

Wednesday 30 January 2013 – Morning

A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

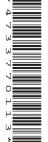
OCR supplied materials:

- Printed Answer Book 4727
- List of Formulae (MF1)

Other materials required:

Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of 16 pages. The Question Paper consists of 4 pages.
 Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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[3]

[5]

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1 Two planes have equations

$$x + 2y + 5z = 12$$
 and $2x - y + 3z = 5$.

- (i) Find the acute angle between the planes.
- (ii) Find a vector equation of the line of intersection of the planes. [4]
- The elements of a group G are the complex numbers a + bi where $a, b \in \{0, 1, 2, 3, 4\}$. These elements are combined under the operation of addition modulo 5.
 - (i) State the identity element and the order of G. [2]
 - (ii) Write down the inverse of 2 + 4i.
 - (iii) Show that every non-zero element of G has order 5. [3]
- 3 Solve the differential equation $x \frac{dy}{dx} 3y = x^4 e^{2x}$ for y in terms of x, given that y = 0 when x = 1. [8]
- 4 The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$
 and $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix}$

respectively.

- (i) Find the shortest distance between the lines. [5]
- (ii) Find a cartesian equation of the plane which contains l_1 and which is parallel to l_2 . [2]
- 5 (i) Solve the equation $z^5 = 1$, giving your answers in polar form. [2]
 - (ii) Hence, by considering the equation $(z+1)^5 = z^5$, show that the roots of

$$5z^4 + 10z^3 + 10z^2 + 5z + 1 = 0$$

can be expressed in the form $\frac{1}{e^{i\theta}-1}$, stating the values of θ .

- 6 The differential equation $\frac{d^2y}{dx^2} + 4y = \sin kx$ is to be solved, where k is a constant.
 - (i) In the case k = 2, by using a particular integral of the form $ax \cos 2x + bx \sin 2x$, find the general solution.
 - (ii) Describe briefly the behaviour of y when $x \to \infty$. [2]
 - (iii) In the case $k \neq 2$, explain whether y would exhibit the same behaviour as in part (ii) when $x \to \infty$. [2]

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- 7 Let $S = e^{i\theta} + e^{2i\theta} + e^{3i\theta} + ... + e^{10i\theta}$.
 - (i) (a) Show that, for $\theta \neq 2n\pi$, where *n* is an integer,

$$S = \frac{e^{\frac{1}{2}i\theta} \left(e^{10i\theta} - 1\right)}{2i\sin\left(\frac{1}{2}\theta\right)}.$$
 [4]

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- (b) State the value of S for $\theta = 2n\pi$, where n is an integer. [1]
- (ii) Hence show that, for $\theta \neq 2n\pi$, where *n* is an integer,

$$\cos\theta + \cos 2\theta + \cos 3\theta + \dots + \cos 10\theta = \frac{\sin\left(\frac{21}{2}\theta\right)}{2\sin\left(\frac{1}{2}\theta\right)} - \frac{1}{2}.$$
 [3]

- (iii) Hence show that $\theta = \frac{1}{11}\pi$ is a root of $\cos \theta + \cos 2\theta + \cos 3\theta + ... + \cos 10\theta = 0$ and find another root in the interval $0 < \theta < \frac{1}{4}\pi$.
- 8 A multiplicative group *H* has the elements $\{e, a, a^2, a^3, w, aw, a^2w, a^3w\}$ where *e* is the identity, elements *a* and *w* have orders 4 and 2 respectively and $wa = a^3w$.

(i) Show that
$$wa^2 = a^2w$$
 and also that $wa^3 = aw$.

- (ii) Hence show that each of aw, a^2w and a^3w has order 2. [4]
- (iii) Find two non-cyclic subgroups of H of order 4, and show that they are not cyclic. [4]

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