OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS **4727**

Further Pure Mathematics 3

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet
  (sent with general stationery)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Friday 28 January 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(i)** Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + xy = x\mathrm{e}^{\frac{1}{2}x^2},$$

giving your answer in the form $y = \mathrm{f}(x)$. **[4]**

(ii) Find the particular solution for which $y = 1$ when $x = 0$. **[2]**

2 Two intersecting lines, lying in a plane p, have equations

$$\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{-3} \quad \text{and} \quad \frac{x-1}{-1} = \frac{y-3}{2} = \frac{z-4}{4}.$$

(i) Obtain the equation of p in the form $2x - y + z = 3$. **[3]**

(ii) Plane q has equation $2x - y + z = 21$. Find the distance between p and q. **[3]**

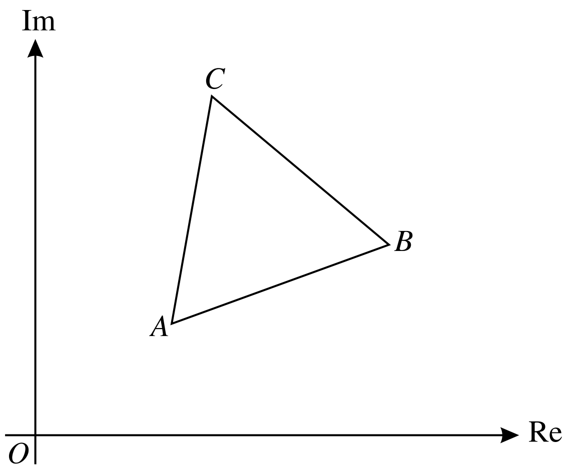
3 **(i)** Express $\sin\theta$ in terms of $\mathrm{e}^{\mathrm{i}\theta}$ and $\mathrm{e}^{-\mathrm{i}\theta}$ and show that

$$\sin^4\theta \equiv \tfrac{1}{8}(\cos 4\theta - 4\cos 2\theta + 3).$$ **[4]**

(ii) Hence find the exact value of $\displaystyle\int_0^{\frac{1}{6}\pi} \sin^4\theta \,\mathrm{d}\theta$. **[4]**

4 The cube roots of 1 are denoted by 1, ω and ω^2, where the imaginary part of ω is positive.

(i) Show that $1 + \omega + \omega^2 = 0$. **[2]**



In the diagram, ABC is an equilateral triangle, labelled anticlockwise. The points A, B and C represent the complex numbers z_1, z_2 and z_3 respectively.

(ii) State the geometrical effect of multiplication by ω and hence explain why $z_1 - z_3 = \omega(z_3 - z_2)$. **[4]**

(iii) Hence show that $z_1 + \omega z_2 + \omega^2 z_3 = 0$. **[2]**

5 **(i)** Find the general solution of the differential equation

$$3\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 5\frac{\mathrm{d}y}{\mathrm{d}x} - 2y = -2x + 13.$$ **[7]**

(ii) Find the particular solution for which $y = -\frac{7}{2}$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = 0$. **[5]**

(iii) Write down the function to which y approximates when x is large and positive. **[1]**

6 Q is a multiplicative group of order 12.

(i) Two elements of Q are a and r. It is given that r has order 6 and that $a^2 = r^3$. Find the orders of the elements a, a^2, a^3 and r^2. **[4]**

The table below shows the number of elements of Q with each possible order.

Order of element	1	2	3	4	6
Number of elements	1	1	2	6	2

G and H are the non-cyclic groups of order 4 and 6 respectively.

(ii) Construct two tables, similar to the one above, to show the number of elements with each possible order for the groups G and H. Hence explain why there are no non-cyclic proper subgroups of Q. **[5]**

7 Three planes Π_1, Π_2 and Π_3 have equations

$$\mathbf{r}.(\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 5, \qquad \mathbf{r}.(\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 6, \qquad \mathbf{r}.(\mathbf{i} + 5\mathbf{j} - 12\mathbf{k}) = 12,$$

respectively. Planes Π_1 and Π_2 intersect in a line l; planes Π_2 and Π_3 intersect in a line m.

(i) Show that l and m are in the same direction. **[5]**

(ii) Write down what you can deduce about the line of intersection of planes Π_1 and Π_3. **[1]**

(iii) By considering the cartesian equations of Π_1, Π_2 and Π_3, or otherwise, determine whether or not the three planes have a common line of intersection. **[4]**

[Question 8 is printed overleaf.]

8 The operation $*$ is defined on the elements (x, y), where $x, y \in \mathbb{R}$, by

$$(a, b) * (c, d) = (ac, ad + b).$$

It is given that the identity element is $(1, 0)$.

(i) Prove that $*$ is associative. **[3]**

(ii) Find all the elements which commute with $(1, 1)$. **[3]**

(iii) It is given that the particular element (m, n) has an inverse denoted by (p, q), where

$$(m, n) * (p, q) = (p, q) * (m, n) = (1, 0).$$

Find (p, q) in terms of m and n. **[2]**

(iv) Find all self-inverse elements. **[3]**

(v) Give a reason why the elements (x, y), under the operation $*$, do not form a group. **[1]**