



**ADVANCED GCE  
MATHEMATICS**

Further Pure Mathematics 3

**4727**

Candidates answer on the answer booklet.

**OCR supplied materials:**

- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Friday 28 January 2011  
Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

2

- 1 (i) Find the general solution of the differential equation

$$\frac{dy}{dx} + xy = xe^{\frac{1}{2}x^2},$$

giving your answer in the form  $y = f(x)$ . [4]

- (ii) Find the particular solution for which  $y = 1$  when  $x = 0$ . [2]

- 2 Two intersecting lines, lying in a plane  $p$ , have equations

$$\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{-3} \quad \text{and} \quad \frac{x-1}{-1} = \frac{y-3}{2} = \frac{z-4}{4}.$$

- (i) Obtain the equation of  $p$  in the form  $2x - y + z = 3$ . [3]

- (ii) Plane  $q$  has equation  $2x - y + z = 21$ . Find the distance between  $p$  and  $q$ . [3]

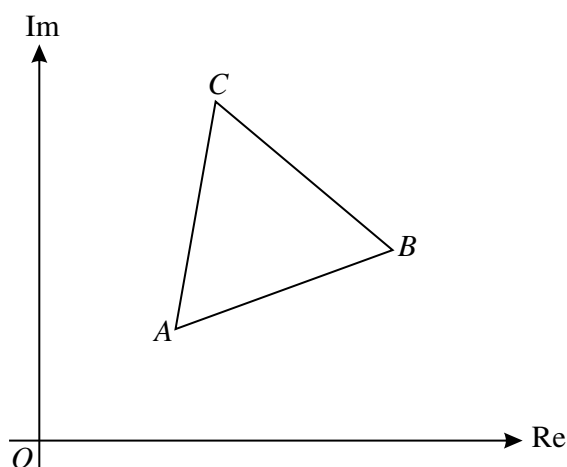
- 3 (i) Express  $\sin \theta$  in terms of  $e^{i\theta}$  and  $e^{-i\theta}$  and show that

$$\sin^4 \theta \equiv \frac{1}{8}(\cos 4\theta - 4 \cos 2\theta + 3). \quad [4]$$

- (ii) Hence find the exact value of  $\int_0^{\frac{1}{6}\pi} \sin^4 \theta \, d\theta$ . [4]

- 4 The cube roots of 1 are denoted by 1,  $\omega$  and  $\omega^2$ , where the imaginary part of  $\omega$  is positive.

- (i) Show that  $1 + \omega + \omega^2 = 0$ . [2]



In the diagram,  $ABC$  is an equilateral triangle, labelled anticlockwise. The points  $A$ ,  $B$  and  $C$  represent the complex numbers  $z_1$ ,  $z_2$  and  $z_3$  respectively.

- (ii) State the geometrical effect of multiplication by  $\omega$  and hence explain why  $z_1 - z_3 = \omega(z_3 - z_2)$ . [4]

- (iii) Hence show that  $z_1 + \omega z_2 + \omega^2 z_3 = 0$ . [2]

## 3

- 5 (i) Find the general solution of the differential equation

$$3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = -2x + 13. \quad [7]$$

- (ii) Find the particular solution for which  $y = -\frac{7}{2}$  and  $\frac{dy}{dx} = 0$  when  $x = 0$ . [5]

- (iii) Write down the function to which  $y$  approximates when  $x$  is large and positive. [1]

- 6  $Q$  is a multiplicative group of order 12.

- (i) Two elements of  $Q$  are  $a$  and  $r$ . It is given that  $r$  has order 6 and that  $a^2 = r^3$ . Find the orders of the elements  $a$ ,  $a^2$ ,  $a^3$  and  $r^2$ . [4]

The table below shows the number of elements of  $Q$  with each possible order.

Order of element	1	2	3	4	6
Number of elements	1	1	2	6	2

$G$  and  $H$  are the non-cyclic groups of order 4 and 6 respectively.

- (ii) Construct two tables, similar to the one above, to show the number of elements with each possible order for the groups  $G$  and  $H$ . Hence explain why there are no non-cyclic proper subgroups of  $Q$ . [5]

- 7 Three planes  $\Pi_1$ ,  $\Pi_2$  and  $\Pi_3$  have equations

$$\mathbf{r} \cdot (\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 5, \quad \mathbf{r} \cdot (\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 6, \quad \mathbf{r} \cdot (\mathbf{i} + 5\mathbf{j} - 12\mathbf{k}) = 12,$$

respectively. Planes  $\Pi_1$  and  $\Pi_2$  intersect in a line  $l$ ; planes  $\Pi_2$  and  $\Pi_3$  intersect in a line  $m$ .

- (i) Show that  $l$  and  $m$  are in the same direction. [5]
- (ii) Write down what you can deduce about the line of intersection of planes  $\Pi_1$  and  $\Pi_3$ . [1]
- (iii) By considering the cartesian equations of  $\Pi_1$ ,  $\Pi_2$  and  $\Pi_3$ , or otherwise, determine whether or not the three planes have a common line of intersection. [4]

[Question 8 is printed overleaf.]

## 4

8 The operation  $*$  is defined on the elements  $(x, y)$ , where  $x, y \in \mathbb{R}$ , by

$$(a, b) * (c, d) = (ac, ad + b).$$

It is given that the identity element is  $(1, 0)$ .

(i) Prove that  $*$  is associative. [3]

(ii) Find all the elements which commute with  $(1, 1)$ . [3]

(iii) It is given that the particular element  $(m, n)$  has an inverse denoted by  $(p, q)$ , where

$$(m, n) * (p, q) = (p, q) * (m, n) = (1, 0).$$

Find  $(p, q)$  in terms of  $m$  and  $n$ . [2]

(iv) Find all self-inverse elements. [3]

(v) Give a reason why the elements  $(x, y)$ , under the operation  $*$ , do not form a group. [1]

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