Mark Scheme 4726
January 2006

4726 FP2 MARK SCHEME January 2006 Final Draft

1(i) Use standard $\ln(1+3x) = 3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3}$ — M1 Allow e.g. $3x^2$, 2! etc.; M1 Attempt to simplify $(3x)^2$ etc.

$= 3x - 9x^2/2 + 9x^3$ — A1 cao

(ii) Produce $(1 + x + x^2/2)$ — B1

M1 Mult. 2 reasonable attempts, each of 3 terms (non-zero)

Get $3x - 3x^2/2 + 6x^3$ — A1√ From their series

SC M1 Reasonable attempt at diff. and replace $x = 0$ (2 correct)
M1√ Put their values into correct Maclaurin expansion
A1 cao
(Applies to either/both parts)

2 Write as $f(x) = \pm(x - e^{-x})$ — B1 Or equivalent

So $f'(x) = \pm(1 + e^{-x})$ — B1 Correct from their $f(x)$

Use $x_{n+1} = x_n - f(x_n)/f'(x_n)$ with $x_0 = 0.5$ — M1 Clear evidence of N-R on their f, f'

Get $x_1 = 0.56631$, $x_2 = 0.56714$ — A1√ At least one to 4d.p.

Get $x_3 = 0.567(1)$ — A1 cao to 3 d.p.

3 Use $A/x + (Bx + C)/(x^2 + 2)$ — B1

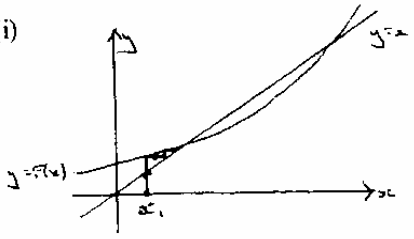
Equate $x+6$ to $A(x^2 + 2) + (Bx+C)x$ (or equiv.) — M1√ Equate to their P.F. (e.g. if B = 0 or C = 0 used)

Use $x = 0$ or equiv. for A (or equate coeff.etc.) — M1√ Include cover-up

Correctly find one of B,C — A1

Get A=3, B=-3,C=1 — A1

4(i)



B1 Line from x_1 to curve

B1 Then to line

B1 Clear explanation; allow use of step/staircase

(ii)(a)Converges to $x=\alpha$ — B1, B1

(b)Diverges (does not give either root) — B1

5 (i) Give $x = -2$ — B1

Attempt to divide out — M1 Giving $y = x+k$; allow $k = 0$ here

Get $y = x + 1$ — A1 Must be =

(ii) Write as quad. $x^2 + x(3 - y) + (3-2y) = 0$ — M1 SC Differentiate M1

Use for real x, $b^2 - 4ac \geq 0$ — M1 Solve $dy/dx=0$ M1

Produce quad. inequality in y — M1 Get 2 x,y values correct A1

Attempt to solve quad. inequality — M1 Attempt at max/min M1

Get A.G. clearly e.g. graph — A1 Justify, e.g. graph, constraints on y A1

6 (i) Use parts to $(-e^{-x}.x^n - \int -e^{-x}.nx^{n-1}\,dx)$	M1 Reasonable attempt e.g. $+e^{-x}$
	A1 cao
Use limits to get e^{-1}	B1 Allow ±
Tidy correctly to A.G.	A1
(ii) Use $I_3 = 3I_2 - e^{-1}$	B1 One such seen
$I_2 = 2I_1 - e^{-1}$	
$I_1 = I_0 - e^{-1}$	
Work out $I_0 = 1 - e^{-1}$ or $I_1 = 1 - 2e^{-1}$	M1,A1
Get $6 - 16e^{-1}$	A1
7 (i) Area under graph $= \int \sqrt{x}\,dx$	B1 Explain RHS (limits need not be specified)
> Sum of areas of rectangles from 1 to $N+1$	B1
Area of each rect. = Width x Height = 1 x $\sqrt{x}$	B1
(ii) Similarly, area under curve from 0 to N	B1
< sum of areas of rect. from 0 to N	B1
Clear explanation of A.G.	B1
(iii) Integrate $x^{0.5}$ and use 2 different sets of limits	M1,M1
Get area between $\frac{2}{3}((N+1)^{1.5}-1)$ and $\frac{2}{3}N^{1.5}$	A1
8 (i) Max. $r = 2$ at $\theta = 0$ and π	B1,B1 Two θ needed (rads only); ignore θ out of range
(ii) Solve $r = 0$ for θ, giving $\theta = \frac{1}{2}\pi$ and $\frac{3}{2}\pi$	M1,A1 Two θ needed (rads only); ignore θ out of range
(iii) Use correct formula with correct r	M1
Expand r	M1
Get $\int A + B\cos 2\theta + C\cos 4\theta\,d\theta$	M1 $C \neq 0$
Integrate their expression correctly	M1√
Get $3\pi/8$	A1 cao
(iv) Express $\cos 2\theta = \cos^2\theta - \sin^2\theta$ or similar	M1
Use $\cos\theta = x/r$ and/or $\sin\theta = y/r$	M1
Simplify to $(x^2 + y^2)^{1.5} = 2x^2$ or similar	M1,A1
9 (i) Correct def^n of cosh x and sinh x	B1,B1
Expand $2.\frac{1}{2}(e^x - e^{-x}).\frac{1}{2}(e^x + e^{-x})$	M1 Reasonable attempt
Clearly get $\frac{1}{2}(e^{2x} - e^{-2x})$ to A.G.	A1
(ii) Attempt to diff. and solve $dy/dx = 0$	M1 Reasonable attempt
Use (ii) to get A cosh x (B sinh x+ C)=0	M1
Clearly see cosh $x > 0$ or similar for one useable factor only	B1
Attempt to solve sinh x = - C/B	M1 Quote or via e^{-x} correctly
Get $x = \ln((3+\sqrt{13})/2)$	A1
Justify one answer only for sinh x = -C/B	B1
Accurate test for MINIMUM	B1 First or second diff^l test with numeric evidence
	B1 Correct value(s) for min.