RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4730:** Mechanics 3

Mark Scheme for January 2011

Oxford Cambridge and RSA Examinations

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1 i	(-)15cos α = (0 –) 0.5x22 or 15sin β = 0.5x22 Impulse makes angle 42.8° (0.748 rads) with negative x-axis	M1 A1 A1 [3]	M1 for using I = Δ(mv) in 'x' direction or for sketching Δ reflecting **I** = m(**v** – **u**) AEF, but angle must be clear
ii	15sin α = 0.5v or 15cos β = 0.5v or $(0.5v)^2 = 15^2 - 11^2$ Correct explicit expression for v Speed is 20.4 ms^{-1}	M1 A1 A1 [3]	For using I = Δ(mv) in 'y' direction or using sketched Δ

2	½ (m)($v^2 - 6^2$) = -(m)g x 0.5 in (i) or ½ (m)($v^2 – 6^2$) = -(m)g x 1 in (ii) v^2 = 26.2 in (i) and 16.4 in (ii) T = 0.4v^2/0.5 in (i) or T + 0.4g = 0.4v^2/0.5 Tension is 21.0N in (i) (20.96) 9.2N in (ii)	M1 A1 M1 A1 A1 A1 [6]	For using the principle of conservation of energy in (i) or (ii) soi For using Newton's second law with a = v^2/L. M1 for either attempt, A1 for both right

3 i	2.8*V* = 1.4x72 Vertical component at *P* is 36 N	M1 A1 [2]	For taking moments about *Q* for *PQ* or for using symmetry
ii	36 + *N* = 72 + 54 Normal component at *R* is 90 N	M1 A1 [2]	For resolving forces vertically on both rods AG
iii	1.44F = 1.2x90 – 0.8x54 or 72x1.4 + 54x3.6 + 1.44F = 90x4 with not more than 1 error in either case Equation correct and leading to F = 45 For using F = μ R Coefficient is 0.5	M1 A1 A1 M1 A1 [5]	For taking moments about *Q* for *QR* or about *P* for the whole structure (all terms needed)

4 i	$0.4(7x0.6) - 0.3x2.8 = 0.4a + 0.3b$ $0.7(7x0.6 + 2.8) = b - a$ Speed of B is $4ms^{-1}$	 M1 A1 M1 A1 M1 A1 [6]	For using the principle of conservation of momentum For using $e(\Delta u) = \Delta v$ For eliminating a from equations
ii	$a = (-)0.9$ Component perp. to l.o.c. is 5.6 $\tan \alpha = 5.6/0.9$ $\alpha = 80.9°$ Angle turned through is $46.0°$ (0.803^c)	B1 B1 M1 A1 A1ft [5]	 For attempting to find α - the angle between the direction of motion of A after collision and the l.o.c. to the left, or $90° - \alpha$ $126.9° - \alpha$

5 i	$2.45e/0.5 = 0.05g$ $(e = 0.1)$ Distance from O is $0.5 + 0.1 = 0.6$m	 M1 A1 A1 [3]	For using $T = \lambda e/L$ and resolving forces vertically accept use of 0.1 to show both sides equal to 0.49 AG
ii	$mg - T = m\ddot{x}$ $0.05g - 2.45(0.1 + x)/0.5 = 0.05\ddot{x}$ $\ddot{x} = -98x$	M1 A1 A1 [3]	For using Newton's second law with 3 terms AG
iii	$a = 0.075$ $n = 7\sqrt{2}$ oe $x = 0.075\cos(7\sqrt{2}\ t)$ $x(0.2) = -0.0298$ $v = -0.075(7\sqrt{2})\sin(7\sqrt{2}\ t)$ $v(0.2) = -0.681$ ➔ velocity is $0.681ms^{-1}$ upwards	B1 B1 M1 A1 M1 A1ft A1 [7]	 accept 9.90 For using $x = a\cos nt$ oe For differentiating $x = a\cos nt$ and using it ft incorrect a and/or n If from $v^2 = n^2(a^2 - x^2)$ the direction must be clearly established

6 i	$112e/4 = 3.5 \times 9.8 \times \frac{40}{49}$ $V^2 = 2 \times 8 \times (4 + 1)$ $V^2 = 80$ $0.5\sqrt{80} = (0.5 + 3.5)u$ Initial speed of combined particles is $\frac{1}{2}\sqrt{5}\ ms^{-1}$	M1 A1 M1 A1 M1 A1 [6]	For using $mg\sin\theta$ and $\lambda e/L$ For using $s = 4 + e$ and $a = 8$ in $v^2 = 2as$, or by energy For using the principle of conservation of momentum AG
ii	Gain in EE = $(112/(2 \times 4))\{(X + 1)^2 - 1^2\}$ Loss of KE = $\frac{1}{2}(0.5 + 3.5) \times 5/4$ Loss of PE = $(0.5 + 3.5) \times 9.8 \times \frac{40}{49}X$ $14(X^2 + 2X) = 2.5 + 32X$ $28X^2 - 8X - 5 = 0$	M1 A1 B1 B1 M1 A1 [6]	For using EE = $\lambda x^2/2L$ For using the principle of conservation of energy AG
OR	$T - mg\sin\theta = -ma$ $\frac{112(x+1)}{4} - 4g\frac{40}{49} = -4a$ $\int (7x - 1)dx = -\int v dv\ (+c)$ $\frac{7x^2}{2} - x = -\frac{v^2}{2} + c$ $c = \frac{5}{8}$ $28X^2 - 8X - 5 = 0$	M1 A1 M1 A1 A1 A1 [6]	For use of $F = ma$ allow one sign slip for A1 Using $a = v\frac{dv}{dx}$ and integrating AG Convincingly

7 i	$0.2g - v^2/2000 = 0.2v(\mathrm{d}v/\mathrm{d}x)$ $(\frac{400v}{3920 - v^2})\frac{dv}{dx} = 1.$	 M1 A1 [2]	For using Newton's second law with $a = v(\mathrm{d}v/\mathrm{d}x)$ AG Convincing, with no slips.
ii	$-200\ln(3920 - v^2) = x + (A)$ $-200\ln(3920) = A$ $x = 200\ln(\frac{3920}{3920 - v^2})$ $e^{x/200} = 3920/(3920 - v^2)$ $v^2 = 3920(1 - e^{-x/200})$ $0 < e^{-x/200} \rightarrow v^2 < 3920$	M1 A1 M1 A1 M1 A1 B1 [7]	For separating variables and integrating For using v(0) = 0 For using inverse ln process AG Convincingly – dep on correct answer
iii	Using $0.2g - v^2/2000 = 0.2a$ $v = 40$ Gain in KE = ½ 0.2x1600 (=160J) $x = 200\ln(\frac{3920}{3920 - 1600})$ (= 104.90) 0.2g x (104.9) – 160 Work done is 45.6 J	M1 A1 B1ft B1ft M1 A1 [6]	 For using WD = loss of PE – gain in KE
OR	Using $0.2g - v^2/2000 = 0.2a$ $v = 40$ $x = 200\ln(\frac{3920}{3920 - 1600})$ (= 104.90...) WD = $\int \frac{v^2}{2000}dx + c$ = $\int \frac{3920}{2000}(1 - e^{-x/200})\,\mathrm{d}x$ = $3920/2000(x + 200e^{(-x/200)} - 392$ Work done is 45.6 J	M1 A1 B1ft M1 A1 A1 [6]	 Use of WD = $\int F\mathrm{d}x$ and subst for v^2

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