

2	1(4)	2(14) 3(14)	14+3=17 16+2=18	17
	2(1)	3(12) 4(12)	20+2=22 15+4=19	19
	2(3)	1(23) 4(23)	23+3=26 16+4=20	20
	2(4)	1(24) 3(24)	14+3=17 12+2=14	14
	3(1)	2(13) 4(13)	21+2=23 18+3=21	21
	3(2)	1(23) 4(23)	23+2=25 16+3=19	19
	3(4)	1(34) 2(34)	17+2=19 13+2=15	15
	4(1)	2(14) 3(14)	14+4=18 16+3=19	18
	4(2)	1(24) 3(24)	14+6=20 12+3=15	15
	4(3)	1(34) 2(34)	17+6=23 13+4=17	17
1	1	2(1) 3(1) 4(1)	19+3=22 21+2=23 18+6=24	22
	2	1(2) 3(2) 4(2)	21+3=24 19+2=21 15+4=19	19
	3	1(3) 2(3) 4(3)	24+2=26 20+2=22 17+3=20	20
	4	1(4) 2(4) 3(4)	17+6=23 14+4=18 15+3=18	18
0	0	1 2 3 4	22+10=32 19+14=33 20+12=32 18+17=35	32

M1: Action column correct for stage 2 (at least 14 of the 20 correct)
A1: All suboptimal min values transferred correctly from stage 3
M1: All times transferred correctly from table for stage 2
A1: All suboptimal min column correct for stage 2 — **[4]**

Follow through their suboptimal min values from stage 2 for the method marks

M1: Suboptimal min values transferred correctly from stage 2
M1: Suboptimal min column correct for stage 1 from their stage 2 values
A1: Totally correct table (cao) — **[3]**

Kite – lark – nightjar – moorhen
(or moorhen – nightjar –lark – kite)

B1: cao (names must be used, allow letters but not numbers)

Minimum journey time = 32 minutes

B1: 32 cao — **[2]**

Total = 13

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