



General Certificate of Education  
Advanced Subsidiary Examination  
January 2012

## Mathematics

## MFP1

### Unit Further Pure 1

Tuesday 17 January 2012 9.00 am to 10.30 am

**For this paper you must have:**

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

**Time allowed**

- 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The quadratic equation

$$2x^2 + 7x + 8 = 0$$

has roots  $\alpha$  and  $\beta$ .

(a) Write down the values of  $\alpha + \beta$  and  $\alpha\beta$ . (2 marks)

(b) Show that  $\alpha^2 + \beta^2 = \frac{17}{4}$ . (2 marks)

(c) Find a quadratic equation, with integer coefficients, which has roots

$$\frac{1}{\alpha^2} \text{ and } \frac{1}{\beta^2} \quad (5 \text{ marks})$$

2 Show that only one of the following improper integrals has a finite value, and find that value:

(a)  $\int_8^{\infty} x^{-\frac{2}{3}} dx;$

(b)  $\int_8^{\infty} x^{-\frac{4}{3}} dx.$  (5 marks)

3 (a) Solve the following equations, giving each root in the form  $a + bi$ :

(i)  $x^2 + 9 = 0;$  (1 mark)

(ii)  $(x + 2)^2 + 9 = 0.$  (1 mark)

(b) (i) Expand  $(1 + x)^3$ . (1 mark)

(ii) Express  $(1 + 2i)^3$  in the form  $a + bi$ . (3 marks)

(iii) Given that  $z = 1 + 2i$ , find the value of

$$z^* - z^3 \quad (2 \text{ marks})$$



- 4 (a) Use the formulae for  $\sum_{r=1}^n r^2$  and  $\sum_{r=1}^n r^3$  to show that

$$\sum_{r=1}^n r^2(4r - 3) = kn(n + 1)(2n^2 - 1)$$

where  $k$  is a constant.

(5 marks)

- (b) Hence evaluate

$$\sum_{r=20}^{40} r^2(4r - 3)$$

(2 marks)

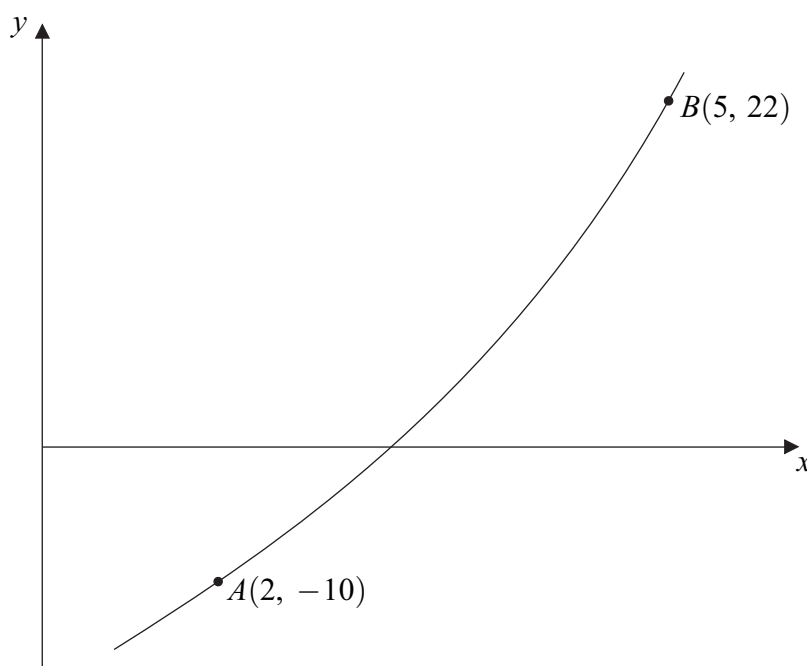
- 5 The diagram below (not to scale) shows a part of a curve  $y = f(x)$  which passes through the points  $A(2, -10)$  and  $B(5, 22)$ .

- (a) (i) On the diagram, draw a line which illustrates the method of linear interpolation for solving the equation  $f(x) = 0$ . The point of intersection of this line with the  $x$ -axis should be labelled  $P$ . (1 mark)

- (ii) Calculate the  $x$ -coordinate of  $P$ . Give your answer to one decimal place. (3 marks)

- (b) (i) On the same diagram, draw a line which illustrates the Newton–Raphson method for solving the equation  $f(x) = 0$ , with initial value  $x_1 = 2$ . The point of intersection of this line with the  $x$ -axis should be labelled  $Q$ . (1 mark)

- (ii) Given that the gradient of the curve at  $A$  is 8, calculate the  $x$ -coordinate of  $Q$ . Give your answer as an exact decimal. (2 marks)



Turn over ►



6 Find the general solution of each of the following equations:

(a)  $\tan\left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}};$  (4 marks)

(b)  $\tan^2\left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{1}{3}.$  (3 marks)

7 A hyperbola  $H$  has equation

$$\frac{x^2}{9} - y^2 = 1$$

(a) Find the equations of the asymptotes of  $H$ . (1 mark)

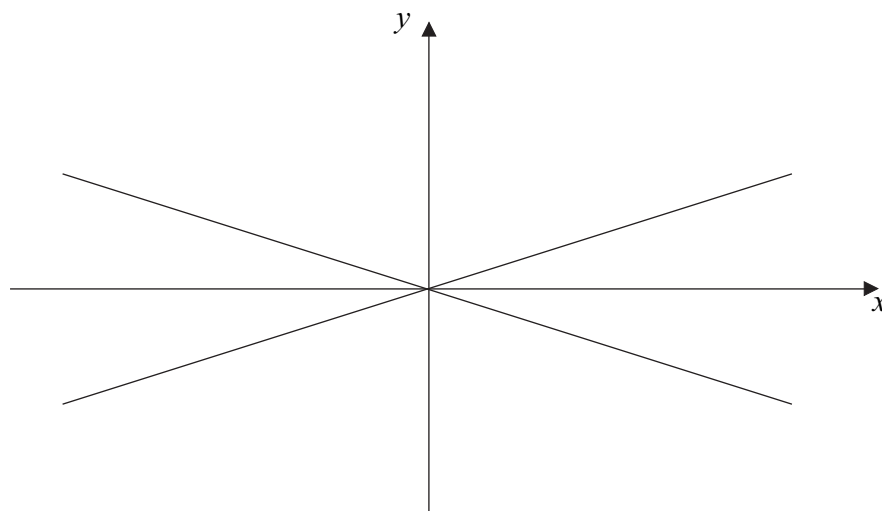
(b) The asymptotes of  $H$  are shown in the diagram opposite. On the same diagram, sketch the hyperbola  $H$ . Indicate on your sketch the coordinates of the points of intersection of  $H$  with the coordinate axes. (3 marks)

(c) The hyperbola  $H$  is now translated by the vector  $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$ .

(i) Write down the equation of the translated curve. (2 marks)

(ii) Calculate the coordinates of the two points of intersection of the translated curve with the line  $y = x$ . (4 marks)

(d) From your answers to part (c)(ii), deduce the coordinates of the points of intersection of the original hyperbola  $H$  with the line  $y = x - 3$ . (2 marks)



**8** The diagram below shows a rectangle  $R_1$  which has vertices  $(0, 0)$ ,  $(3, 0)$ ,  $(3, 2)$  and  $(0, 2)$ .

**(a)** On the diagram, draw:

**(i)** the image  $R_2$  of  $R_1$  under a rotation through  $90^\circ$  clockwise about the origin; *(1 mark)*

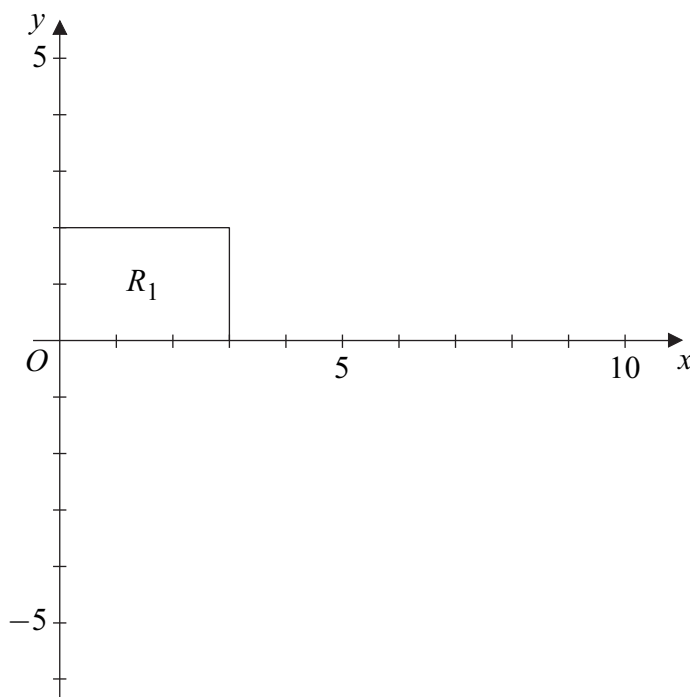
**(ii)** the image  $R_3$  of  $R_2$  under the transformation which has matrix

$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \qquad \qquad \qquad (3 \text{ marks})$$

**(b)** Find the matrix of:

**(i)** the rotation which maps  $R_1$  onto  $R_2$ ; *(1 mark)*

**(ii)** the combined transformation which maps  $R_1$  onto  $R_3$ . *(3 marks)*



**Turn over ►**



9 A curve has equation

$$y = \frac{x}{x-1}$$

(a) Find the equations of the asymptotes of this curve. (2 marks)

(b) Given that the line  $y = -4x + c$  intersects the curve, show that the  $x$ -coordinates of the points of intersection must satisfy the equation

$$4x^2 - (c+3)x + c = 0 \quad (3 \text{ marks})$$

(c) It is given that the line  $y = -4x + c$  is a tangent to the curve.

(i) Find the two possible values of  $c$ .

(No credit will be given for methods involving differentiation.) (3 marks)

(ii) For each of the two values found in part (c)(i), find the coordinates of the point where the line touches the curve. (4 marks)

